Parton Momentum Distribution Prior to Equilibrium

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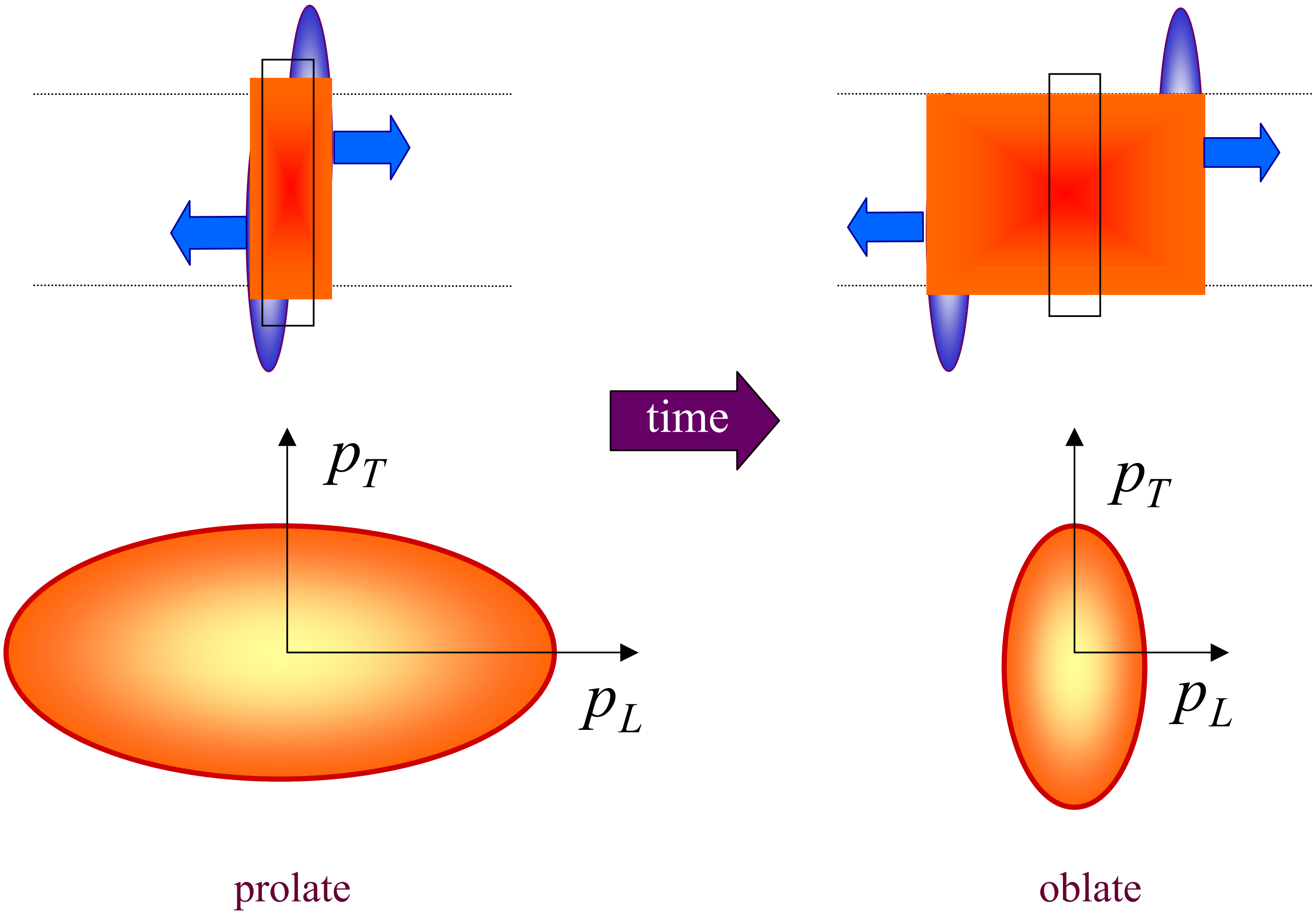
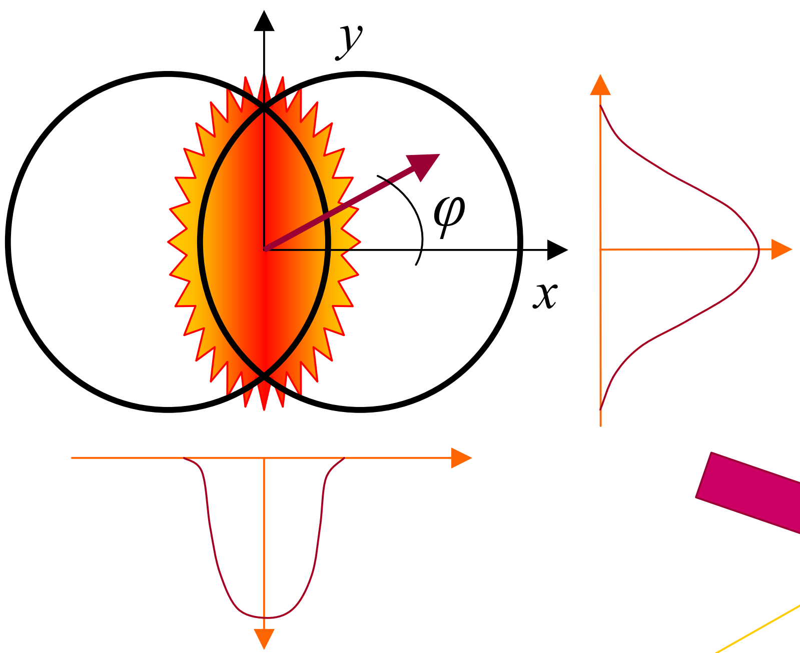
Evolution of Parton Momentum Distribution



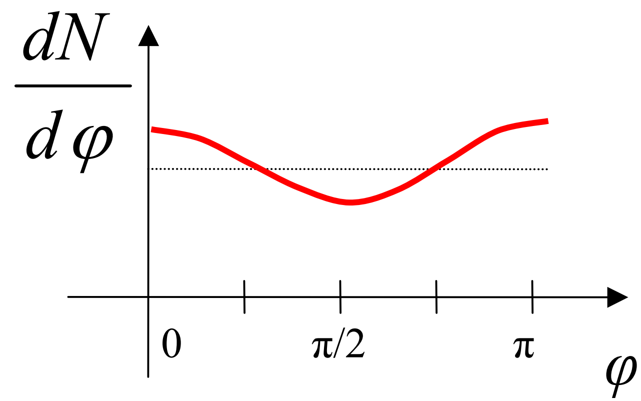
Elliptic Flow & Equilibration

Success of hydrodynamic models in describing elliptic flow



Hydrodynamics

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\nabla\right)\mathbf{v} = -\frac{\nabla p}{\rho}$$



Hydrodynamics requires local thermodynamical equilibrium!

