What is ‘‘Elliptic Flow?’’

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(for the STAR collaboration)

Montreal

July, 2007

Agenda

- ***Azimuth autocorrelations***
- ***Nonflow and minijets***
- ***Quadrupole (flow) systematics***
- ***A-A eccentricity models***
- ***Universal quadrupole trends***
- ***Flow fluctuations***
- ***Quadrupole y_t dependence***
- ***What is elliptic flow?***

Autocorrelations and Power Spectra

$$\rho(\phi) = \sum_{i=1}^{n} r_i\, \delta(\phi - \phi_i) = \sum_{m=-\infty}^{\infty} \frac{\mathbf{Q}_m}{2\pi} \cdot \vec{u}(m\phi) \qquad \mathbf{Q}_m = \sum_{i=1}^{n} r_i\, \vec{u}(m\phi_i)$$

FT ⟶ ***RT*** ⟶

$$\rho_A(\phi_\Delta) \equiv \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi\; \rho(\phi)\rho(\phi + \phi_\Delta) = \frac{Q_0^2}{[2\pi]^2} + 2\sum_{m=1}^{\infty} \frac{Q_m^2}{[2\pi]^2} \cos(m\phi_\Delta)$$

FT ⟶

power spectrum: $Q_m^2 = n\left\langle r^2 \right\rangle + n(n-1)\left\langle r^2 \cos(m\phi_\Delta)\right\rangle$

Wiener-Khintchine theorem

RT ⟶

$V_m^2 = n(n-1)v_m^2$ ***signal***

azimuth autocorrelation:

$$\rho_A(\phi_\Delta) = \frac{n\left\langle r^2 \right\rangle}{2\pi} \delta(\phi_\Delta) + \frac{V_0^2}{[2\pi]^2} + 2\sum_{m=1}^{\infty} \frac{V_m^2}{[2\pi]^2} \cos(m\phi_\Delta)$$

arXiv:0704.1674 ***2D random walk*** ***correlations***

Conventional EP Flow Analysis

single-particle density:

$$\rho(\phi) = \rho_0 \left\{ 1 + 2\sum_{m=1}^{\infty} v_m \cos(m[\phi - \Psi_r]) \right\}$$

Ψ_r **:** true reaction plane

not observable

note

assumes only sinusoidal 'flow' components

1. Event-wise 'flow' vectors: $\mathbf{Q}_m = \sum_{i=1}^{n} r_i \, \vec{u}(m\phi_i)$
2. *Event-plane* angle from $\mathbf{Q}_m$: $\Psi_m = \frac{1}{m} \tan^{-1} \frac{Q_{my}}{Q_{mx}}$
3. Ensemble average: $v_m^{obs} = \overline{\langle \cos(m[\phi - \Psi_m]) \rangle}$
4. Correct for *event-plane resolution*: $v_m = v_m^{obs} / \overline{\cos(m(\Psi_m - \Psi_r))}$

v_m Relation to Azimuth Autocorrelation

azimuth autocorrelation

$$\rho_A(\phi_\Delta) = \frac{n}{2\pi}\delta(\phi_\Delta) + \frac{V_0^2}{[2\pi]^2} + 2\sum_{m=1}^{\infty}\frac{V_m^2}{[2\pi]^2}\cos(m\phi_\Delta)$$

assumes only ‘flow’ components

$$V_m^2 \equiv \sum_{j\neq i=1}^{n,n-1}\vec{u}(m\phi_i)\cdot\vec{u}(m\phi_j) = \bar{n}^2 v_m^2$$

v_2\{2\}

$$\mathbf{Q}'_{mi} = \sum_{j\neq i}^{n-1}\vec{u}(m\phi_j)$$

remove ‘autocorrelation’

$$= n\left\{\frac{1}{n}\sum_{i=1}^{n}\vec{u}(m\phi_i)\cdot\frac{\sum_{j\neq i}^{n-1}\vec{u}(m\phi_j)}{Q'_m}\right\}\frac{Q'_m}{V_m}V_m$$

