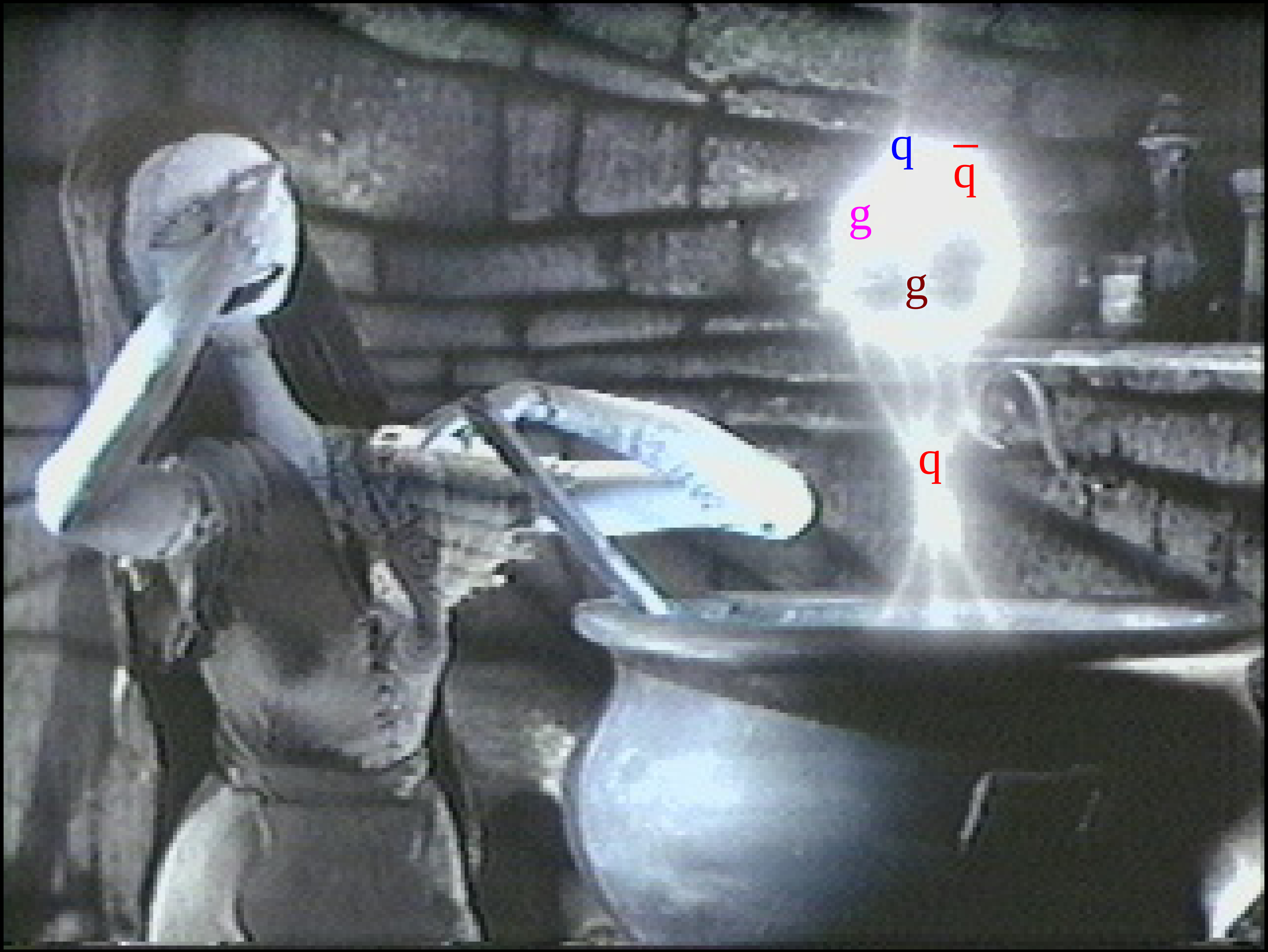

q
$\bar{q}$
g
g
q

QGP thermalization at weak coupling

Peter Arnold

A simple question:

What is the (local) thermalization time for QGPs in heavy ion collisions for arbitrarily high energy collisions, where $\alpha_s << 1$?

A much simpler question:

How does that time depend on α_s?

$$t_{\rm eq} \sim \frac{\alpha_{\rm s}^{-??}}{\text{momentum scale}}$$

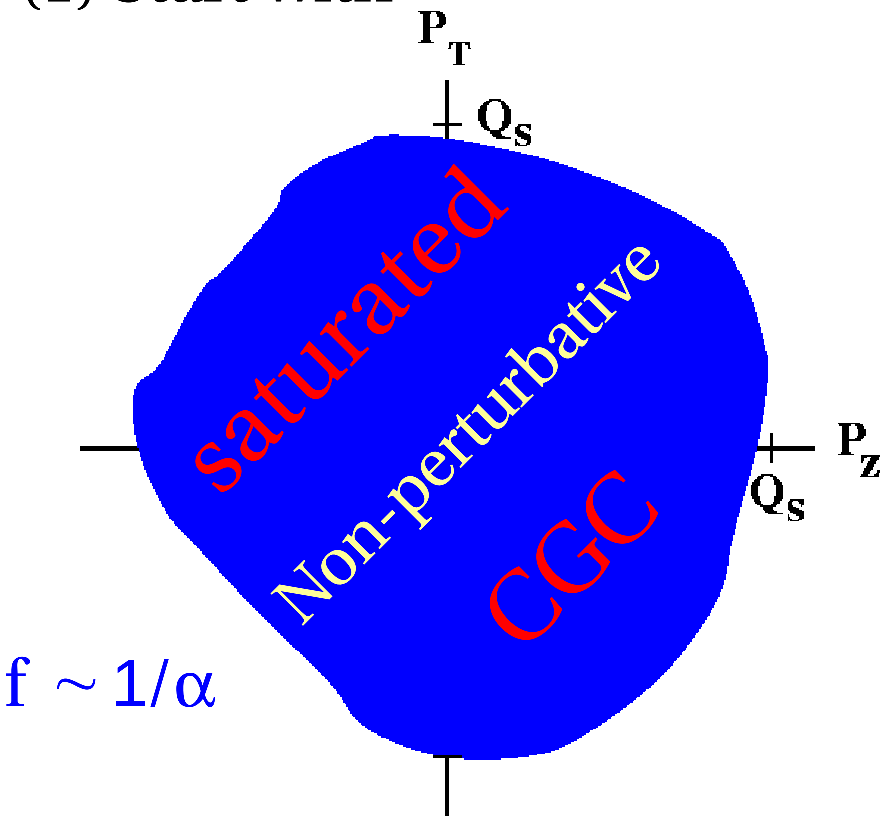
A theoretical outrage:

We do not know even the power **??** of α_s .

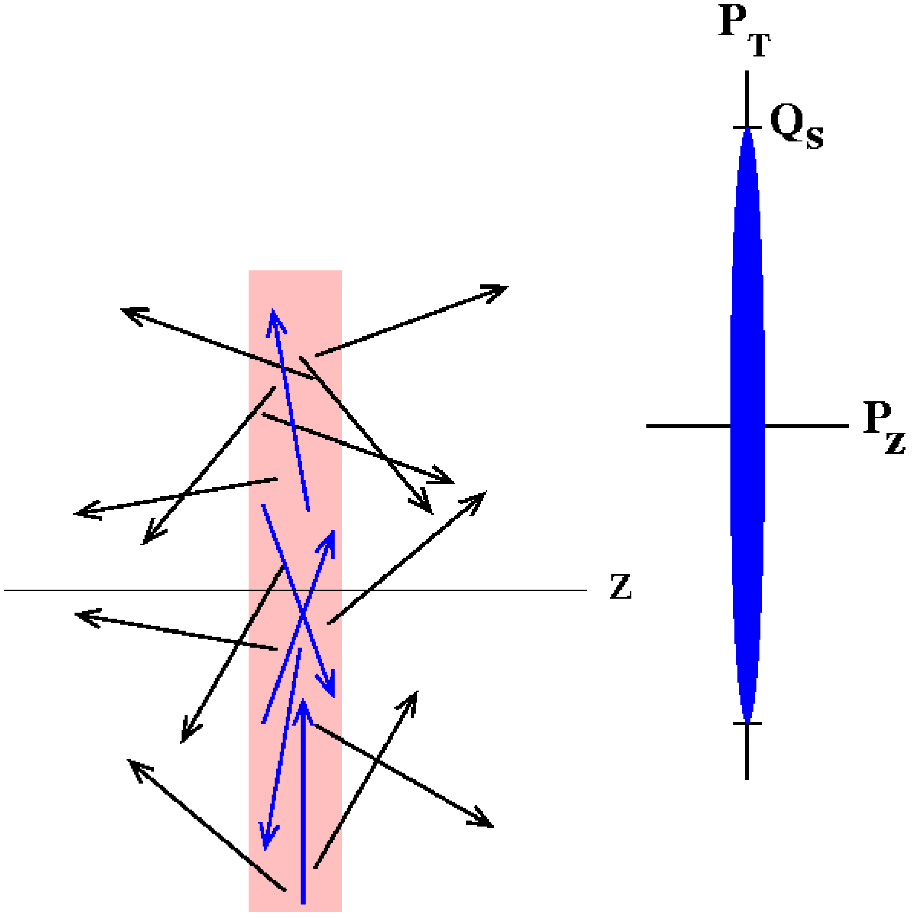
Review of bottom-up thermalization

(Baier, Mueller, Schiff, Son '00)