Equilibration Time

$$v_2 \sim \varepsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle}$$

Eccentricity decays due to the free streaming!

$\varepsilon \searrow \;\Rightarrow\; v_2 \searrow$ ➡ $t_{eq} \leq 1 \text{ fm}/c$

time of equilibration

U. Heinz, AIP Conf. Proc.**739,** 163 (2004)

Decay of Eccentricity

free-streaming model

$$f(\mathbf{p},\mathbf{r},t) \sim \exp\left[-\frac{(x-v_x t)^2}{2\sigma_x^2} - \frac{(y-v_y t)^2}{2\sigma_y^2} - \frac{(x-v_x t)^2}{2\sigma_x^2} - \frac{Y^2}{2\Delta Y^2}\right]\frac{P(p_T)}{p_T^2\,\mathrm{ch}\,Y}$$

$m=0$

$$\begin{cases} \langle x^2 \rangle = \int \frac{d^3p}{(2\pi)^3} \int d^3r\, x^2\, f(\mathbf{p},\mathbf{r},t) = \sigma_x^2 + \alpha t^2 \\ \langle y^2 \rangle = \int \frac{d^3p}{(2\pi)^3} \int d^3r\, y^2\, f(\mathbf{p},\mathbf{r},t) = \sigma_y^2 + \alpha t^2 \end{cases}$$

$$\varepsilon(t) = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle} = \frac{\varepsilon(0)}{1 + \frac{\alpha}{R_T^2} t^2}$$

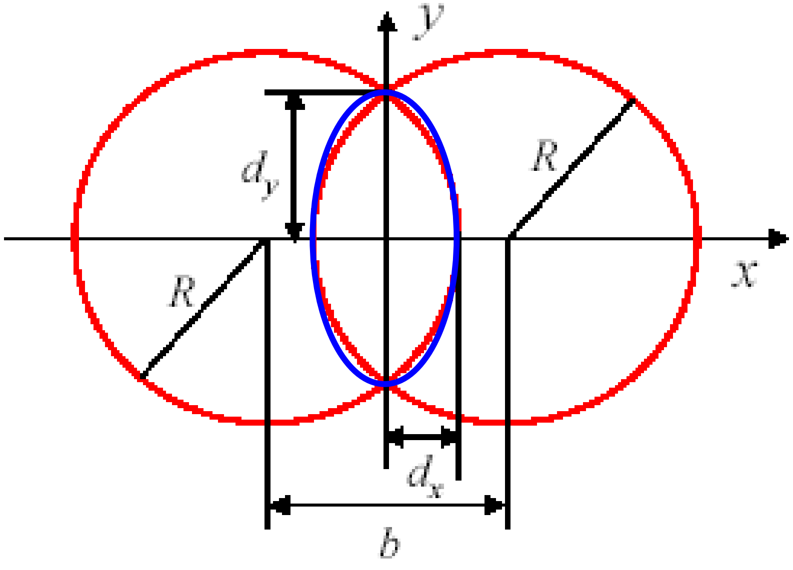
$$\varepsilon(0) = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2}, \qquad R_T^2 = \frac{\sigma_y^2 + \sigma_x^2}{2}$$

$$\alpha = 1/2 \;\; at \;\; Y \approx 0$$

W. Jas and St. Mrówczyński, arXiv:0706.2273 [nucl-th].

Decay of Eccentricity cont.