$$\left\langle\cos(m[\phi-\Psi'_m])\right\rangle$$

(event-plane resolution)$^{-1}$

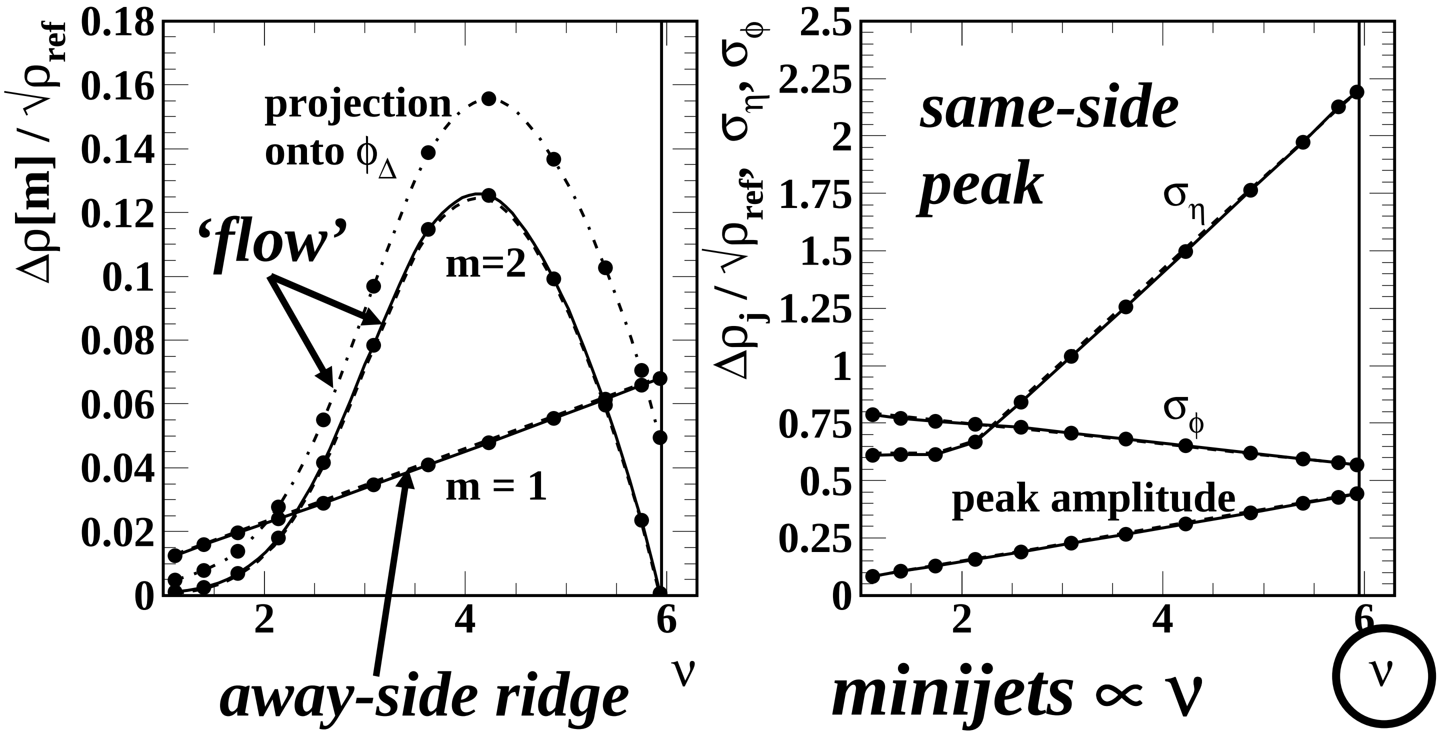
v_m^{obs} ***from standard method***

$$\approx n\sqrt{V_m^2}\left\{\frac{v_m^{obs}}{\overline{\overline{\cos(m[\Psi_m-\Psi_r])}}}\right\}$$

v_2\{EP\}

conventional flow analysis is a 1D autocorrelation in disguise

200 GeV Centrality Trends



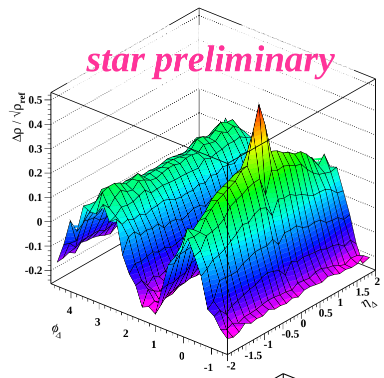
Pearson's covariance

$$\frac{\Delta\rho_A}{\sqrt{\rho_{ref}}}(\phi_\Delta) = \frac{\Delta\rho[0]}{\sqrt{\rho_{ref}}} + 2\sum_{m=1}^{\infty}\frac{\Delta\rho[m]}{\sqrt{\rho_{ref}}}\cos(m\phi_\Delta) + \cdots$$

subtract statistical reference

$$= \frac{\Delta\sigma^2_{n/}}{2\pi} + 2\sum_{m=1}^{\infty}\frac{V_m^2}{2\pi\,\bar{n}}\cos(m\phi_\Delta) + \textbf{NONFLOW}$$