(1) Start with



(2) expansion -> anisotropic

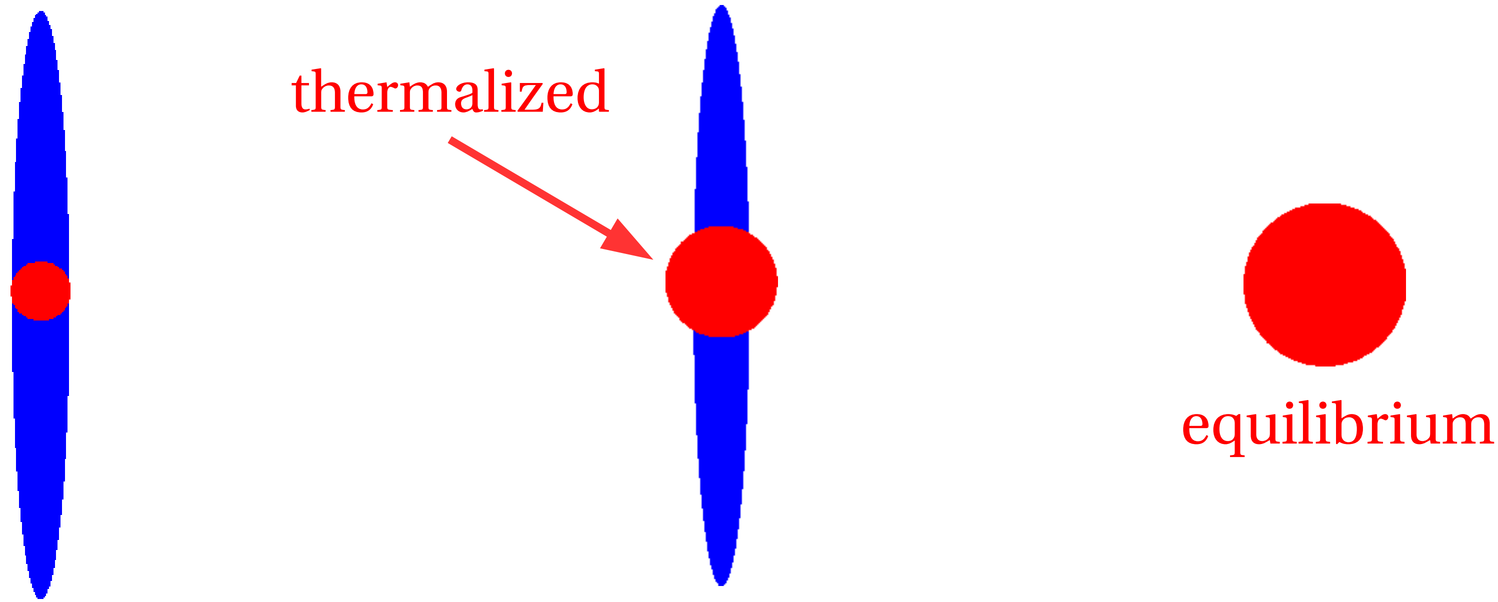


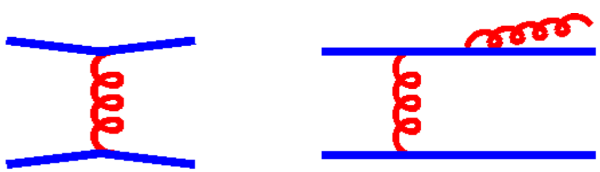
(free expansion would be $p_z \propto 1/\tau$)

In this talk, |p|~Q_s is called hard.

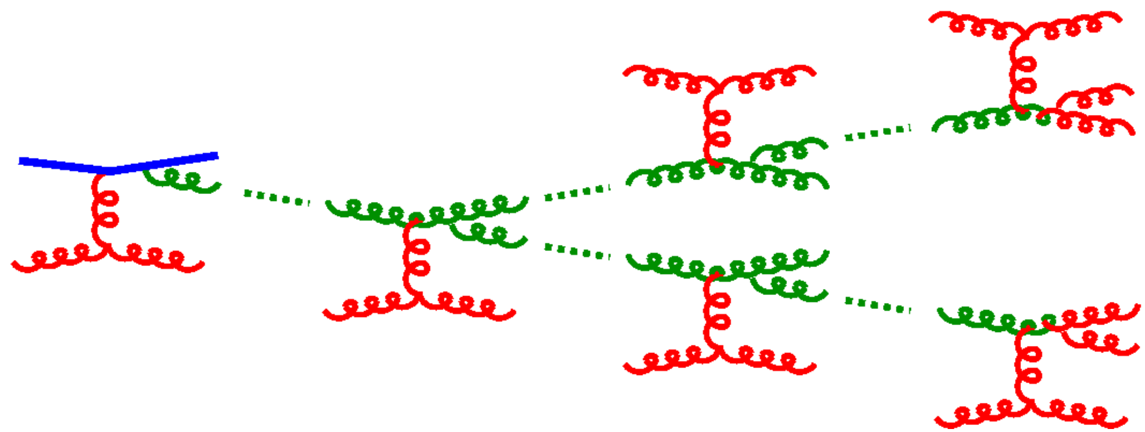
Review of bottom-up thermalization

(Baier, Mueller, Schiff, Son '00)





$p_z \propto \tau^{-1/3}$



broadening --> $f << 1/g^2$

hard particles perturbative

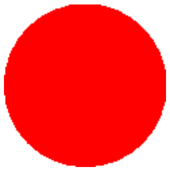
Review of bottom-up thermalization

(Baier, Mueller, Schiff, Son '00)

They found*

$$t_{\mathrm{eq}} \sim \alpha^{-13/5}$$

(in units where $Q_s = 1$)



equilibrium

The Problem: This analysis only considered individual 2-particle collisions and ignored coherent collective effects, namely plasma instabilities.

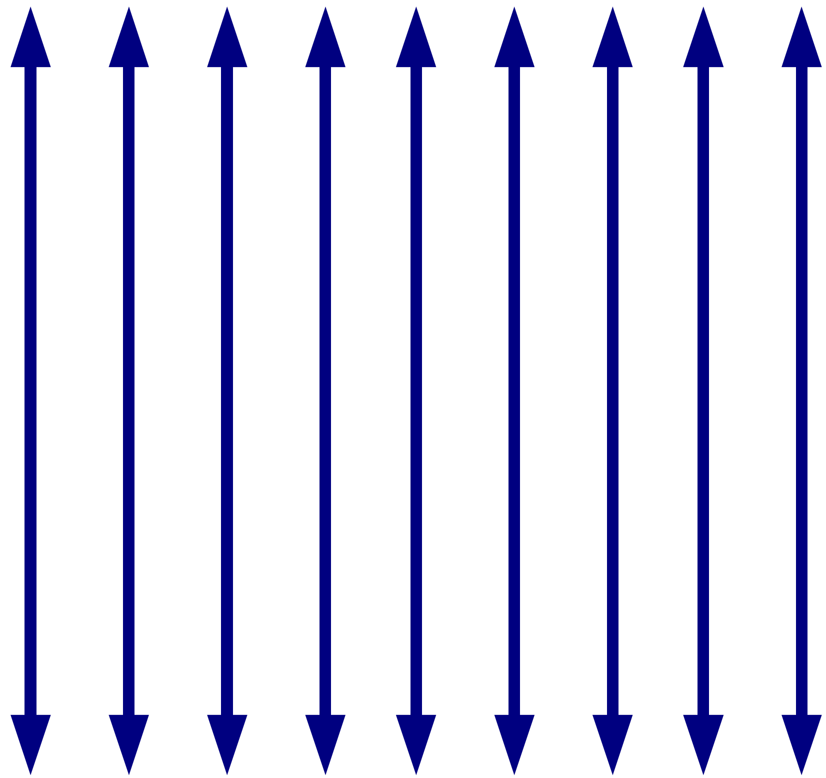
---

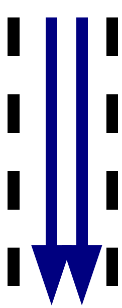
* *Hey, why the funny fraction?*

condition for hard Brem in time τ: $1 \sim \alpha^4 T^3 \tau^2$

conservation of energy: $T^4 \sim 1/\alpha\tau$

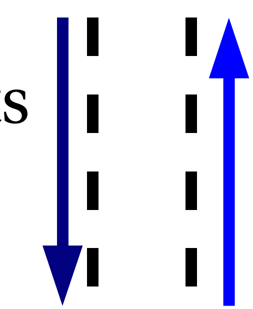
A picture of the Weibel (or filamentation) instability