$$\frac{x^2}{d_x^2} + \frac{y^2}{d_y^2} \le 1$$



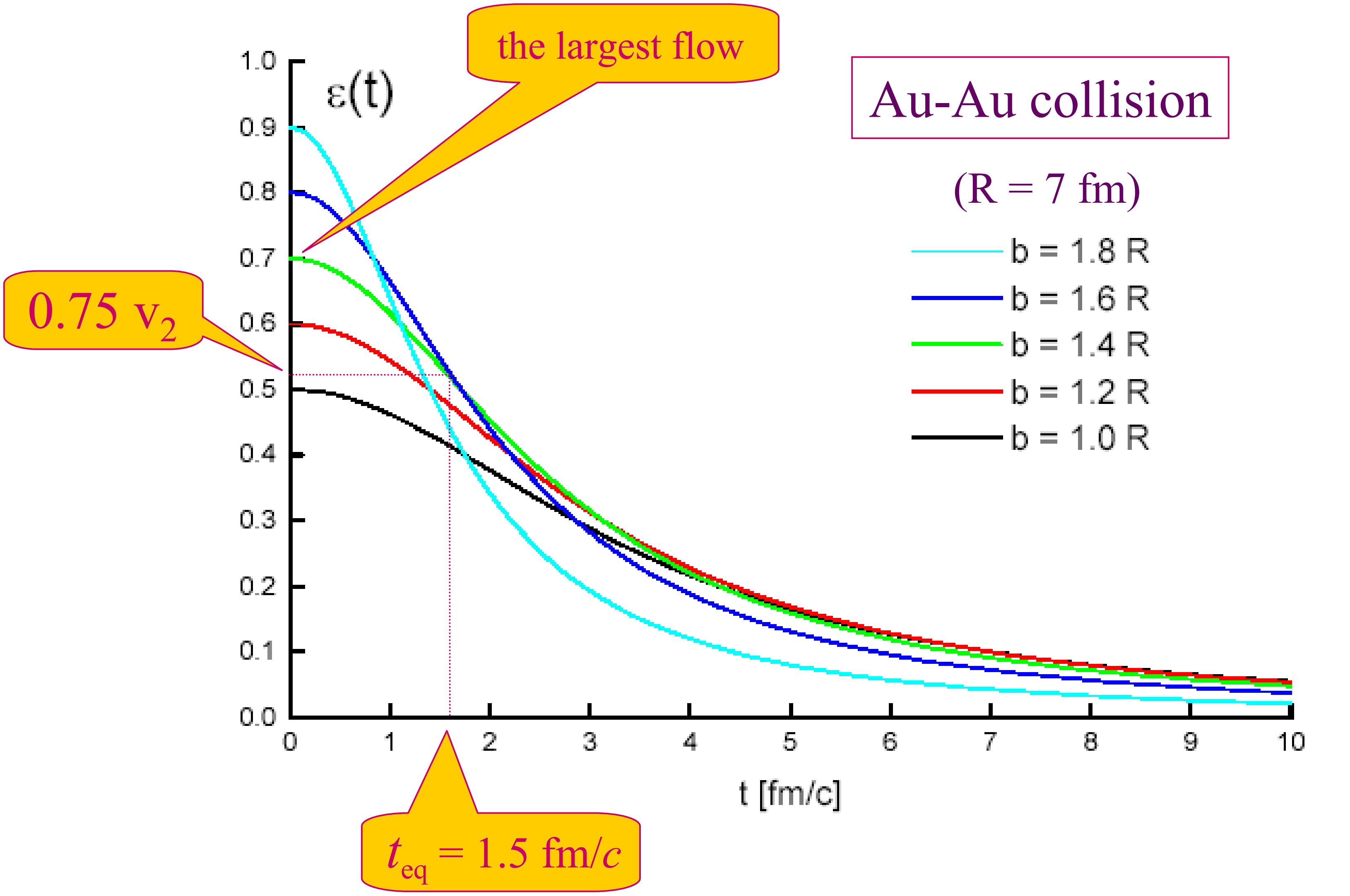
$$\begin{cases} d_x^2 = (R - b/2)^2 \\ d_y^2 = R^2 - b^2/4 \end{cases}$$

$$\begin{cases} \sigma_x = \sqrt{\langle x^2 \rangle} = d_x / 2 \\ \sigma_y = \sqrt{\langle y^2 \rangle} = d_y / 2 \end{cases}$$

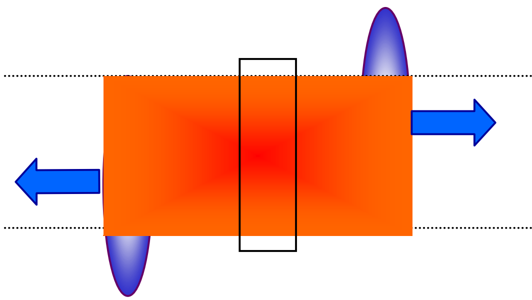
$$\varepsilon(0) = \frac{b}{2R}$$

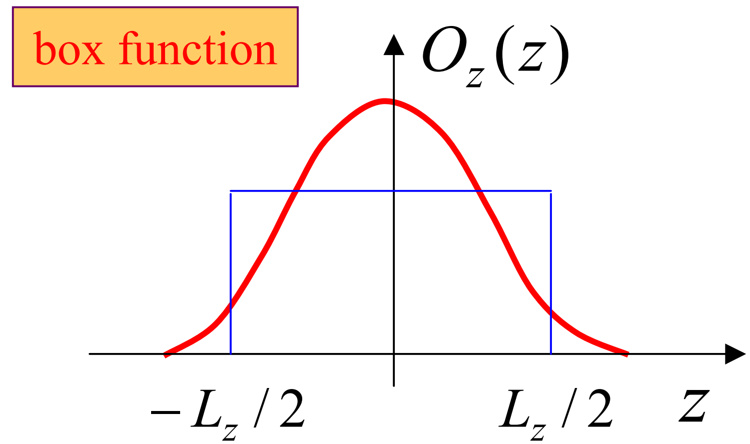
$$R_T^2 = \frac{R^2}{4}\left(1 - \varepsilon(0)\right)$$

Decay of Eccentricity cont.



Momentum Distribution in a Box





$$\int_{-L_z/2}^{L_z/2} dz = \int_{-\infty}^{+\infty} dz\, O_z(z), \qquad \int_{-L_z/2}^{L_z/2} dz\, z^2 = \int_{-\infty}^{+\infty} dz\, z^2 O_z(z)$$

$$O(\mathbf{r}) = O_x(x)\, O_y(y)\, O_z(z) = \left(\frac{6}{\pi}\right)^{3/2} \exp\left[-\frac{6x^2}{L_x^2} - \frac{6y^2}{L_y^2} - \frac{6z^2}{L_z^2}\right]$$