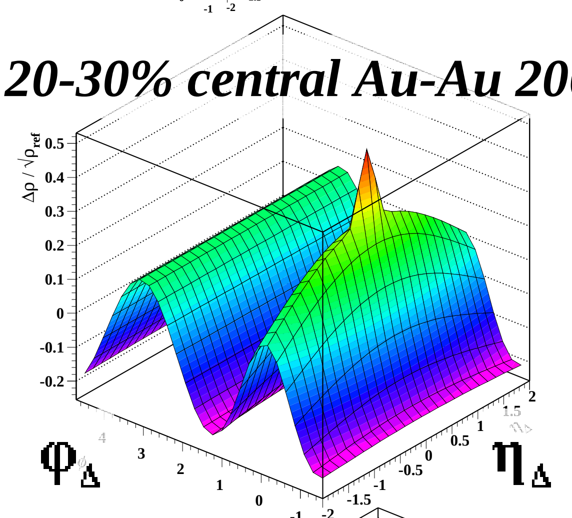
Modeling 2D Autocorrelations



Michael Daugherty data histograms

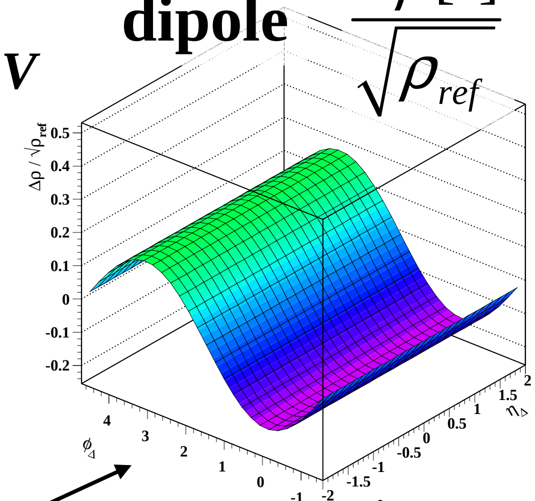
David Kettler model fits

20-30% central Au-Au 200 GeV



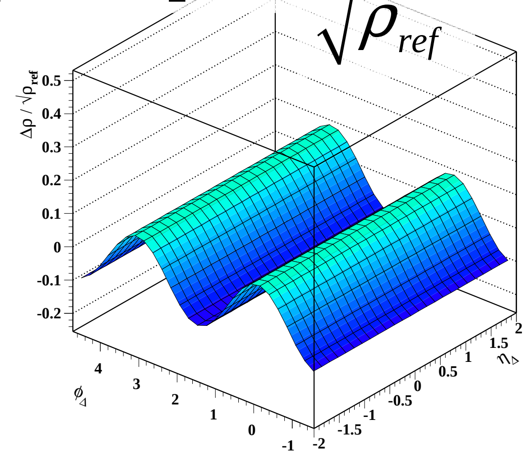
=

dipole $\frac{\Delta\rho[1]}{\sqrt{\rho_{ref}}}$



+

quadrupole $\frac{\Delta\rho[2]}{\sqrt{\rho_{ref}}}$

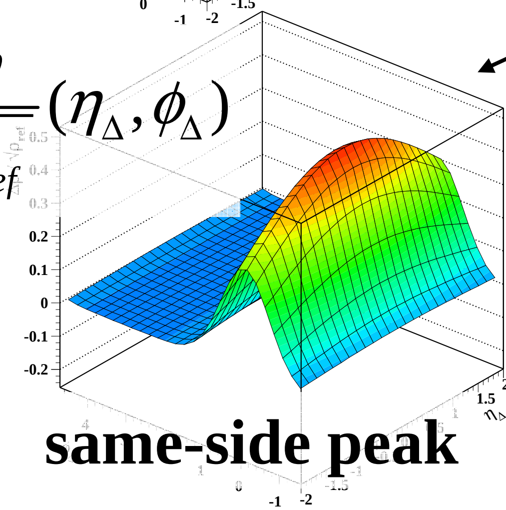


large

small

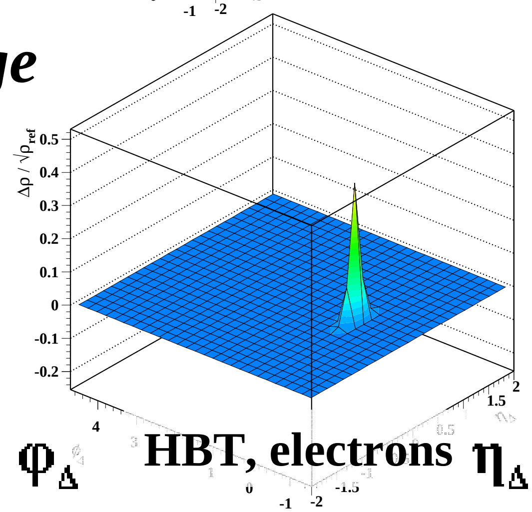
$\frac{\Delta\rho}{\sqrt{\rho_{ref}}}(\eta_\Delta,\phi_\Delta)$

