Surface Symmetry Energy

P. Danielewicz, MSU-NSCL

- Weizsäcker Binding-Energy Formula
- Modified Energy Formula
- Skin Size vs Asymmetry & Separation-Energy Difference
- Asymmetry Oscillations
- Microscopic Background
- Conclusions

Weizsäcker Formula

Nuclear energy: $\propto A$

$$E = -a_V\,A + a_S\,A^{2/3} + a_C\,\frac{Z^2}{A^{1/3}} + a_A\,\frac{(N-Z)^2}{A}$$

No surface symmetry energy...

Surface energy: $E_S = a_S\,A^{2/3} = \dfrac{a_S}{4\pi\,r_0^2}\,4\pi\,r_0^2\,A^{2/3} = \dfrac{a_S}{4\pi\,r_0^2}\,S$

$$\frac{E_S}{S} = \sigma = \frac{a_S}{4\pi\,r_0^2} \quad \text{(tension – work per area)}$$

$\rightarrow$ Because nucleons at the surface less bound, creating surface requires work.

Symmetry energy reduces the binding, so, as n-p asymmetry increases, the work to create surface should drop

$$\sigma = \frac{\partial\,E_S}{\partial\,S} \quad \searrow$$

(in the general definition of tension)

σ as microscopic should depend on a microscopic quantity characterizing neutron-proton (n-p) asymmetry $\rightarrow \mu_a$

$$\mu_a = \frac{\partial E}{\partial (N - Z)}$$

Since tension should drop no matter whether more neutrons or protons $\rightarrow$ quadratic in chemical potential

$$\sigma = \sigma_0 - \gamma \, \mu_a^2$$

Surface energy E_S must then also depend on $\mu_a \ldots$

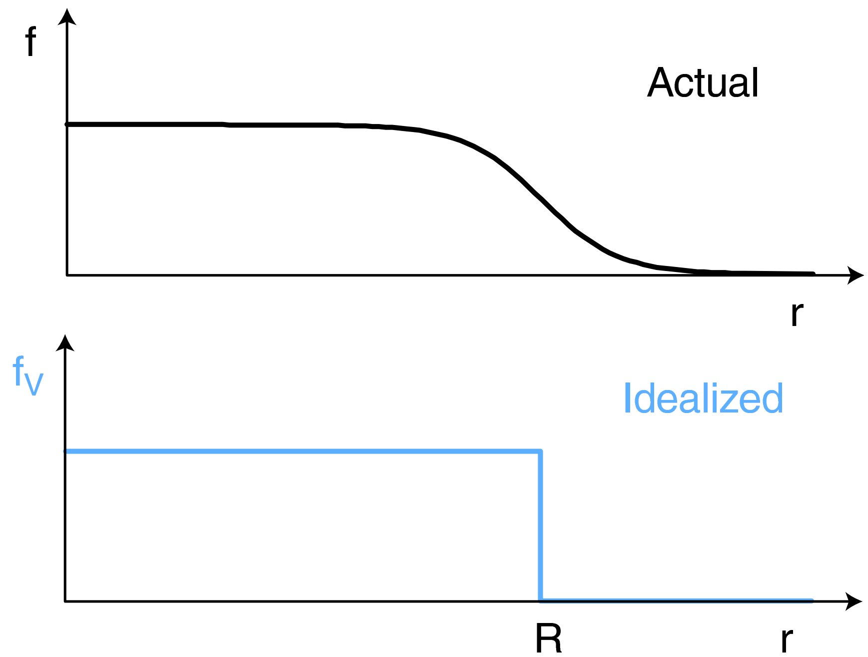
Thermodynamic consistency then requires:
Surface must contain n-p excess!

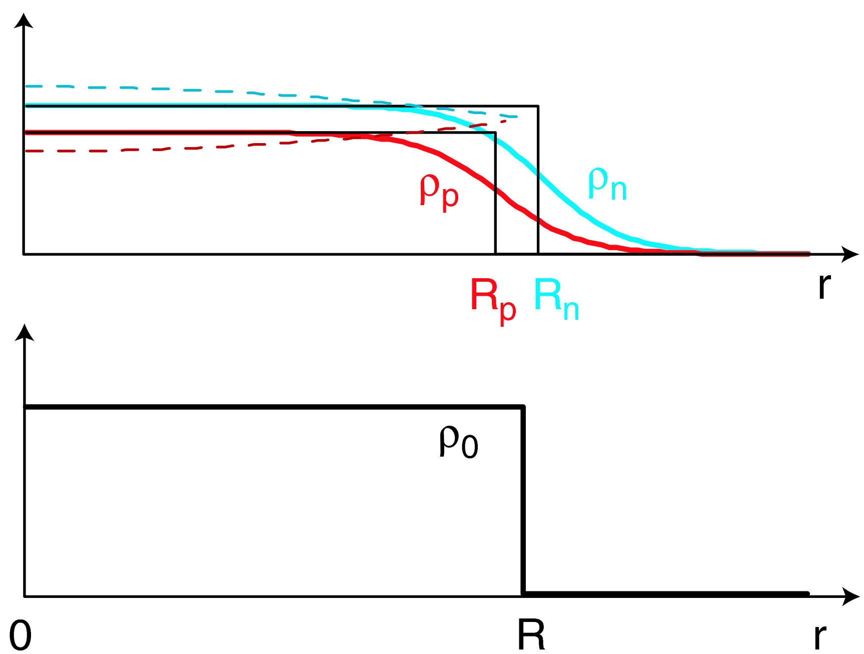
$$(N_S - Z_S) \propto \mu_a$$

Surface energy must be quadratic in the excess and/or μ_a.
?How can surface hold particles?!