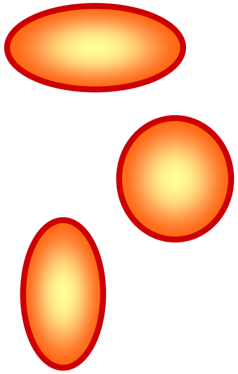
Evolution of Momentum Distribution

$$\rho(t) \equiv \frac{2\langle p_L^2 \rangle}{\langle p_T^2 \rangle}$$

$\rho > 1$ prolate

$\rho = 1$ spherical

$\rho < 1$ oblate

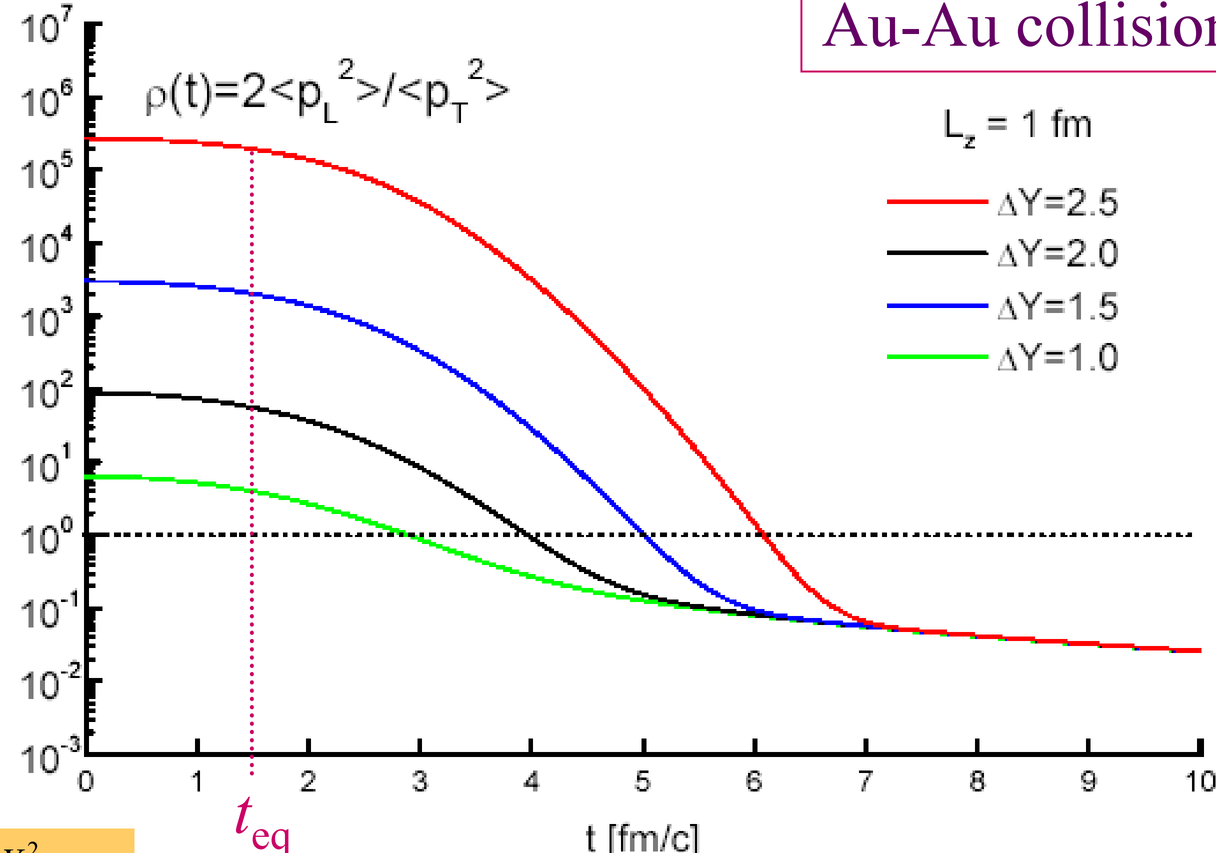


$$\begin{cases} \langle p_i^2 \rangle = \dfrac{1}{N} \displaystyle\int d^3r\, O(\mathbf{r}) \int \dfrac{d^3p}{(2\pi)^3}\, p_i^2\, f(\mathbf{p},\mathbf{r},t), \quad i = L, T \\[2ex] N = \displaystyle\int d^3r\, O(\mathbf{r}) \int \dfrac{d^3p}{(2\pi)^3}\, f(\mathbf{p},\mathbf{r},t) \end{cases}$$

Evolution of Momentum Distribution cont.

$$\sigma_x = \sigma_y = L_x = L_y = 3 \text{ fm}, \quad \sigma_z = L_z = 1 \text{ fm}$$

Au-Au collision

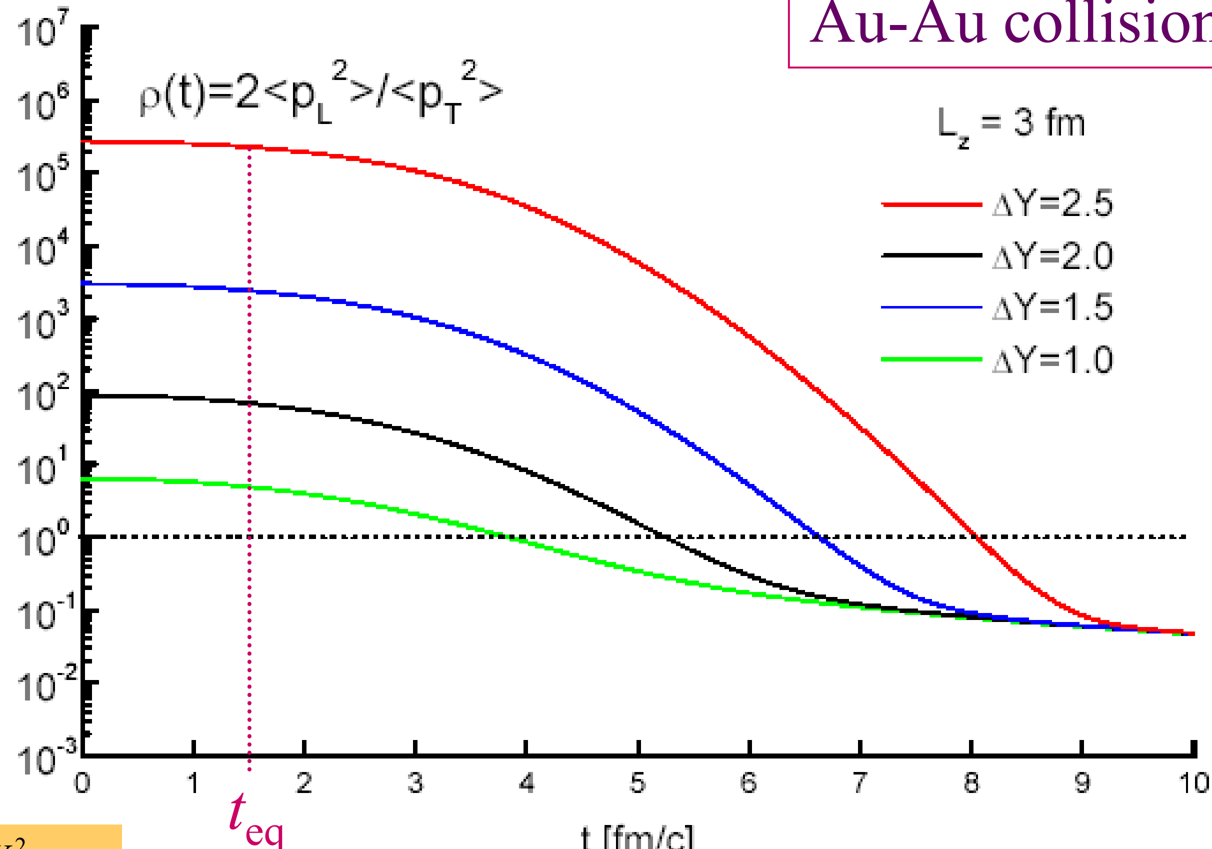


$$\rho(0) = e^{2\Delta Y^2} - 1$$

Evolution of Momentum Distribution cont.