+



same-side peak

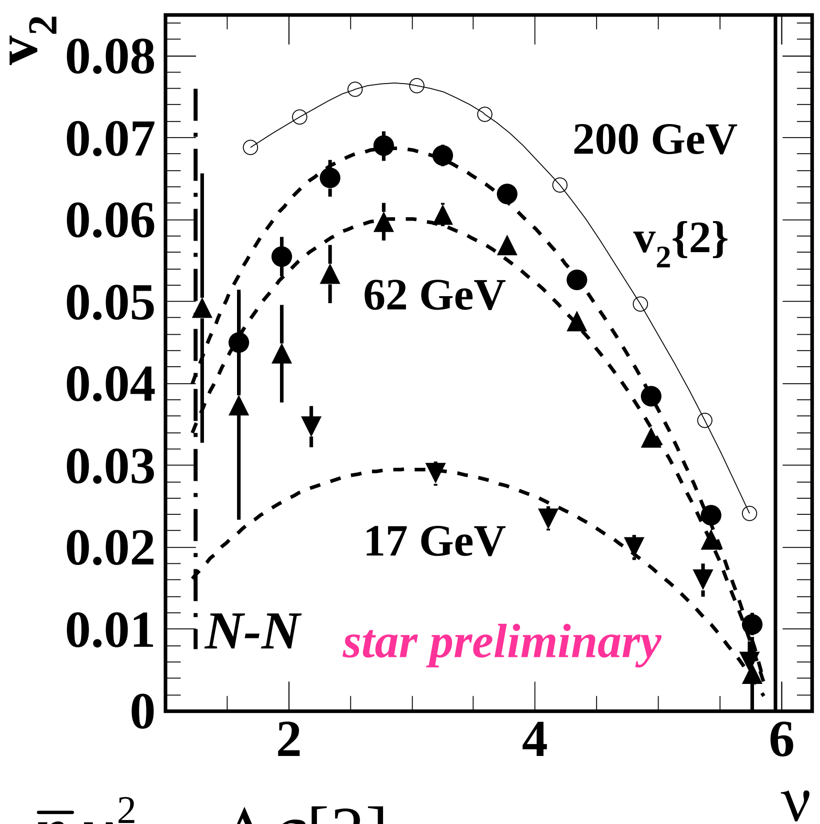
+



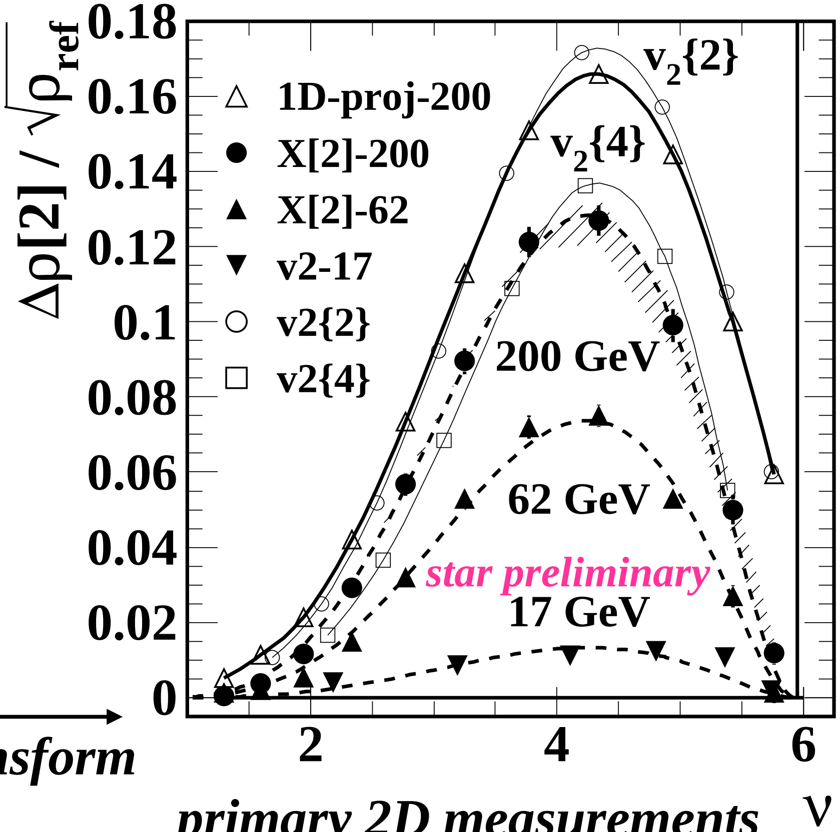
HBT, electrons

additional 1D Gaussian on η_Δ, negligible for central collisions

Quadrupole Centrality Systematics



transform