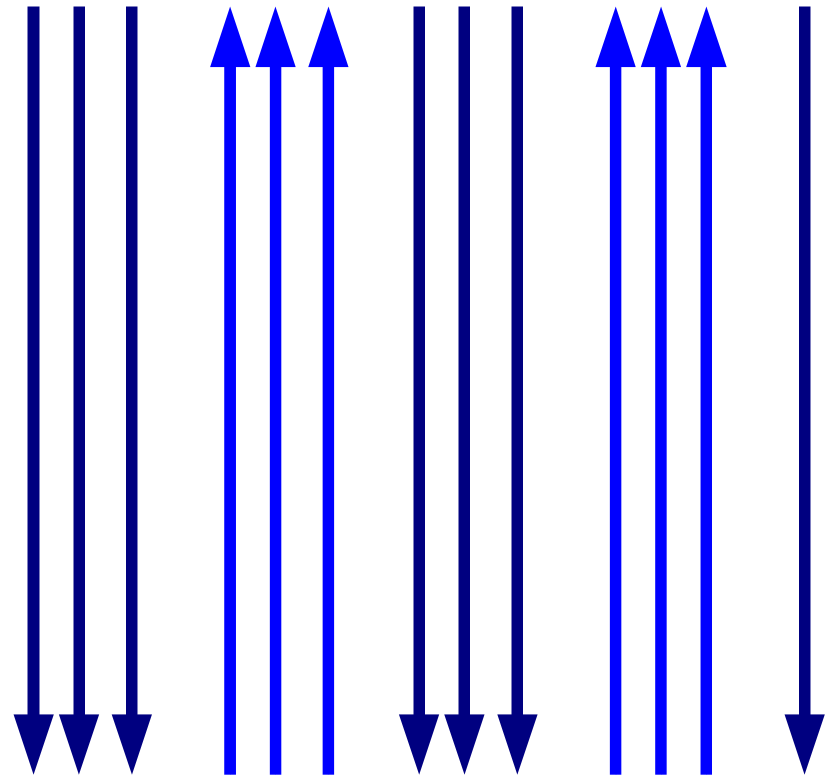


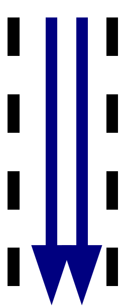
Parallel currents attract

Opposite currents repel



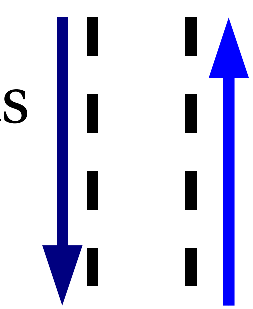
A picture of the Weibel (or filamentation) instability



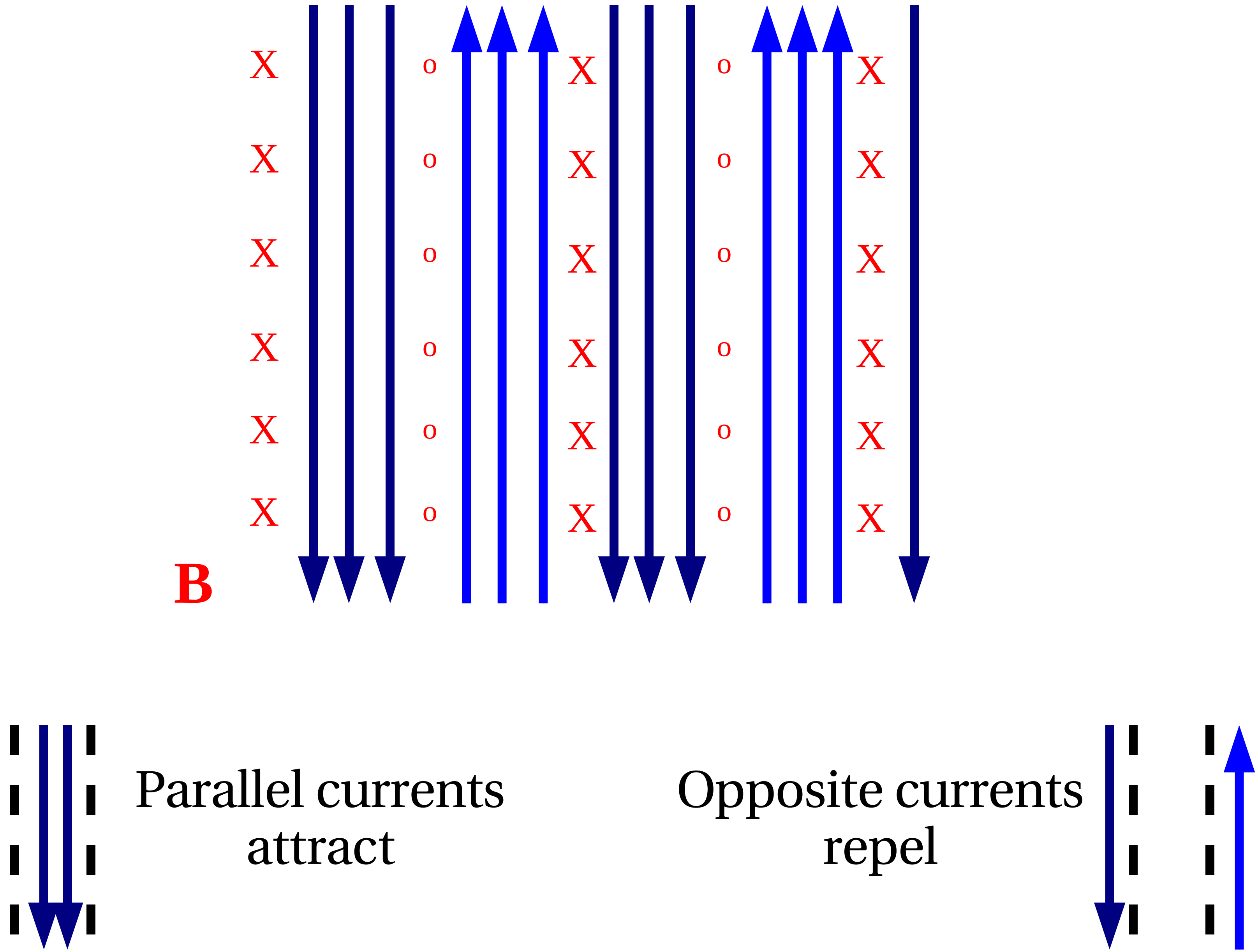


Parallel currents attract

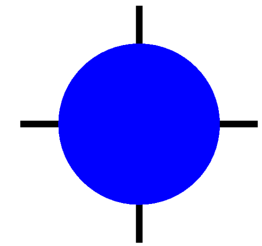
Opposite currents repel

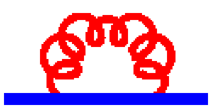
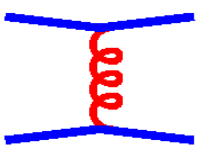


A picture of the Weibel (or filamentation) instability



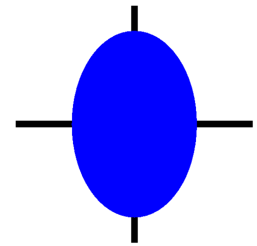
Scales

Review of thermal equilibrium: 

	hard particle f	1
	hard particle momenta	T
	plasmon mass m	gT
	particle collision rate (small angles, color randomizes)	g^2T
	Expansion rate	$\frac{T^2}{M_{\mathrm{Pl}}}$

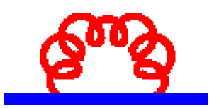
Scales

O(1) **distorted** thermal:

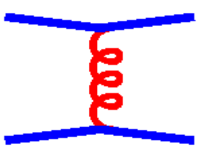


hard particle f — 1

hard particle momenta — T



plasmon mass m — gT
also **instability growth rate**



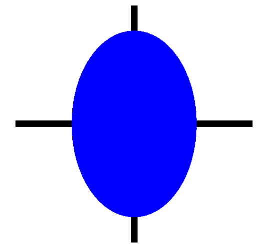
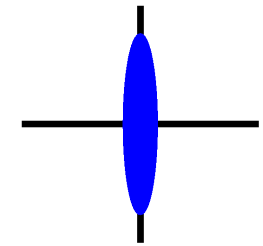
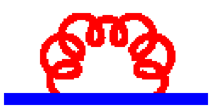
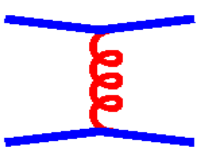
particle collision rate — $g^2 T$
(small angles, color randomizes)