$$\sigma_x = \sigma_y = L_x = L_y = 3 \text{ fm}, \quad \sigma_z = 1 \text{ fm}, \quad L_z = 3 \text{ fm},$$

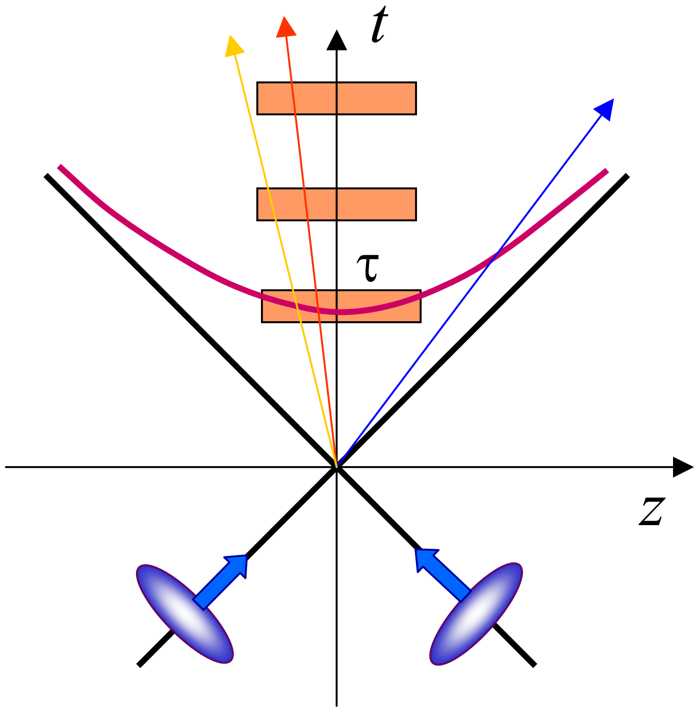
Au-Au collision



$$\rho(0) = e^{2\Delta Y^2} - 1$$

Effect of finite formation time

$$f(\mathbf{p},\mathbf{r},t) \rightarrow \Theta\left(t\sqrt{1-\mathrm{v}_z^2}-\tau\right) f(\mathbf{p},\mathbf{r},t)$$



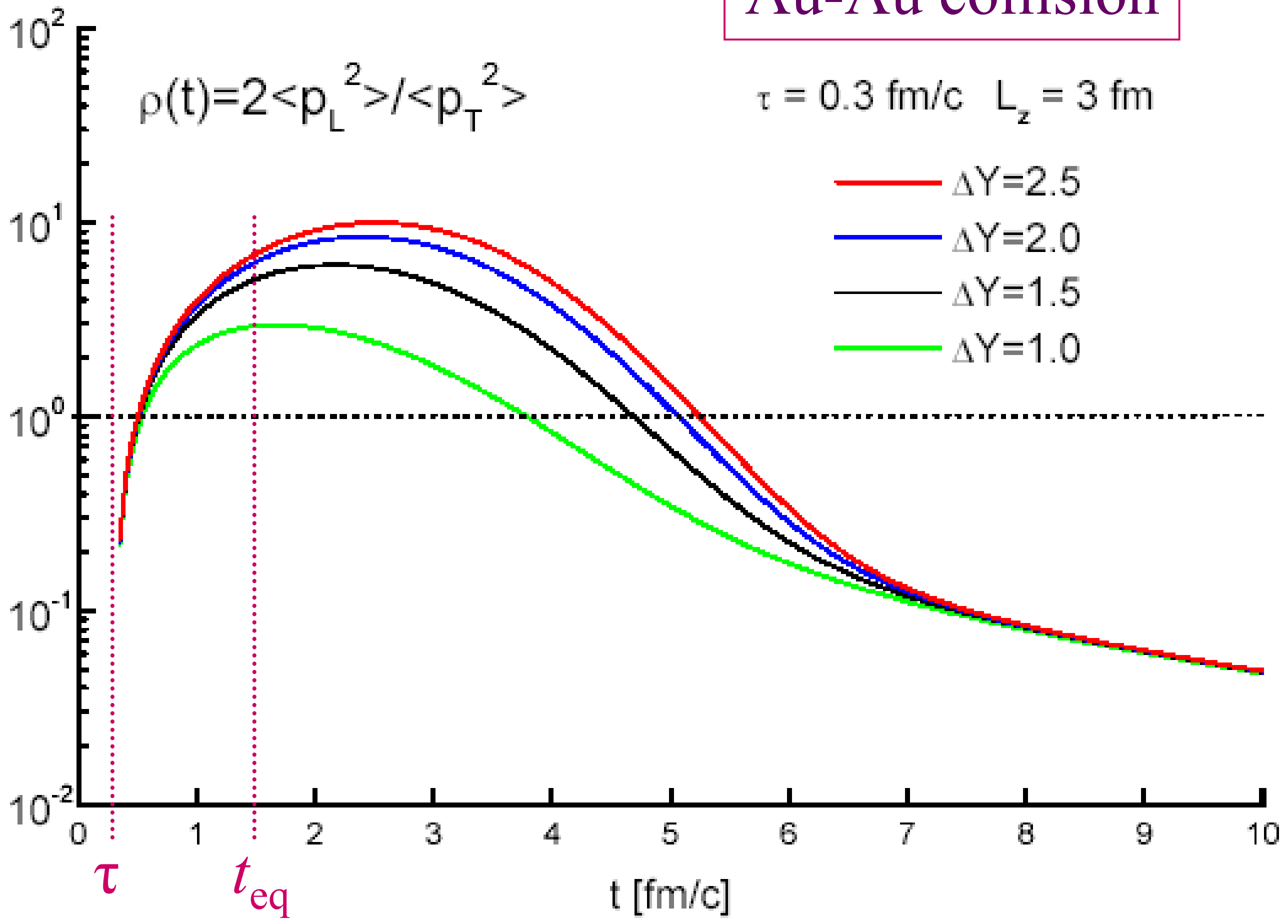
$t\sqrt{1-\mathrm{v}_z^2}$ - parton's proper time