Gibbs definition for surface quantities - difference between actual and idealized where volume contribution only: $F_S = F - F_V$

result depends on surface position R
$\rightarrow A_S = A - A_V = 0$





2-component system: surfaces for neutrons and protons may be displaced.

Net surface position set demanding: $A_S = 0$.

However, $N_S - Z_S \neq 0$!

With thermodynamic consistency resolved, $\sigma = \sigma_0 - \gamma\, \mu_a^2$ yields for surface energy

$$E_S = \sigma_0\, \mathcal{S} + \gamma\, \mu_a^2\, \mathcal{S} = E_S^0 + \frac{1}{4\gamma}\, \frac{(N_S - Z_S)^2}{\mathcal{S}} = E_S^0 + \beta\, \frac{(N_S - Z_S)^2}{A^{2/3}}$$

Volume: $$E_V = E_V^0 + \alpha\, \frac{(N_V - Z_V)^2}{A} \quad \text{(mass formula)}$$

Net Energy & Asymmetry: $E = E_S + E_V\,,\; N - Z = N_S - Z_S + N_V - Z_V$

Capacitor analogy: $q_X = N_X - Z_X, \quad E_X = E_X^0 + \frac{q_X^2}{2C_X}$

$$C_S = \frac{A^{2/3}}{2\beta}, \quad C_V = \frac{A}{2\alpha}$$

Minimal energy under the surface-volume asymmetry partition – energy of capacitors in parallel:

$$E = E^0 + \frac{q^2}{2C} = E^0 + \frac{(N - Z)^2}{\frac{A}{\alpha} + \frac{A^{2/3}}{\beta}}$$

The partition: $$\frac{q_S}{q_V} = \frac{C_S}{C_V} \quad \Leftrightarrow \quad \frac{N_S - Z_S}{N_V - Z_V} = \frac{\alpha}{\beta}\, A^{-1/3}$$

Modified Energy Formula

$$E = -a_V\, A + a_S\, A^{2/3} + a_C\, \frac{Z^2}{A^{1/3}} + \alpha\, \frac{(N-Z)^2}{A}\, \frac{1}{1+\frac{\alpha}{\beta}\, A^{-1/3}}$$

Regular formula: $\frac{\alpha}{\beta} = 0$ – surface not accepting excess ($\beta = \infty$)

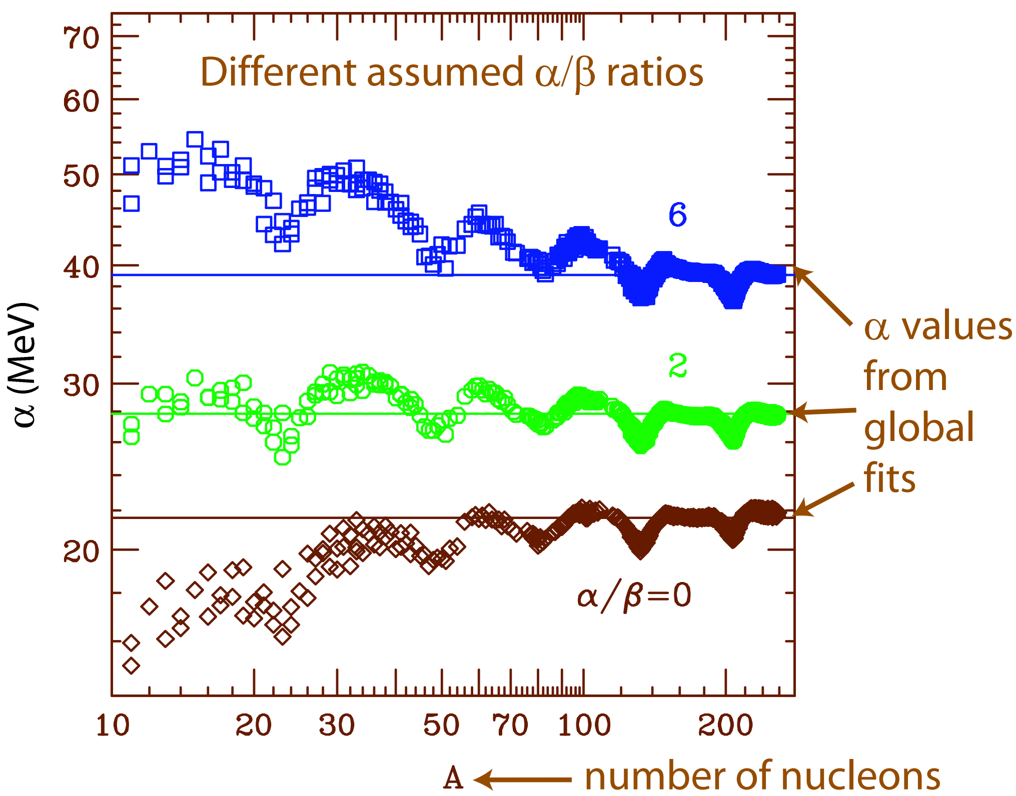
$\alpha \equiv a_a$ for $\frac{\alpha}{\beta} = 0$ or $A \to \infty$

Any need for modification?!

Test: After a global fit, invert the formula using measured E for individual nuclei to get $\alpha\,(a_a)$ locally.

α from a local inversion should represent, on the average the α from a global fit.