Expansion rate — $\dfrac{T^2}{M_{\rm Pl}}$

growth rate >> collision rate >> expansion rate

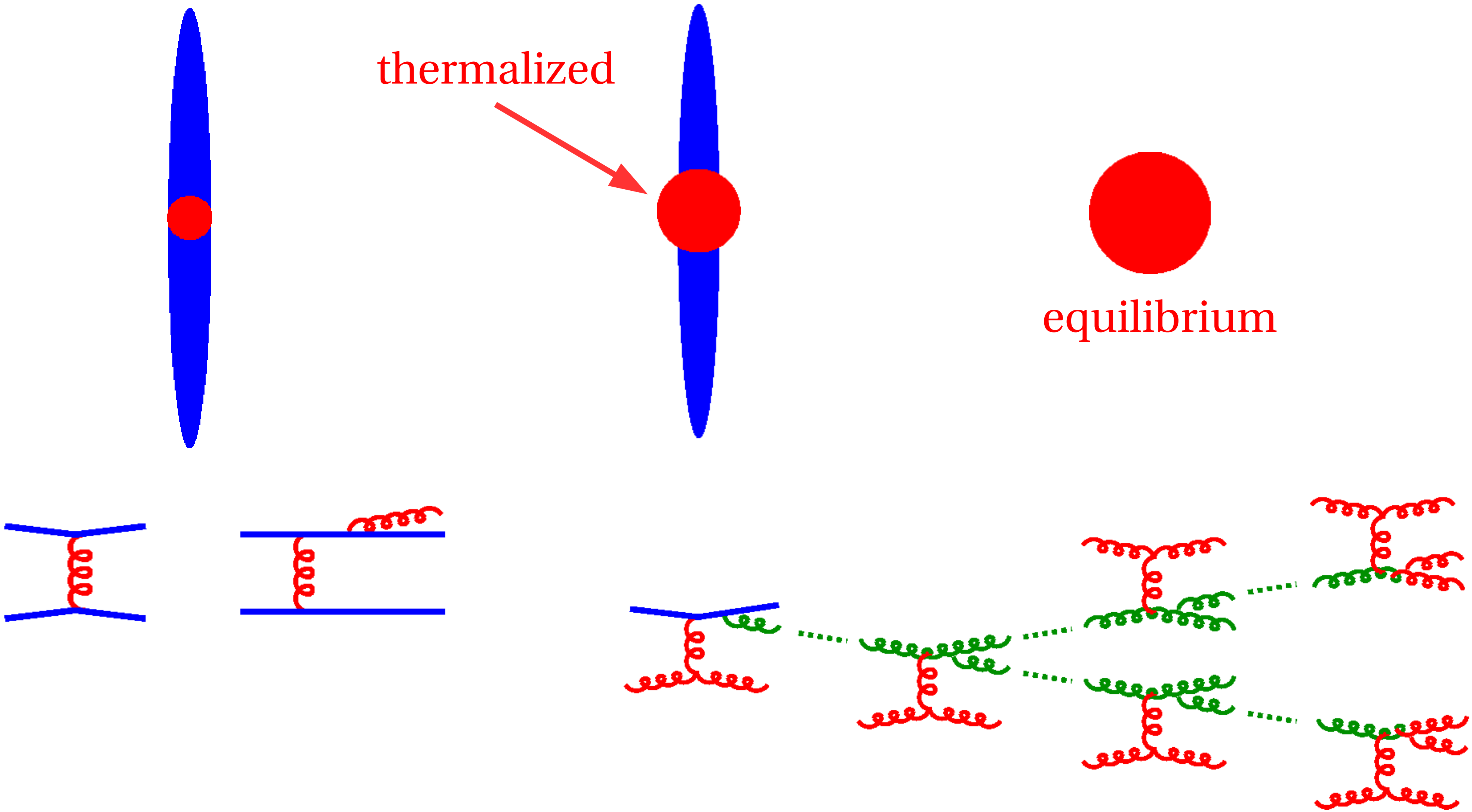
Scales

$1 \ll \tau \ll g^{-3}$
units $Q_s = 1$

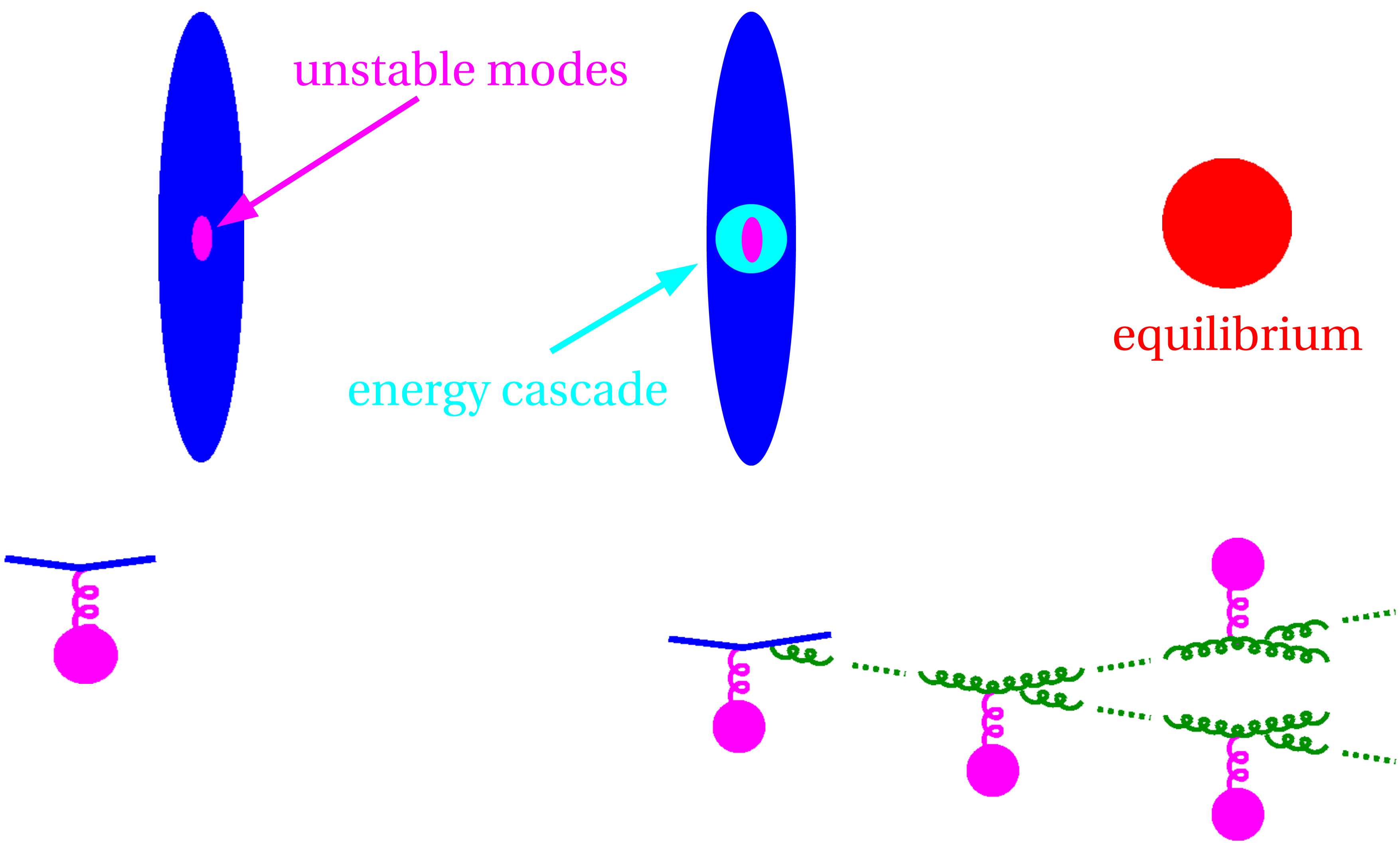
	O(1) distorted thermal *vs.* **first stage original bottom-up**		
	hard particle f	1	$\gg 1$
	hard particle momenta	T	1
	plasmon mass m also **instability growth rate**	gT	$\tau^{-1/2}$
	particle collision rate (small angles, color randomizes)	$g^2 T$	$\tau^{-2/3}$
	Expansion rate	$\dfrac{T^2}{M_{\rm Pl}}$	τ^{-1}

growth rate >> collision rate >> expansion rate

How is bottom-up modified?

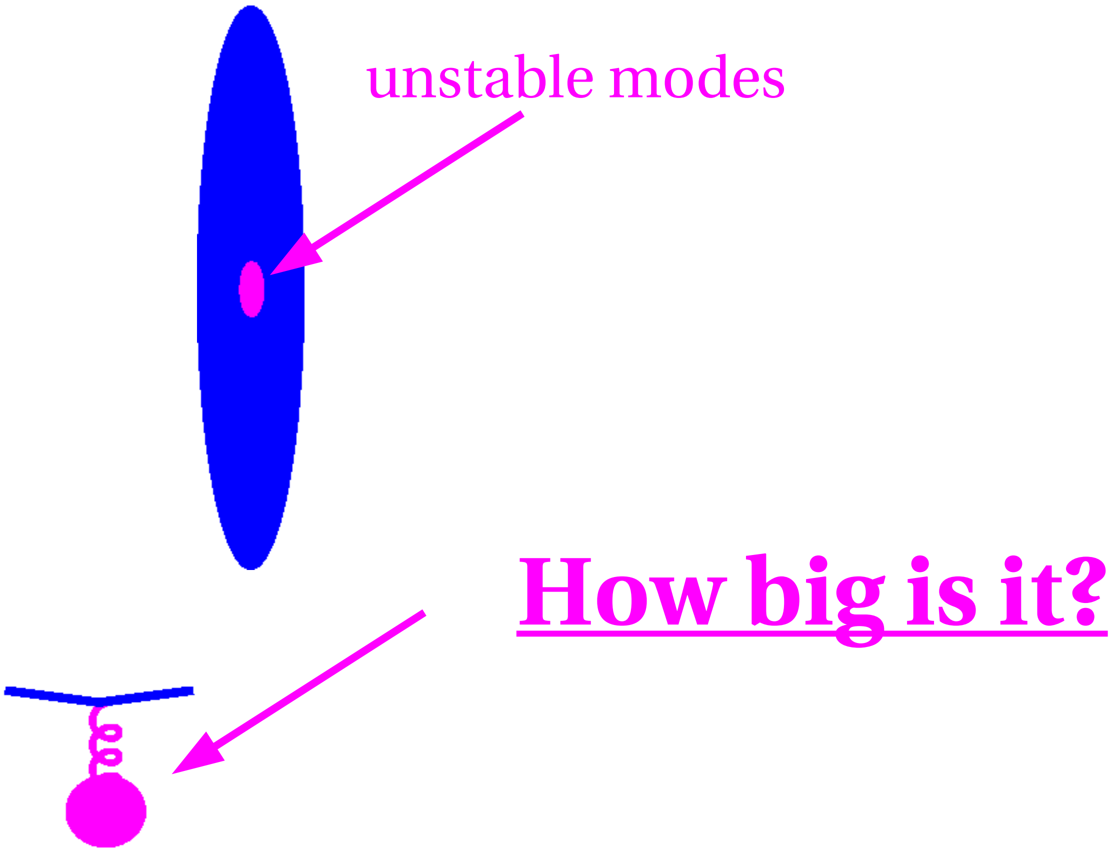


How is bottom-up modified?