$$-Y_{\max} < Y < Y_{\max}$$

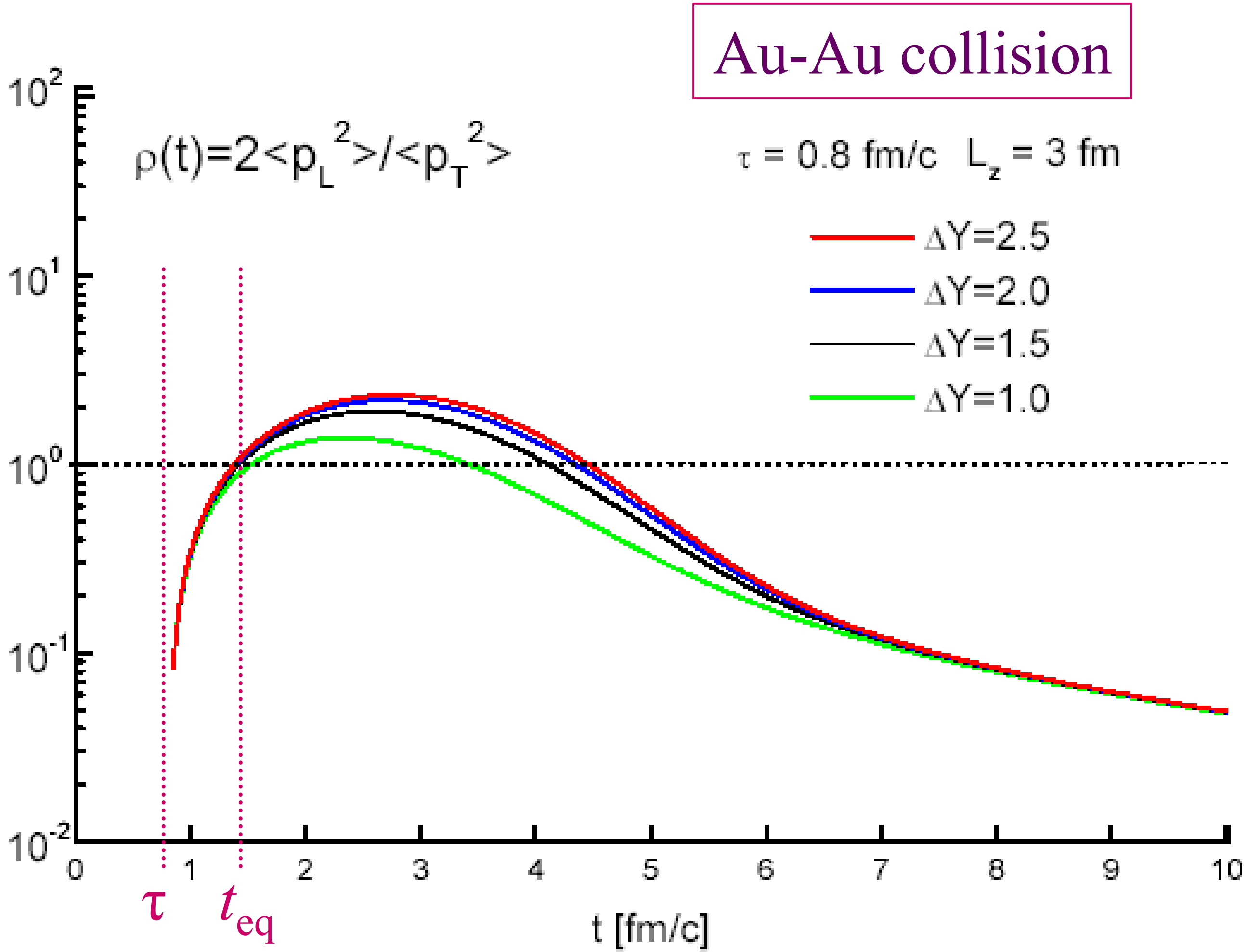
$$\operatorname{th} Y_{\max} = \frac{1}{t}\sqrt{t^2-\tau^2}$$

Effect of finite formation time cont.

Au-Au collision



Effect of finite formation time cont.



Conclusion

Prior to equilibrium the momentum distribution is prolate not oblate

Is the conclusion reliable?

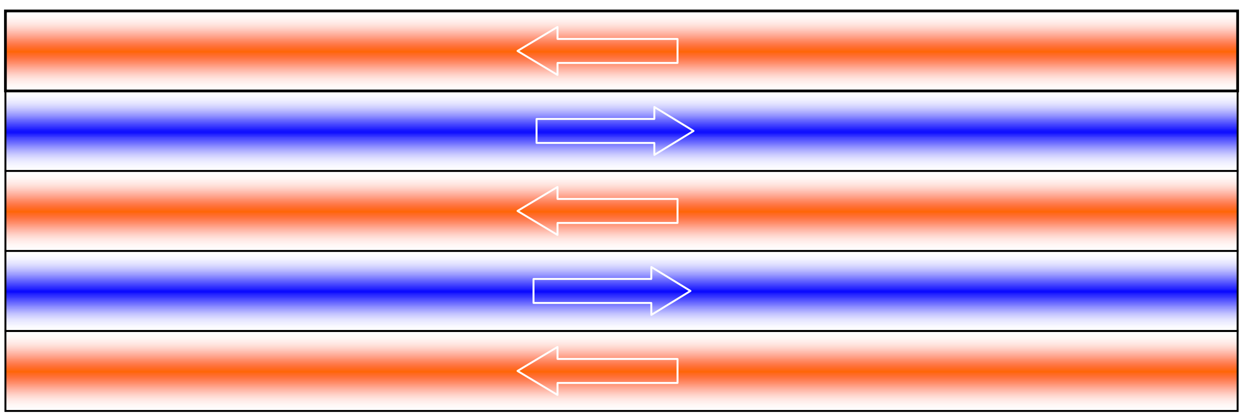
Why the momentum configuration is important?