$\alpha/\beta \sim 2$ best



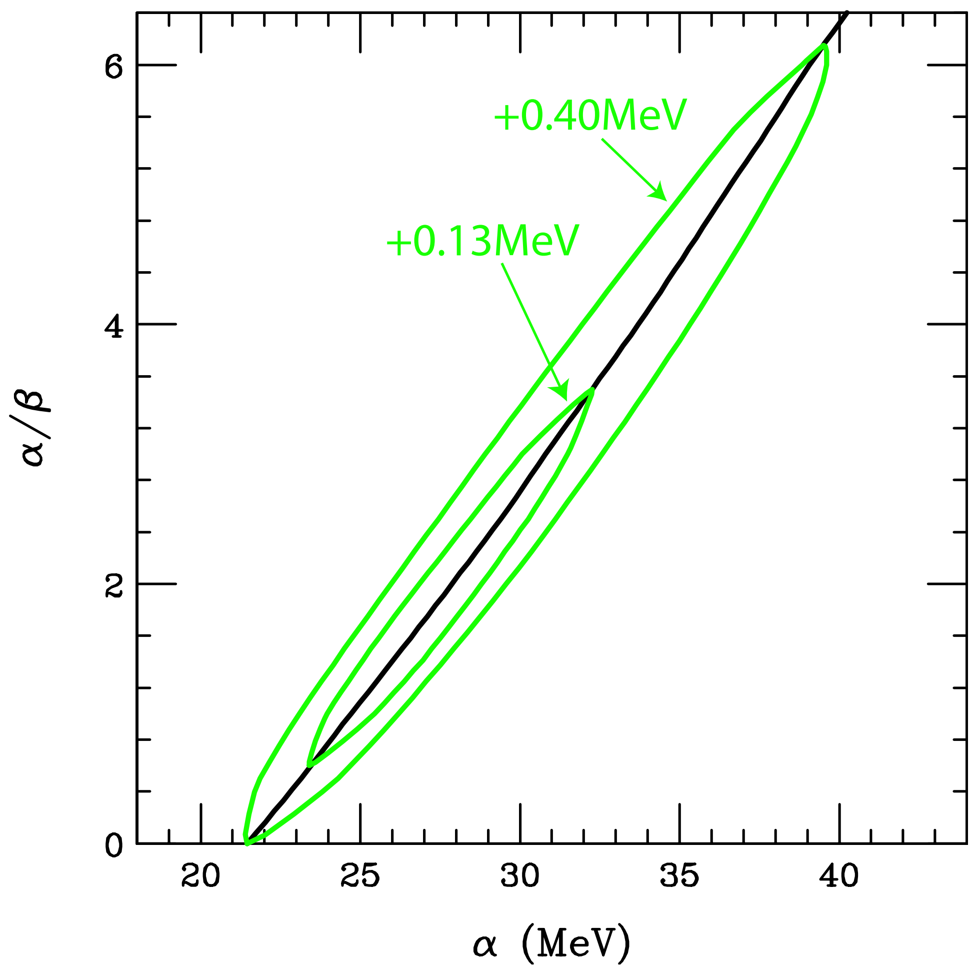
Best-fit parameters in the α/β-α (vol/sur-vol) plane.

Strong correlation valley.

$$a_a = 21\,\mathrm{MeV} \rightarrow \frac{\alpha}{1+\frac{\alpha}{\beta}\,A^{-1/3}}$$

Largest number of asymmetric nuclei at $A \sim 200$. Valley:

$$21\,\mathrm{MeV} = \frac{\alpha}{1+\frac{\alpha}{\beta}\,200^{-1/3}}$$



$\Delta(E_{th} - E_{exp})$ for light asymmetric ($|N-Z|/A > 0.2$) nuclei:
7.4 MeV for $\alpha/\beta = 0$
2.3 MeV for $\alpha/\beta \sim 2$ (similar to that for all nuclei ~ 2 MeV)

Asymmetry Skins

The energy formula predicts different neutron and proton radii.

For heavy nuclei a correction due to Coulomb forces that push protons out

$$E = E_0 + E_V + E_S + E_C \qquad E_C = \frac{e^2}{4\pi\epsilon_0} \frac{1}{R} \left(\frac{3}{5} Z_V^2 + Z_V Z_S + \frac{1}{2} Z_S^2 \right)$$

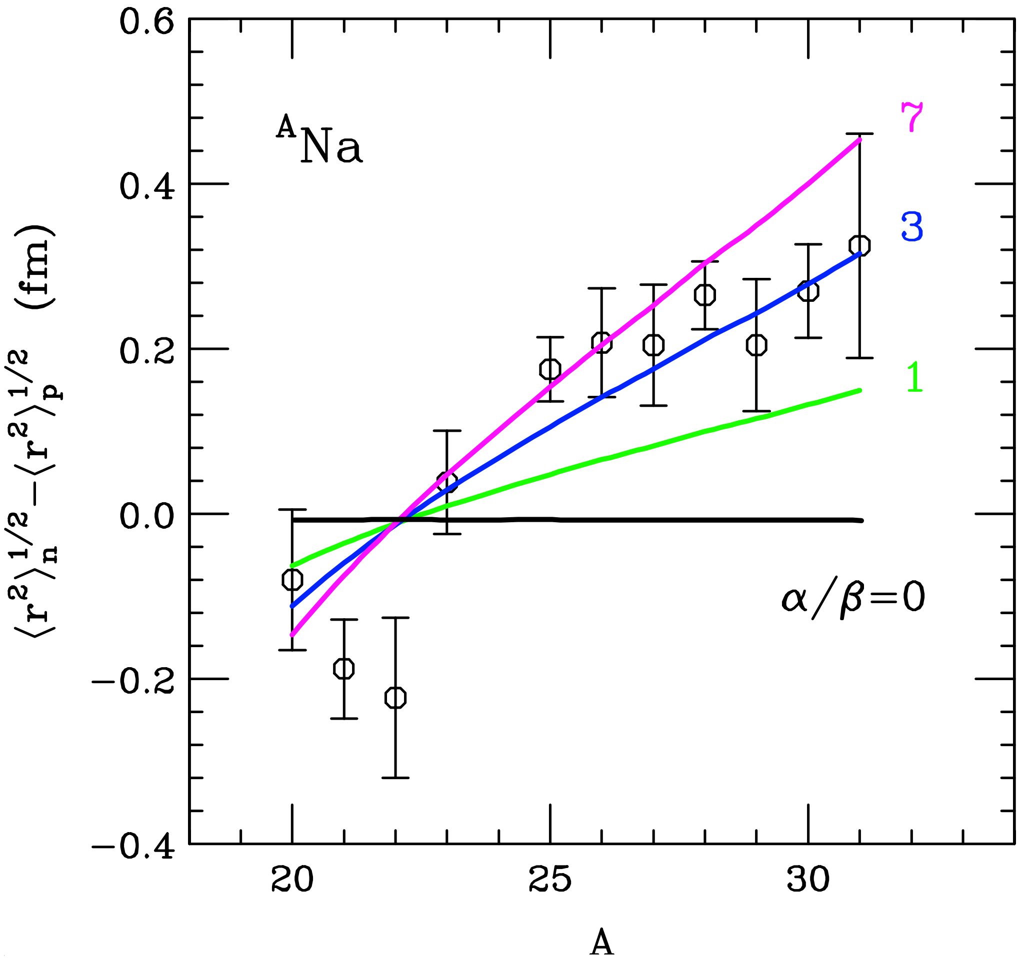
From the modified minimalization, analytic difference of rms radii:

$$\frac{\langle r^2 \rangle_n^{1/2} - \langle r^2 \rangle_p^{1/2}}{\langle r^2 \rangle^{1/2}} = \frac{A}{6NZ} \frac{N-Z}{1 + \frac{\beta}{\alpha} A^{1/3}} - \frac{a_C}{168\alpha} \frac{A^{5/3}}{N} \frac{\frac{10}{3} + \frac{\beta}{\alpha} A^{1/3}}{1 + \frac{\beta}{\alpha} A^{1/3}}$$

The Coulomb correction (2^{nd} term) favors larger proton radii...

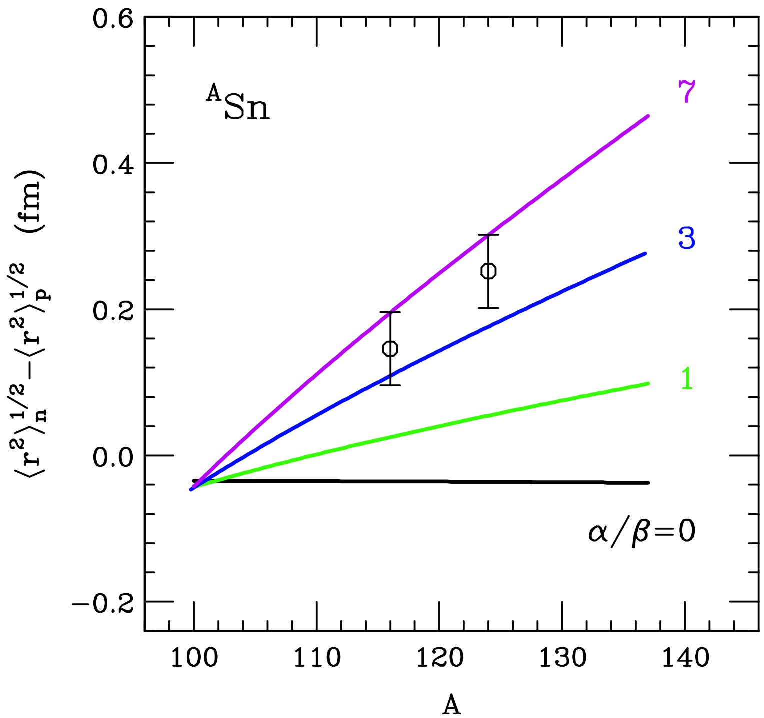
Measurements of n-p skin sizes difficult: two different probes required.
E.g. electrons + protons, $\pi^+ + \pi^-$, protons + neutrons

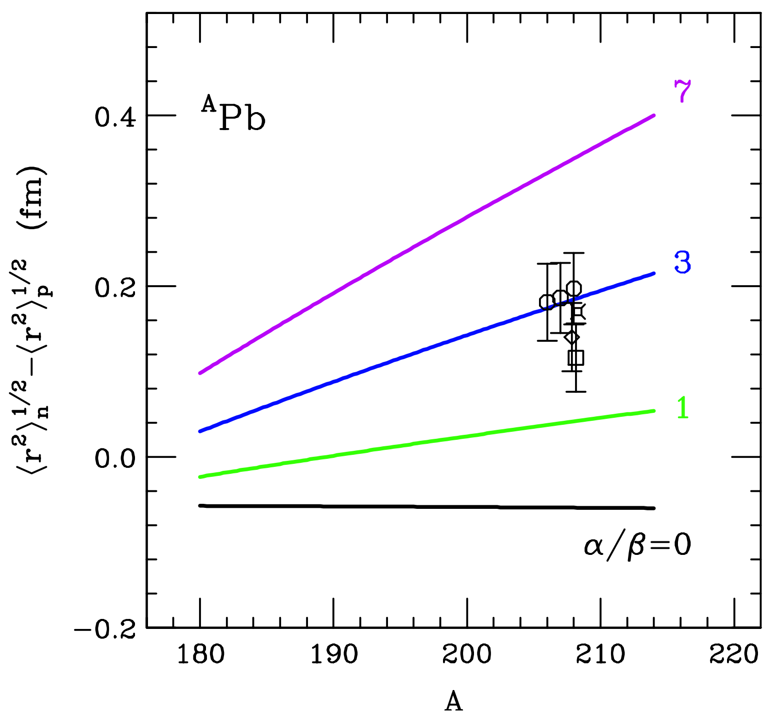
Comparison of measured n-p skin sizes (Suzuki *et al.* '95 - symbols) to the formula (lines), for different Na isotopes



difference between the rms n and p radii vs A

More comparisons to data:



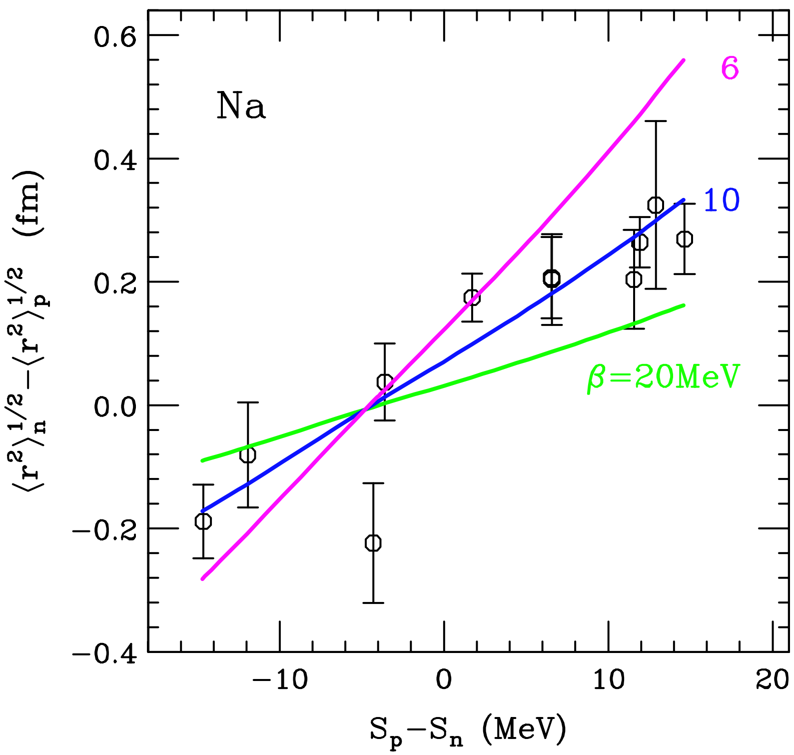


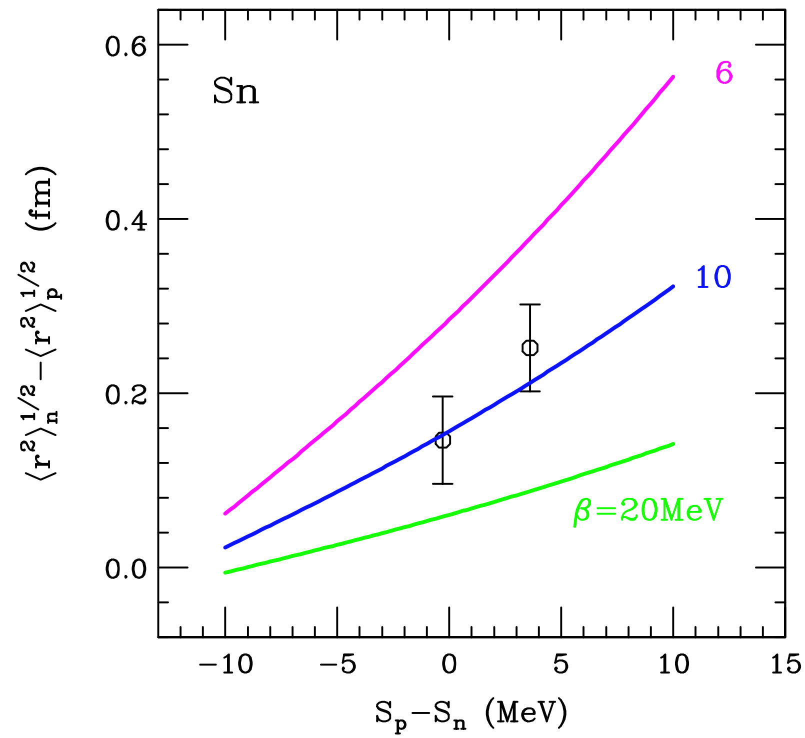
Skin size vs charge and mass numbers tests the symmetry parameter ratio α/β

$$\frac{q_S}{q} = \frac{C_S}{C} \quad \Leftrightarrow \quad \frac{N_S - Z_S}{N - Z} = \frac{1}{1 + \frac{\beta}{\alpha} A^{1/3}}$$

Skin size vs proton-neutron separation energy difference (chem pot conjugate to $N - Z$) measures surface symmetry parameter β:

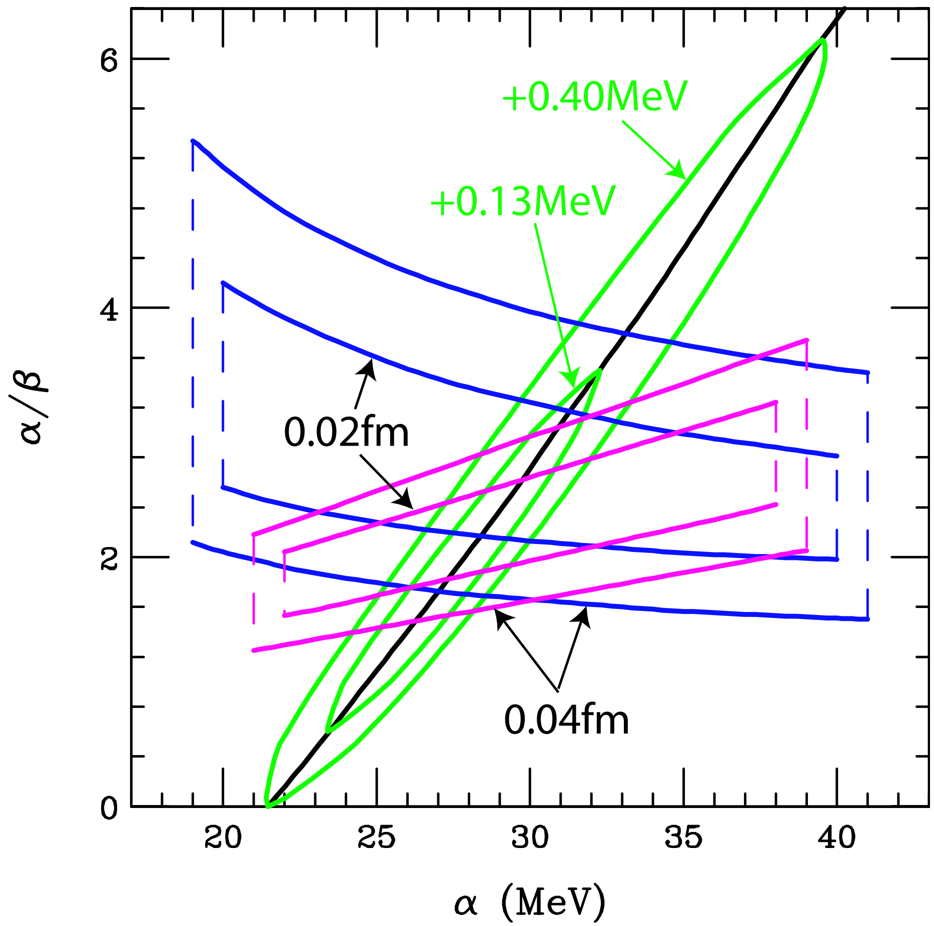
$$q_S = C_S\, V \quad \Leftrightarrow \quad N_S - Z_S = \frac{A^{2/3}}{2\beta} \times \frac{1}{2}(S_p - S_n)$$





difference between the rms n and p radii vs difference between p and n separation energies plane of α/β (vol/sur) vs α (vol)

Results from global fits to skin dependencies + from fit to masses



Conclusions:

$$27\,\mathrm{MeV} \lesssim \alpha \lesssim 31\,\mathrm{MeV}$$

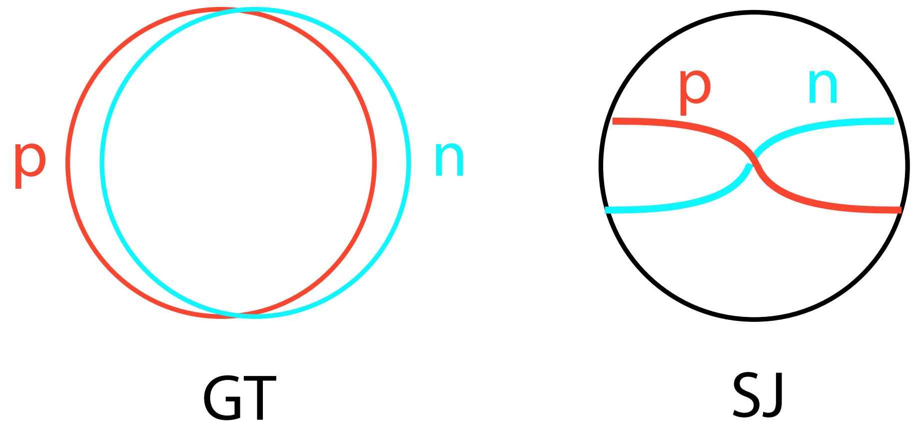
$$2.0 \lesssim \alpha/\beta \lesssim 2.8$$

$$11\,\mathrm{MeV} \lesssim \beta \lesssim 14\,\mathrm{MeV}$$

Asymmetry Oscillations

Movement of neutrons vs protons - giant resonances visible in excitation cross sections

Two classical models of the simplest giant dipole resonance (GDR)



Goldhaber-Teller (GT): n & p distributions oscillate against each other as rigid entities:

$$E_{GDR} = \hbar\Omega \propto \sqrt{A^{2/3}/A} = A^{-1/6}$$

Steinwedel-Jensen (SJ): Standing wave of n-p in the interior with vanishing flux at the surface

$$E_{GDR} = \hbar c_a/\lambda \propto A^{-1/3}$$

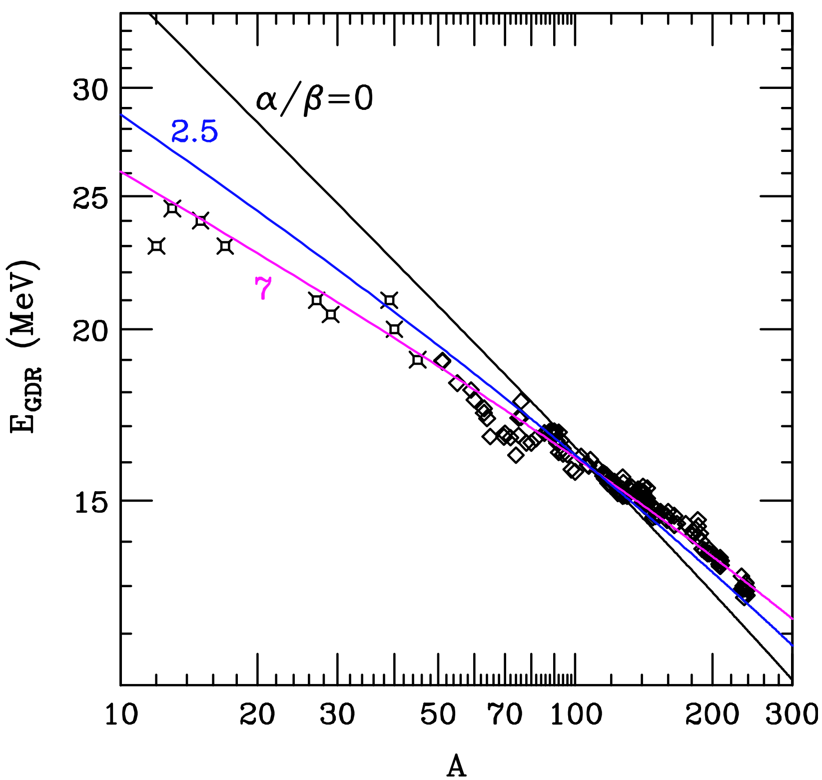
GT model: $\alpha \to \infty$ SJ model: $\beta \to \infty$