Mueller, Shoshi, Wong:
Could it segue back to traditional bottom-up?

How is bottom-up modified?



Problem: Suppose an anisotropic distribution of plasma particles generates a plasma instabilities with wave numbers of order *m*. How big do the associated magnetic fields grow?

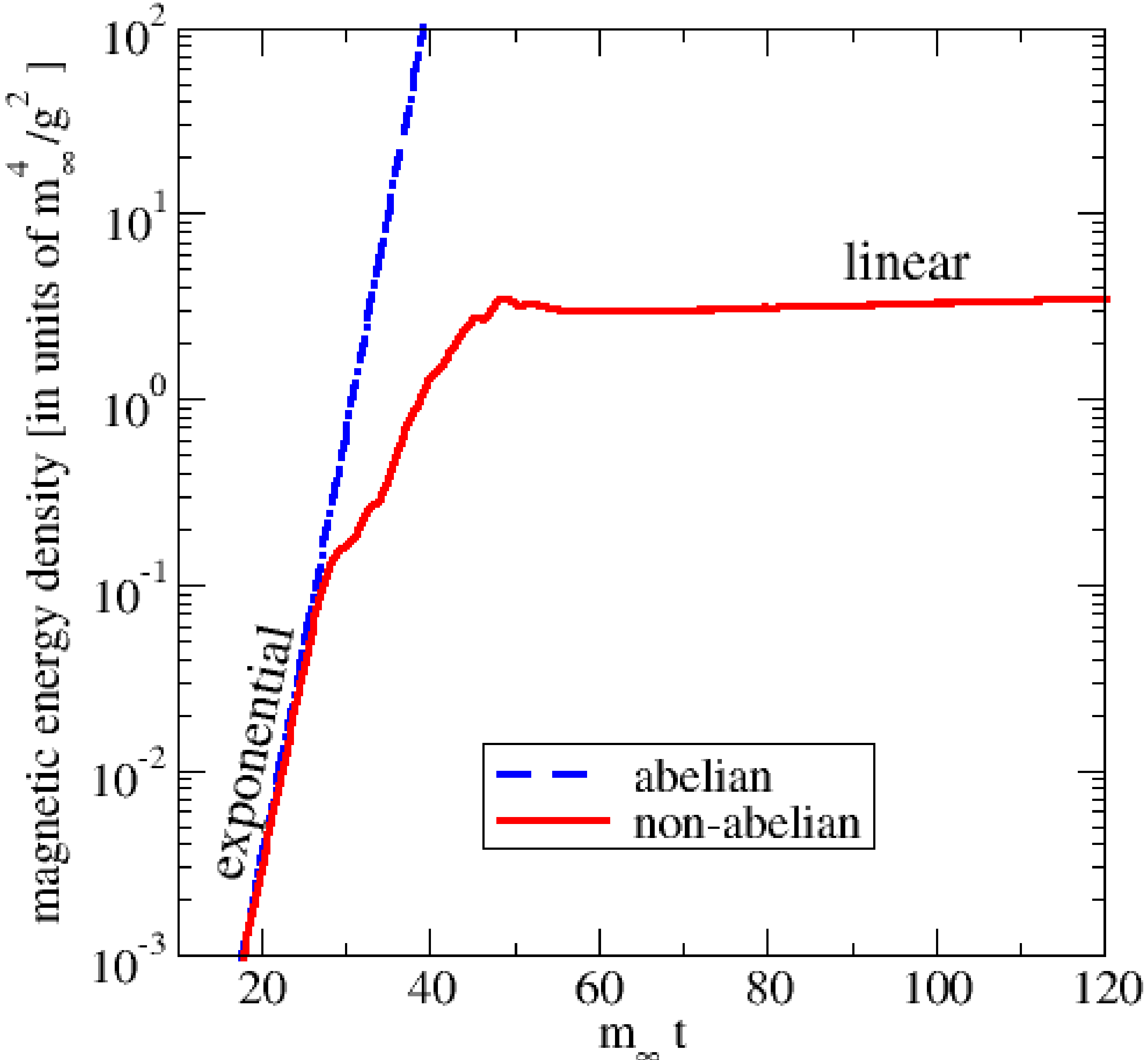
Answer for moderate anisotropy:

$$B_* \sim \frac{m^2}{g} \qquad \text{(QCD)}$$

This is the value of B at which *non-abelian* self-interaction of the magnetic field becomes important.*

---

* To see this, put $k \sim m$ into a covariant derivative $D \sim i(k - gA)$. The gA is non-perturbative when $A \sim k/g$, corresponding to $B \sim kA \sim k^2/g$.



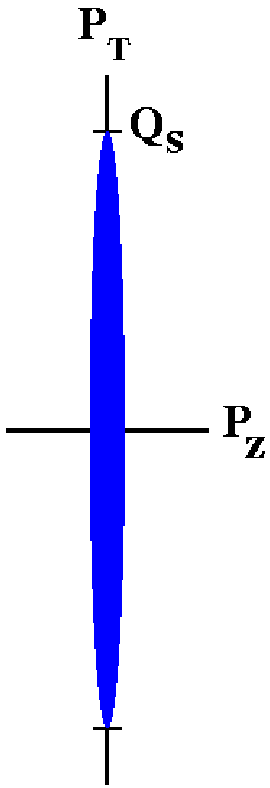
Arnold, Moore, Yaffe '05

Rebhan, Romatschke, Strickland '05

Bodeker, Rummukainen '07

Problem: Suppose an anisotropic distribution of plasma particles generates a plasma instabilities with wave numbers of order *m*. How big do the associated magnetic fields grow?

Various guesses for extreme anisotropy:



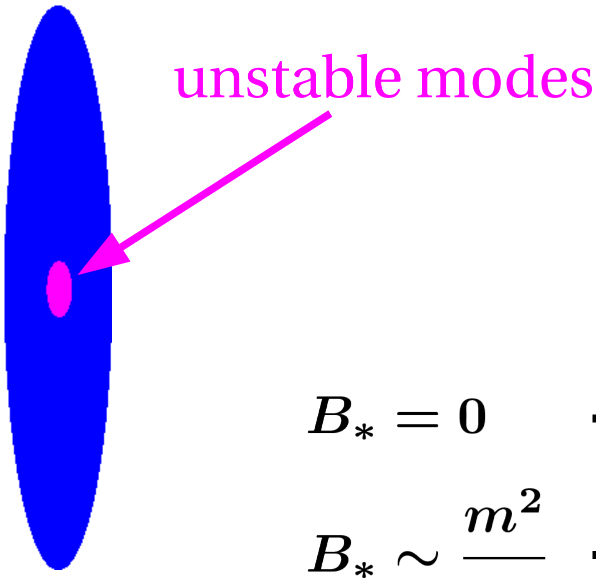
$$\theta \sim p_z/p \ll 1$$

$$B_* \sim \frac{m^2}{g}$$ (same as moderate case)

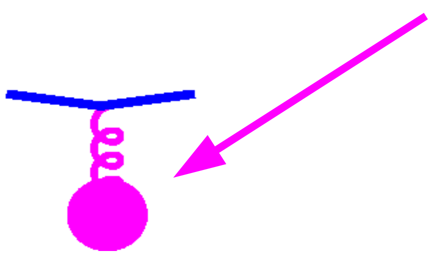
$$B_* \sim \frac{m^2}{g\theta}$$ Arnold & Moore '05