Seeds of instability

$$\left\langle j_a^{\mu}(x) \right\rangle = 0$$ **but current fluctuations are finite**

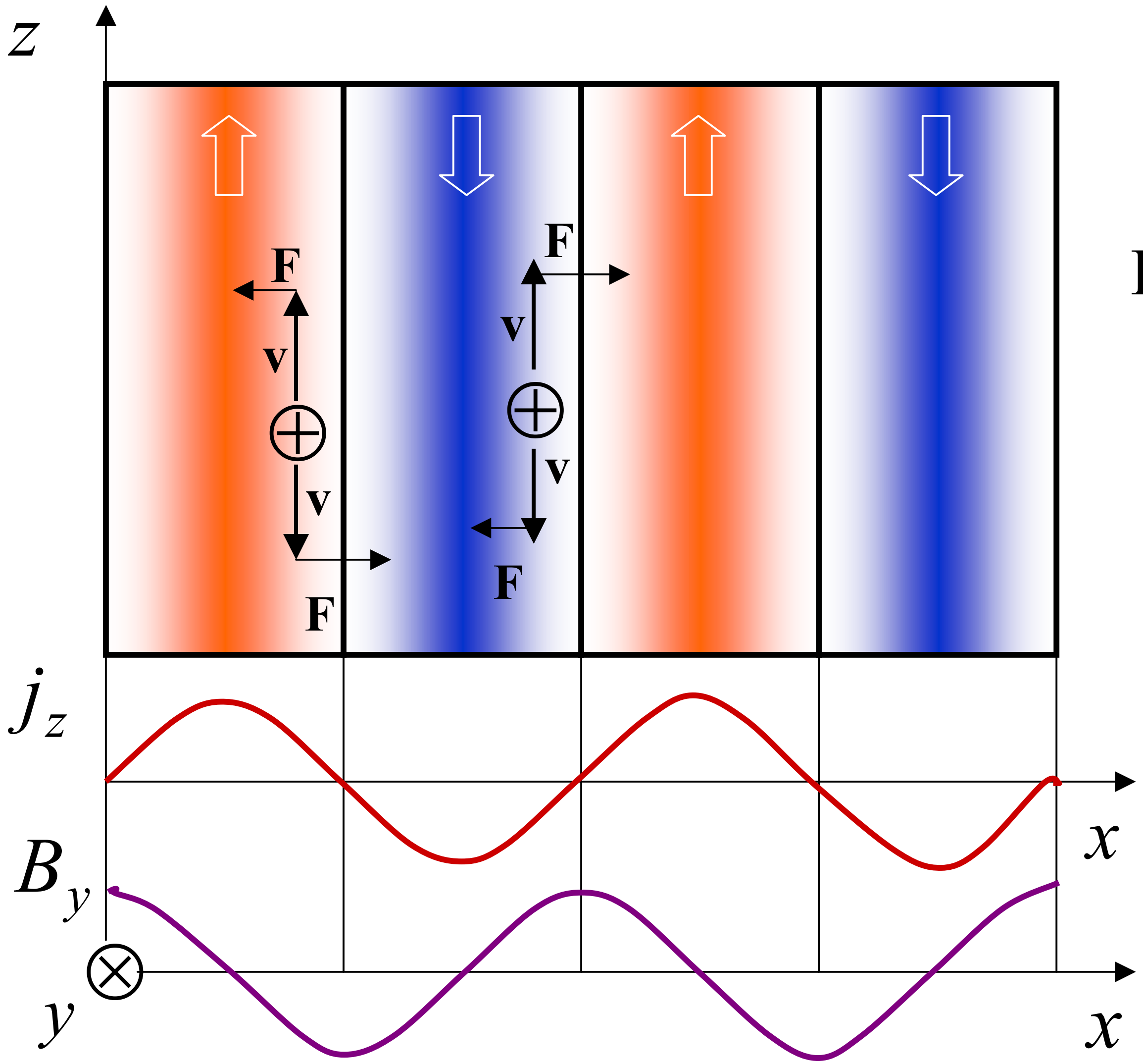
$$\left\langle j_a^{\mu}(x_1)\, j_b^{\nu}(x_2) \right\rangle = \frac{1}{8}\delta^{ab} \int \frac{d^3p}{(2\pi)^3} \frac{p^{\mu}p^{\nu}}{E_p^2} f(\mathbf{p}) \delta^{(3)}(\mathbf{x}-\mathbf{v}t) \neq 0$$

$$x_1 = (t_1, \mathbf{x}_1), \quad x_2 = (t_2, \mathbf{x}_2), \quad x = (t_1 - t_2, \mathbf{x}_1 - \mathbf{x}_2)$$