Realistic model: SJ but asymmetry flux may flow in and out of the surface... The boundary condition produces:

$$qR\, j_1(qR) = \frac{3\beta\, A^{1/3}}{\alpha}\, j_1'(qR)$$

j_1 - spherical Bessel function, typical for waves when spherical symmetry; q - wavenumber, $E_{GDR} = \hbar\, c_a\, q$

As $\beta\, A^{1/3}/\alpha$ changes, the condition changes between that of open and close pipe and the resonance evolves between GT and SJ



Microscopic Background

In the Thomas-Fermi approximation with

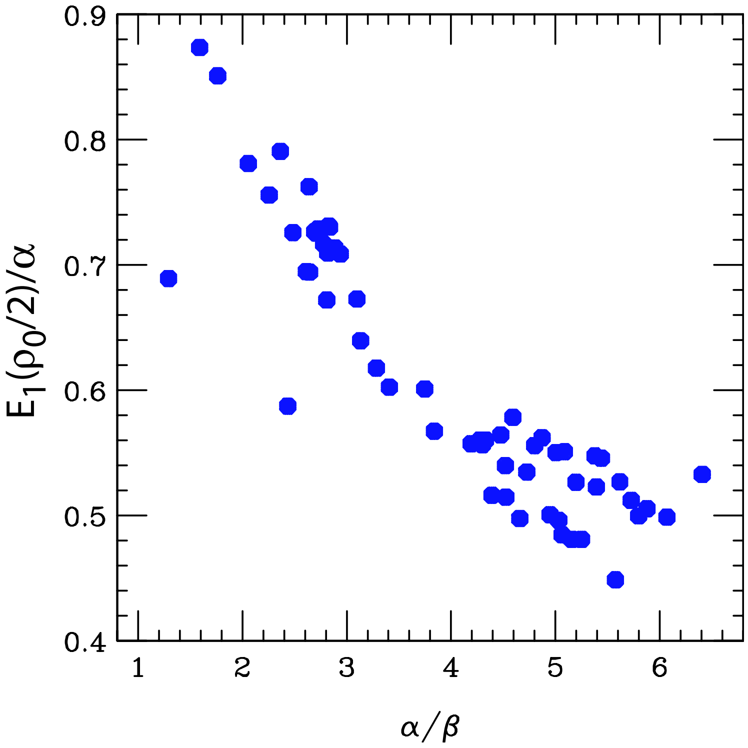
$E = E_0 + \int d^3r\, \rho\, E_1(\rho)\, \left(\frac{\rho_n - \rho_p}{\rho}\right)^2$, where E_1 - symmetry energy $(E_1(\rho_0) = \alpha)$, the Gibbs prescription for semiinfinite matter yields

$$\frac{\alpha}{\beta} = \frac{3}{r_0} \int dx\, \frac{\rho}{\rho_0} \left(\frac{\alpha}{E_1(\rho)} - 1 \right)$$

α/β probes the shape of $E_1(\rho)$!

From $2.0 \lesssim \alpha/\beta \lesssim 2.8$ for mean-field structure calcs (Furnstahl '02 - symbols), we deduce symmetry energy reduction at half the normal density:

$$0.57 \lesssim E_1(\rho_0/2)/\alpha \lesssim 0.83$$



Conclusions

- Adding a single parameter to the standard nuclear binding formula greatly extends access to the physics of neutron-proton asymmetry in nuclei.
- The surface symmetry energy is needed to explain binding of light asymmetric nuclei. In the net energy, the surface and volume symmetry contributions combine as energies of two connected capacitors.
- The finite surface symmetry energy implies existence of asymmetry skins.
- The measured skin sizes limit the ratio of volume-to-surface symmetry coefficients to the range $2.0 \lesssim \alpha/\beta \lesssim 2.8$.

- A combination of the skin and mass information yields for the volume (i.e. infinite-matter) symmetry coefficient $27\,\text{MeV} \lesssim \alpha \lesssim 31\,\text{MeV}$.
- Emergence of the surface symmetry energy is related to a weakening of the symmetry energy with density. The ratio α/β can be used to limit the reduction factor at half the normal density to $0.57 \lesssim E_1(\rho_0/2)/\alpha \lesssim 0.83$.
- Description of giant dipole resonances improves with an inclusion of the surface symmetry energy. The resonances are more of a GT type for light nuclei and of an SJ type for heavy.