$$B_* \sim \frac{m^2}{g\theta^2}$$ limit from Nielsen-Olesen instabilities

Stage I of bottom-up



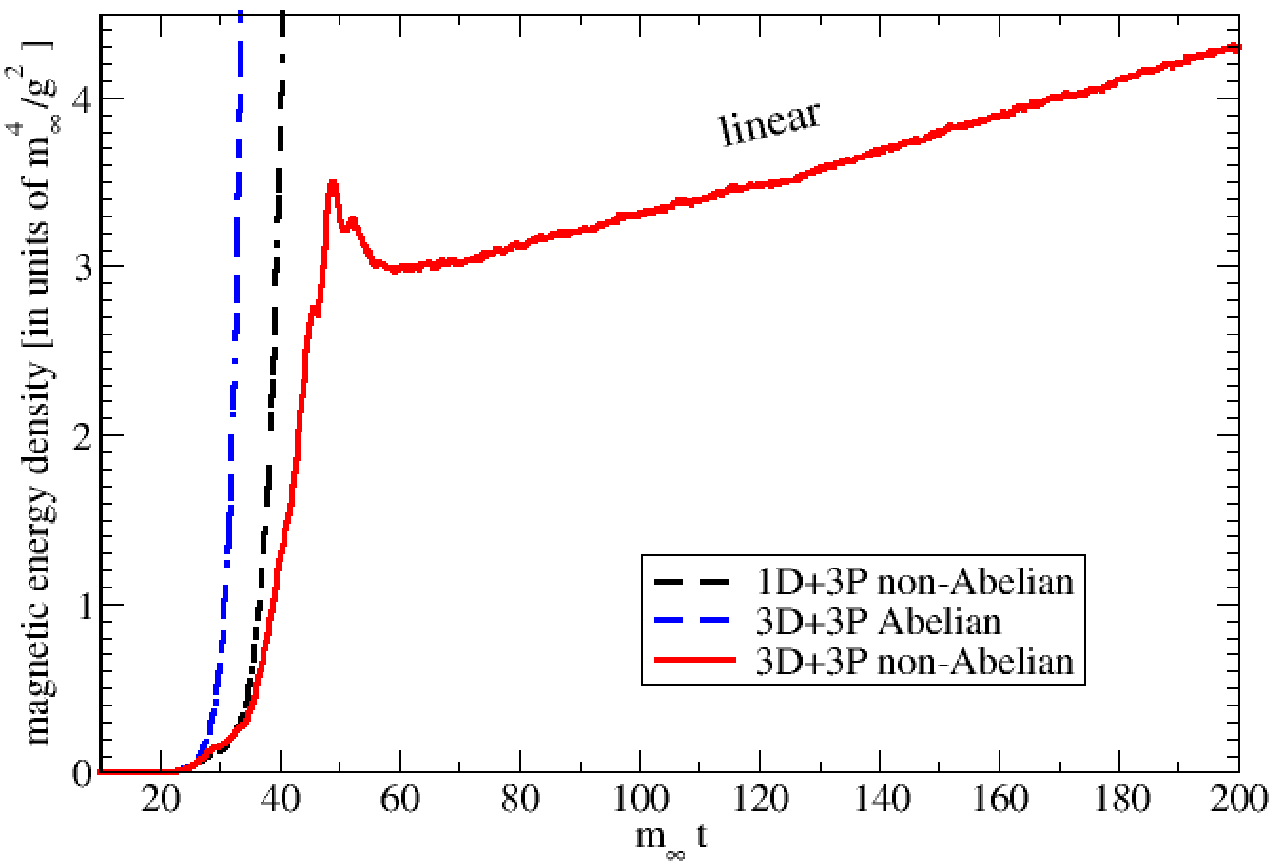
$$\theta \equiv v_z \sim p_z/p$$

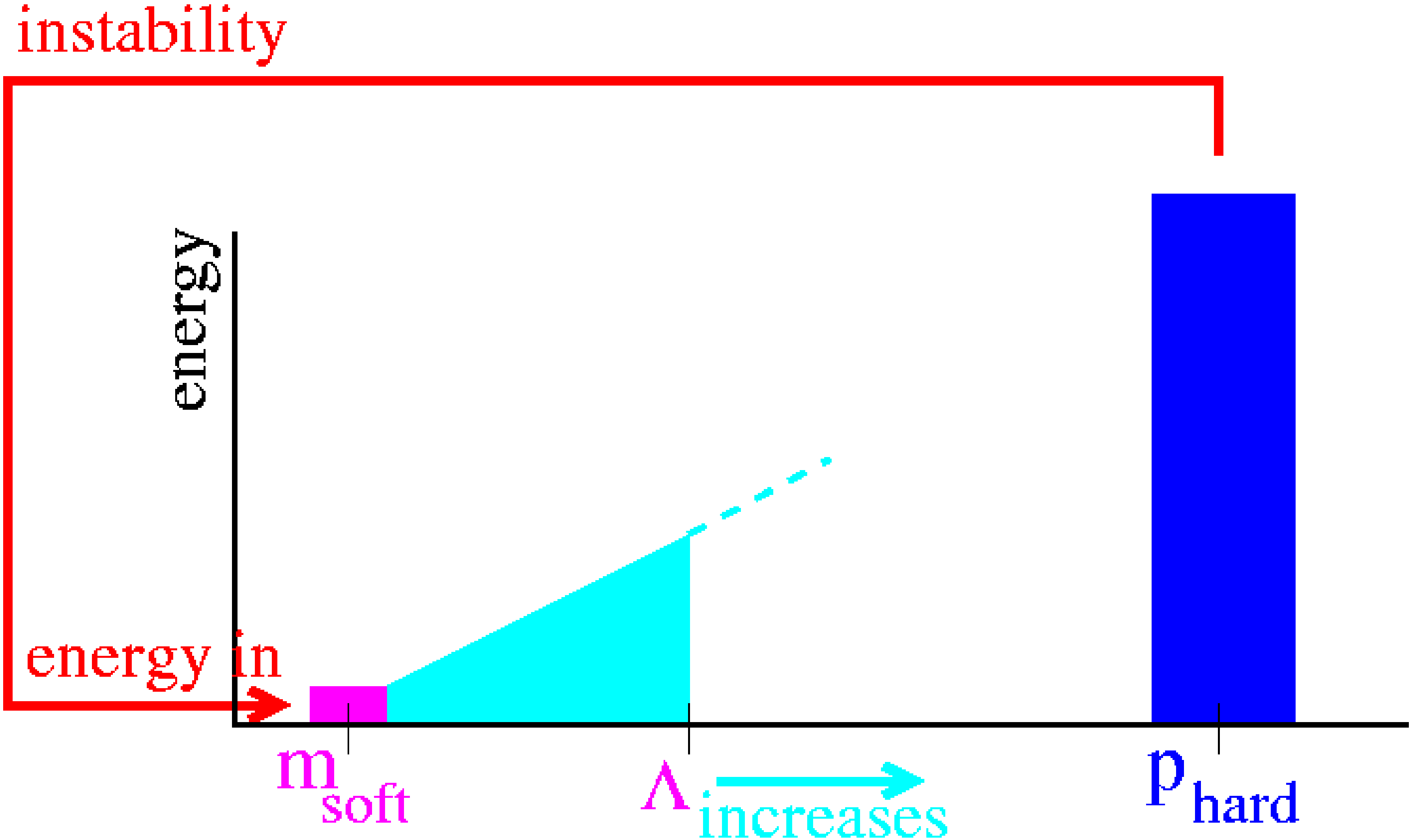


$B_* = 0$ →	$\theta \sim \tau^{-1/3}$	original bottom-up
$B_* \sim \dfrac{m^2}{g}$ →	$\theta \sim \tau^{-1/4}$	Bodeker '05
$B_* \sim \dfrac{m^2}{g\theta}$ →	$\theta \sim \tau^{-1/8}$	Arnold & Moore '05
$B_* \sim \dfrac{m^2}{g\theta^2}$ →	$\theta \sim \tau^{-1/12}$	Nielsen-Olesen limited

How to tell?

Too naive idea: Look at magnetic energy $B^2/2$ at late times and take the square root.





Cascade is gas of perturbative plasmons.

But, using a simple model for how energy flows from the unstable mode into this cascade, one can relate B_* to the *rate* of linear growth in magnetic energy,

Measuring from simulations how the linear growth rate scales with anisotropy, we find [Arnold & Moore '07]

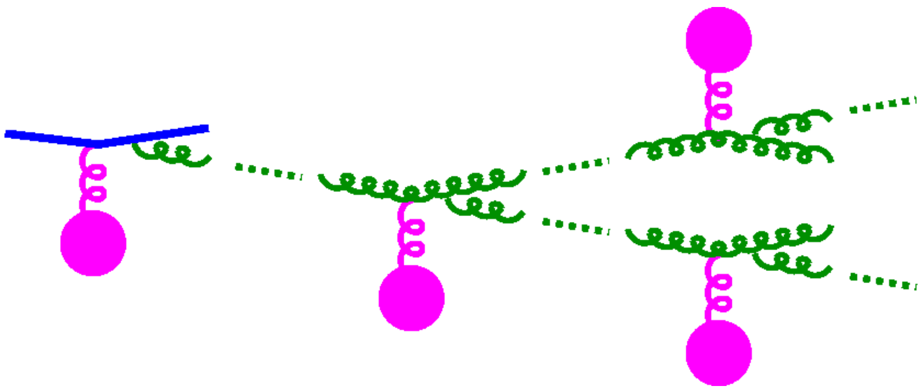
$$B_* \sim \frac{m^2}{g\theta} \quad \longrightarrow \quad \theta \sim \tau^{-1/8}$$ Arnold & Moore

If we accept this, then we now understand the first stage of the bottom-up scenario with instabilities.

What's left?

(1) Verify our understanding of B_* through other measurements.

(2) Figure out how it affects the later stages of bottom-up thermalization:



Extra Slides in case I need 'em

The Vlasov Equations

Traditional QED Plasmas

Describe particles by classical phase space density $f(p,x,t)$.
Describe EM fields by classical gauge fields $A_\mu(x,t)$.

$$\partial_t f + v \cdot \nabla_x f + e\left(E + v \times B\right) \cdot \nabla_p f = 0$$ Collisionless Boltzmann eq.

$$\partial_\mu F^{\mu\nu} = j^\nu = \int_p e\, v^\nu f$$ Maxwell's eqs.