Direction of the momentum surplus

Mechanism of filamentation



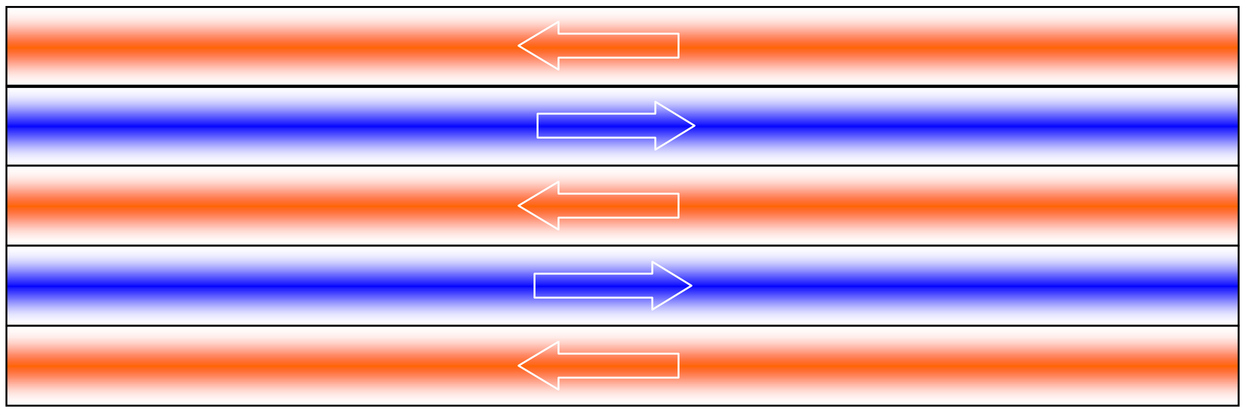
Lorentz force

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

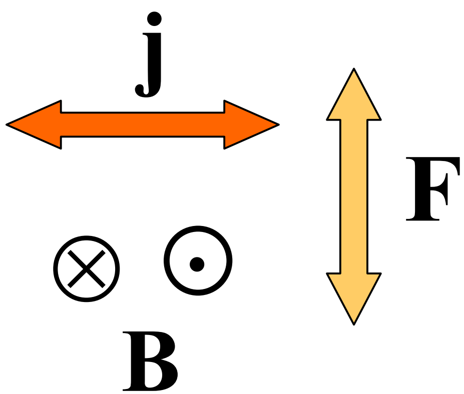
Ampere's law

$$\nabla \times \mathbf{B} = \mathbf{j}$$

Isotropization - particles

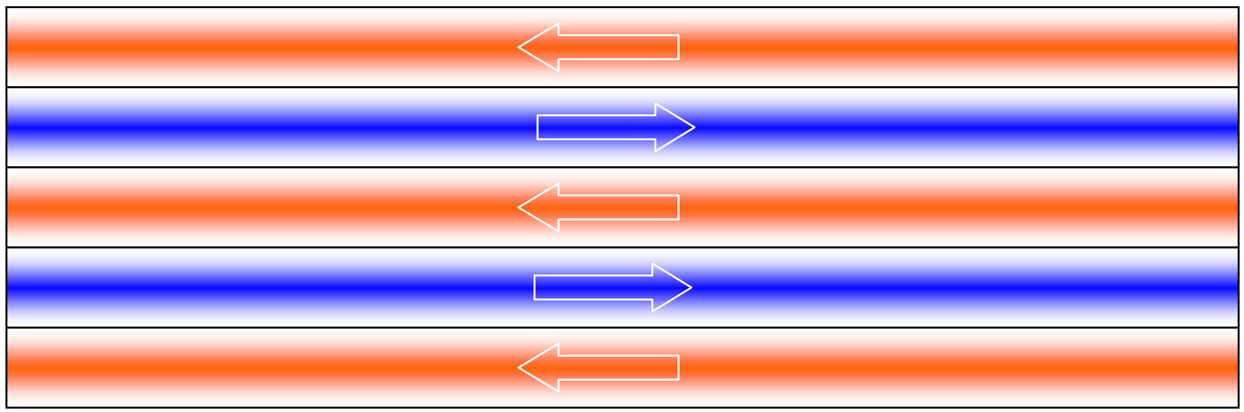


Direction of the momentum surplus

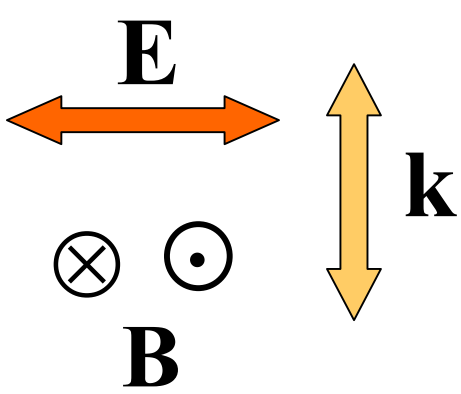


$$\Delta \mathbf{p} = \int dt\, \mathbf{F}$$

Isotropization - fields



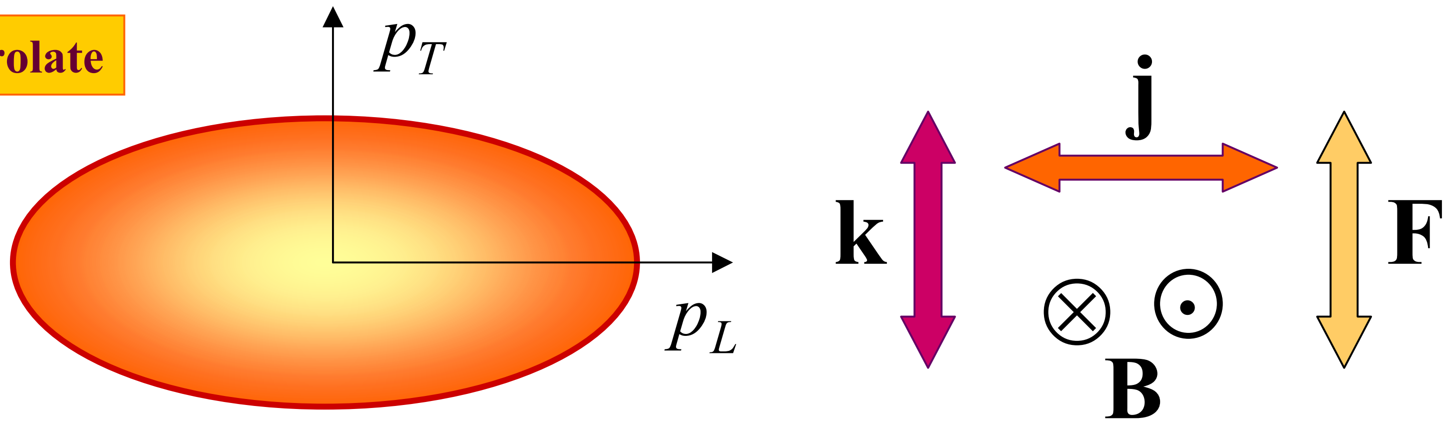
Direction of the momentum surplus



$$\mathbf{P}_{\text{fields}} \sim \mathbf{B}^{a} \times \mathbf{E}^{a} \sim \mathbf{k}$$

Prolate vs. oblate

prolate



oblate