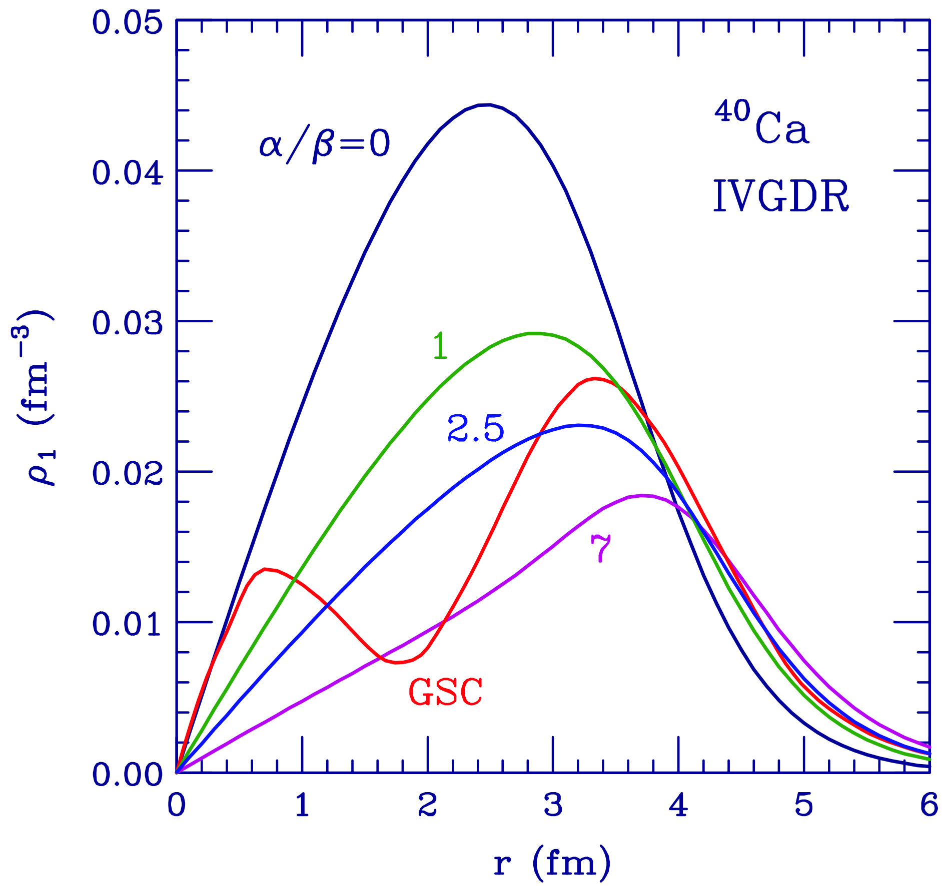
nucl-th/0301050

Local Amplitude ≡ Transition Density

$$\rho_1(r) = \frac{D_V}{\rho_0}\, j_\ell(qr) \left[\rho(r) - \frac{\alpha}{3\beta\, A^{1/3}}\, r\, \frac{d\rho}{dr}\right]$$

Compared to microscopic calculations (Khamerdzhiev et al '97)

GSC, including 2p-2h excitations and ground-state correlations:



Different Mass Formulas

Liquid droplet model (Myers & Swiatecki '69)

$$\begin{aligned} E \; = \; & \left(-a_1 + J\,\overline{\delta}^2 - \frac{1}{2}\,K\,\overline{\epsilon}^2 + \frac{1}{2}\,M\,\overline{\delta}^4\right)\,A \\ & + \left(a_2 + Q\,\tau^2 + a_3\,A^{-1/3}\right)\,A^{2/3} + c_1\,\frac{Z^2}{A^{1/3}}\,\left(1 + \frac{1}{2}\,\tau\,A^{-1/3}\right) \\ & - c_2\,Z^2\,A^{1/3} - c_3\,\frac{Z^2}{A} - c_4\,\frac{Z^{4/3}}{A^{1/3}} \end{aligned}$$

where

$$\overline{\epsilon} = \frac{1}{K}\,\left(-2a_2\,A^{-1/3} + L\,\overline{\delta}^2 + c_1\,\frac{Z^2}{A^{4/3}}\right)\,, \qquad \tau = \frac{3}{2}\,\frac{J}{Q}\,(\overline{\delta} + \overline{\delta}_s)$$

$$\overline{\delta} = \frac{I + \frac{3}{8}\,\frac{c_1}{Q}\,\frac{Z^2}{A^{5/3}}}{1 + \frac{9}{4}\,\frac{J}{Q}\,A^{-1/3}}\,, \qquad \overline{\delta}_s = -\frac{c_1}{12J}\,\frac{Z}{A^{1/3}}\,, \qquad I = \frac{N-Z}{N+Z}$$

$Q = H/(1 - \frac{2}{3}\,P/J)$. Expansion in asymmetry yields results consistent with current, but approach more complex...

The current formula:

$$E = -a_V\, A + a_S\, A^{2/3} + a_C\, \frac{Z^2}{A^{1/3}} + \alpha\, \frac{(N-Z)^2}{A}\, \frac{1}{1+\frac{\alpha}{\beta}\, A^{-1/3}}$$

Liquid drop model [LDM] (Myers & Swiatecki '66)

$$\begin{aligned} E &= -a_V\, \left(1-\kappa_V\, I^2\right)\, A + a_S\, \left(1-\kappa_S\, I^2\right)\, A^{2/3} \\ &\quad + a_C\, \frac{Z^2}{A^{1/3}} - a_4\, \frac{Z^2}{A} \end{aligned}$$

with $I = (N-Z)/A$. LDM corresponds to the expansion in the current formula:

$$\frac{1}{\frac{A}{\alpha} + \frac{A^{2/3}}{\beta}} \simeq \frac{\alpha}{A}\, \left(1 - \frac{\alpha}{\beta}\, A^{-1/3}\right)$$

But that expansion only accurate for $A \gtrsim 1000$, i.e. never!