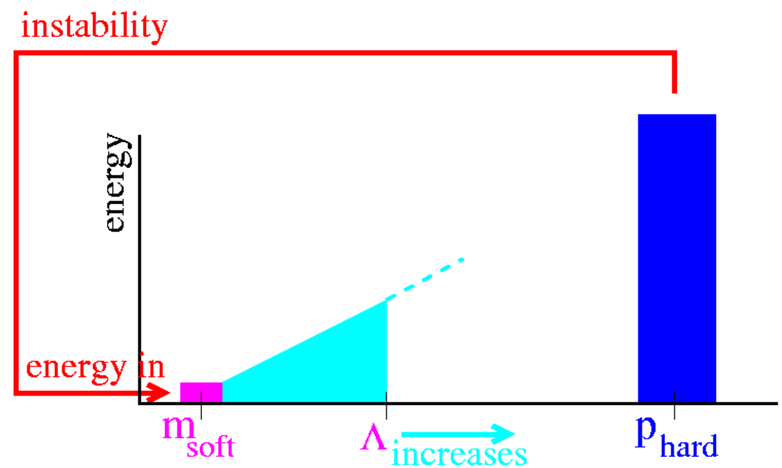
QCD Plasmas

$f(p,x,t)$ becomes a color density matrix.

$\partial_t \rightarrow \partial_t - i e A_0$ and $\nabla_x \rightarrow \nabla_x - i e \mathbf{A}$ above.

Rate of linear energy growth

Imagine half of unstable mode energy is dumped into cascade. Time to recover is ~ $1/\gamma$ ~ $1/m$. So

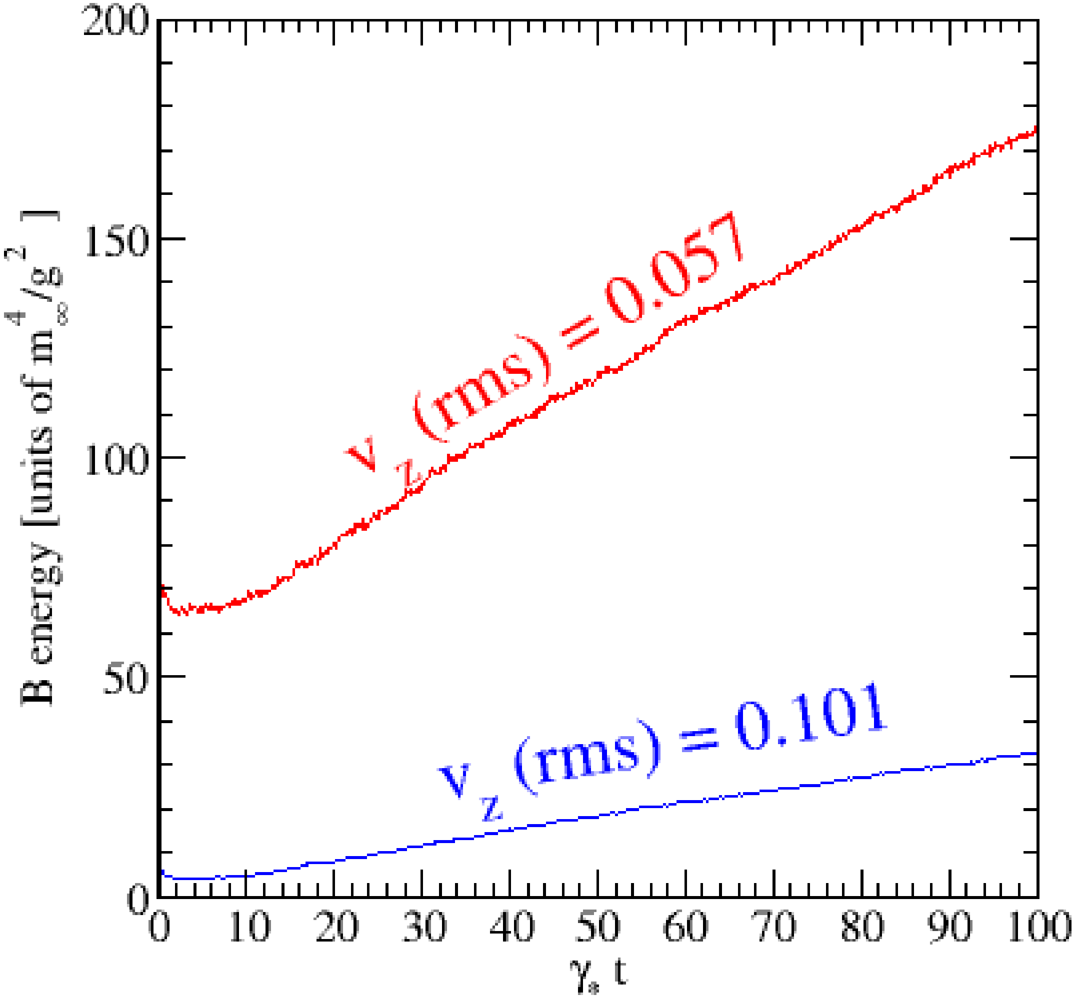


$$\frac{d\epsilon}{dt} \sim \gamma B_*^2$$

$$\epsilon \sim \text{initial} + \gamma B_*^2 t \sim \text{initial} + \gamma \frac{m^4}{g^2 \theta^{2n}} t$$

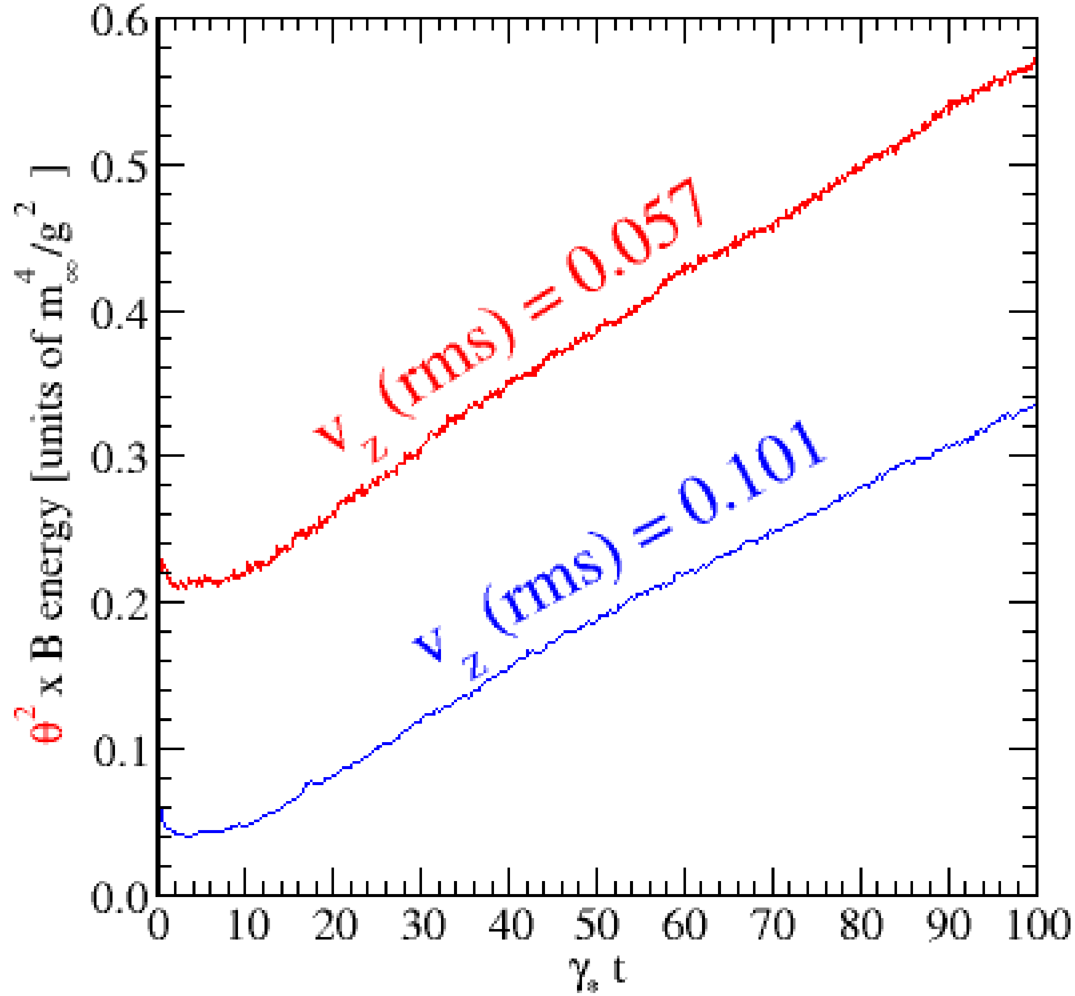
My previous argument was that $n = 1$.

$$\epsilon \quad \sim \quad \text{initial} + \gamma B_*^2 t \quad \sim \quad \text{initial} + \frac{m^4}{g^2 \theta^{2n}} \gamma t$$



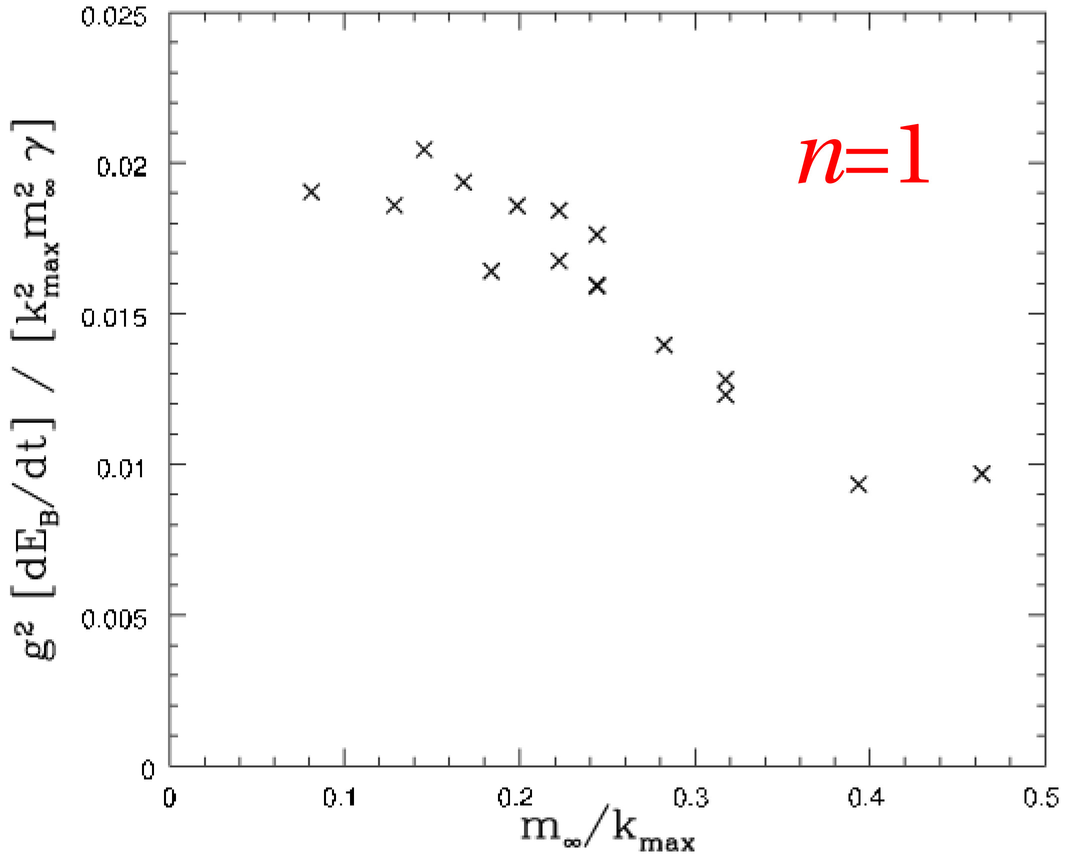
Note: Simulations have used “strong” initial conditions

$$\epsilon \quad \sim \quad \text{initial} + \gamma B_*^2 t \quad \sim \quad \text{initial} + \frac{m^4}{g^2 \theta^{2n}} \gamma t$$



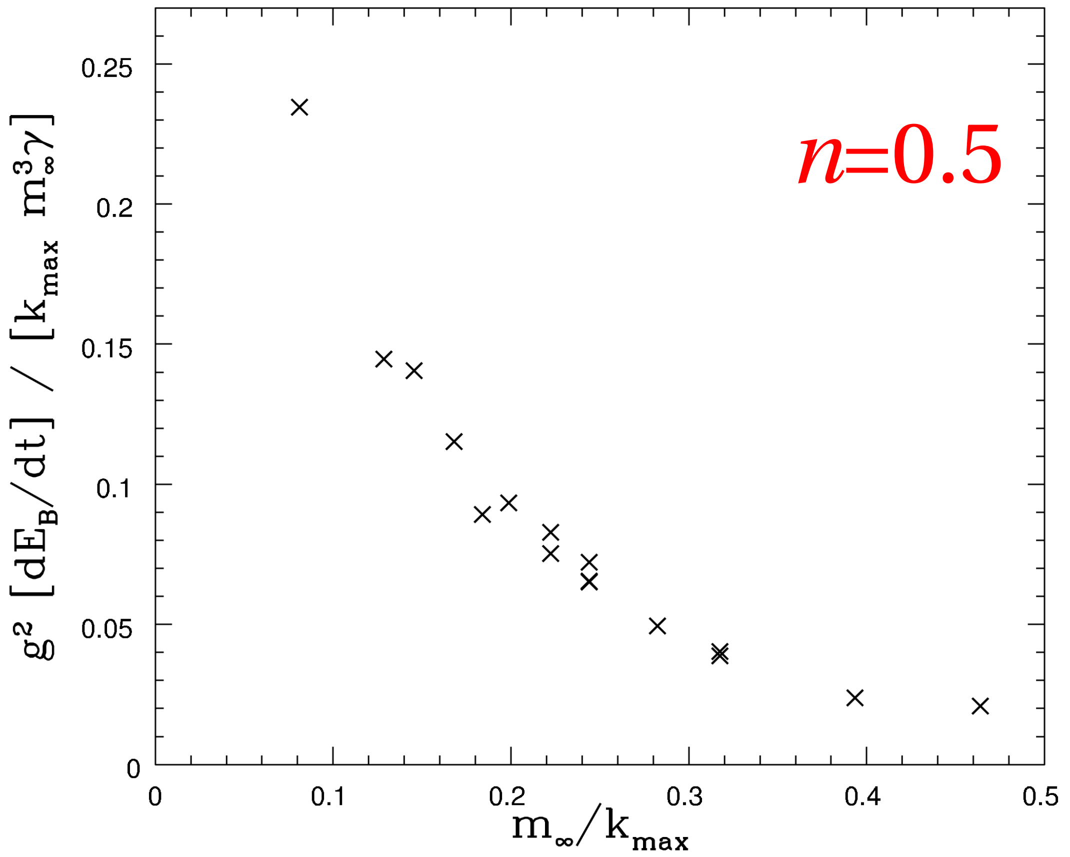
Note: Simulations have used “strong” initial conditions

$$\theta^{2n} \frac{d\epsilon}{dt} \to \text{constant} \quad \text{as} \quad \theta \to 0 \text{ ?}$$



Note: $k_{\max} \sim m / \theta$

$$\theta^{2n} \frac{d\epsilon}{dt} \to \text{constant} \quad \text{as} \quad \theta \to 0 \; ?$$



Note: $k_{max} \sim m\,/\,\theta$

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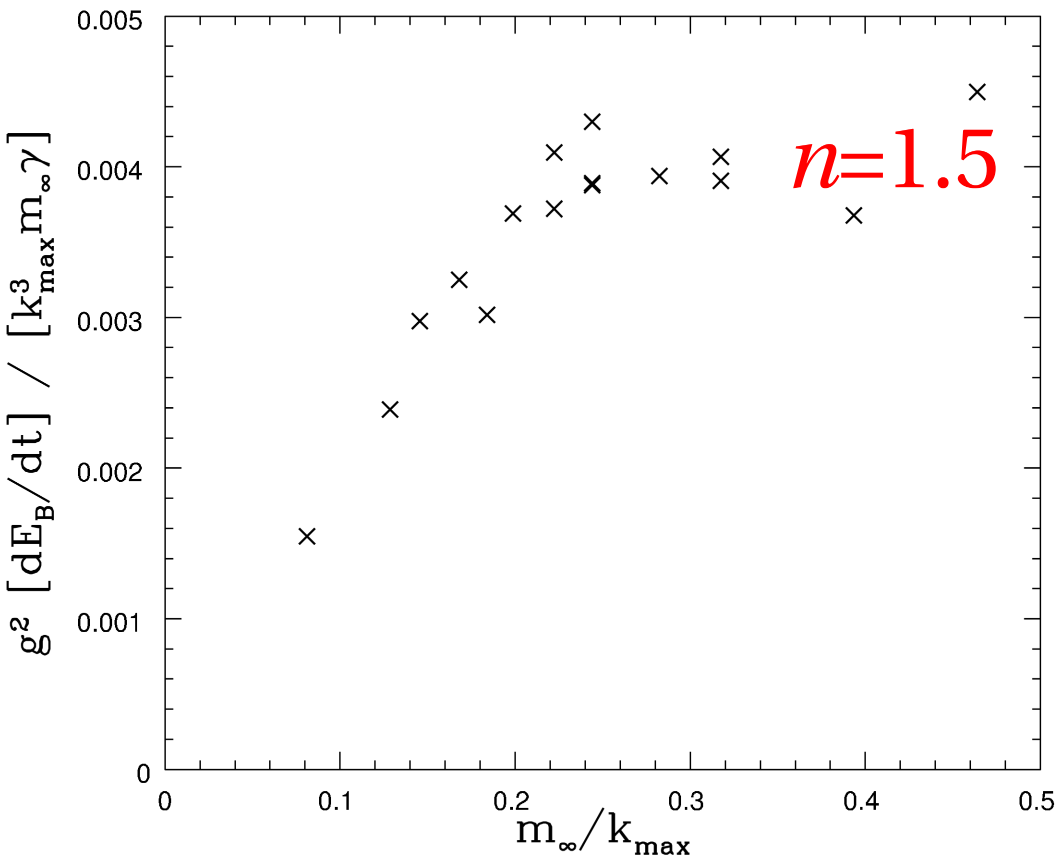

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