primary 2D measurements

$$\frac{\bar{n}\, v_2^2}{2\pi} \equiv \frac{\Delta\rho[2]}{\sqrt{\rho_{ref}}}$$

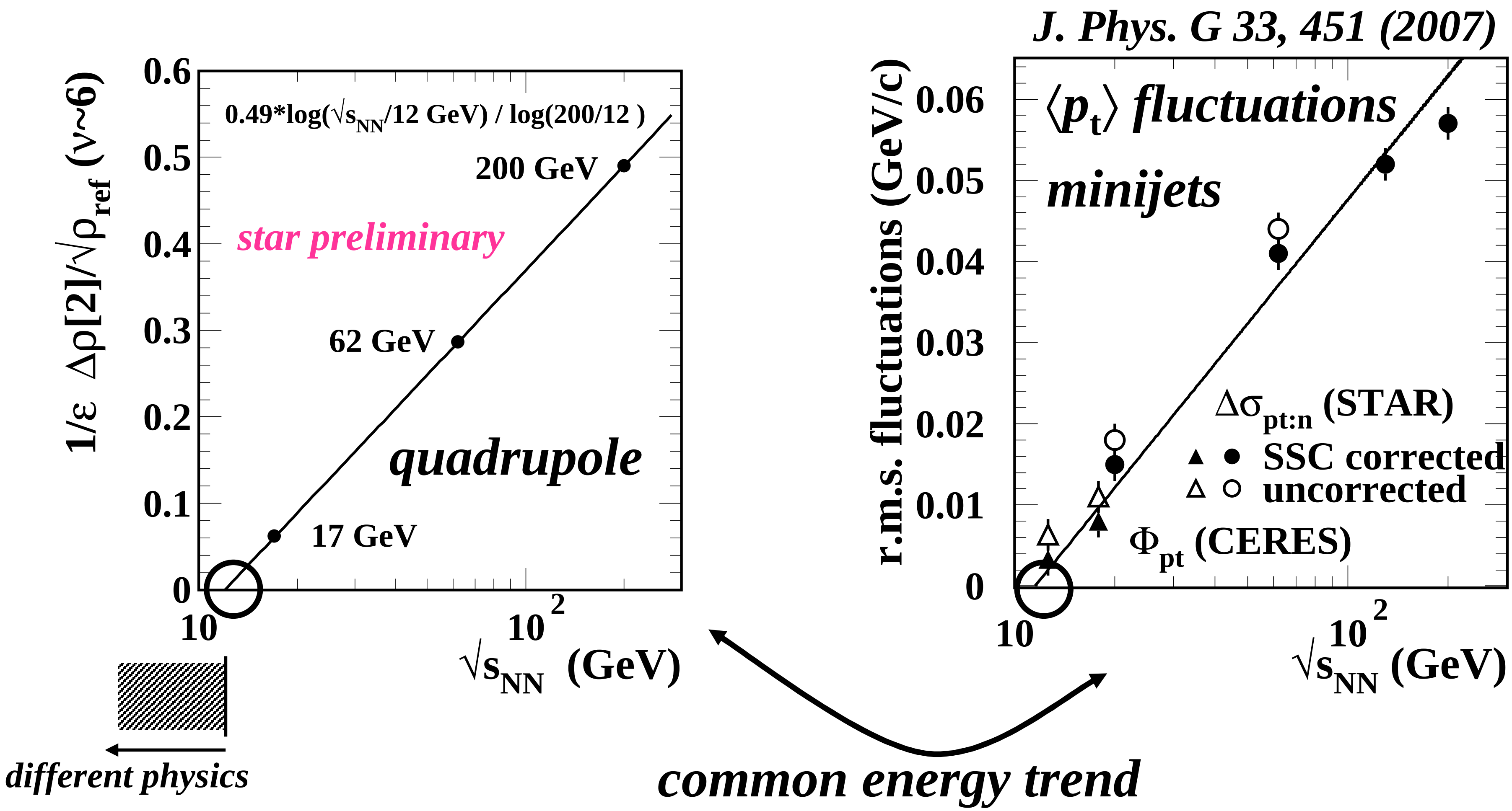
2D autocorrelation model fits

David Kettler

***dashed curves:** all have common shape –*

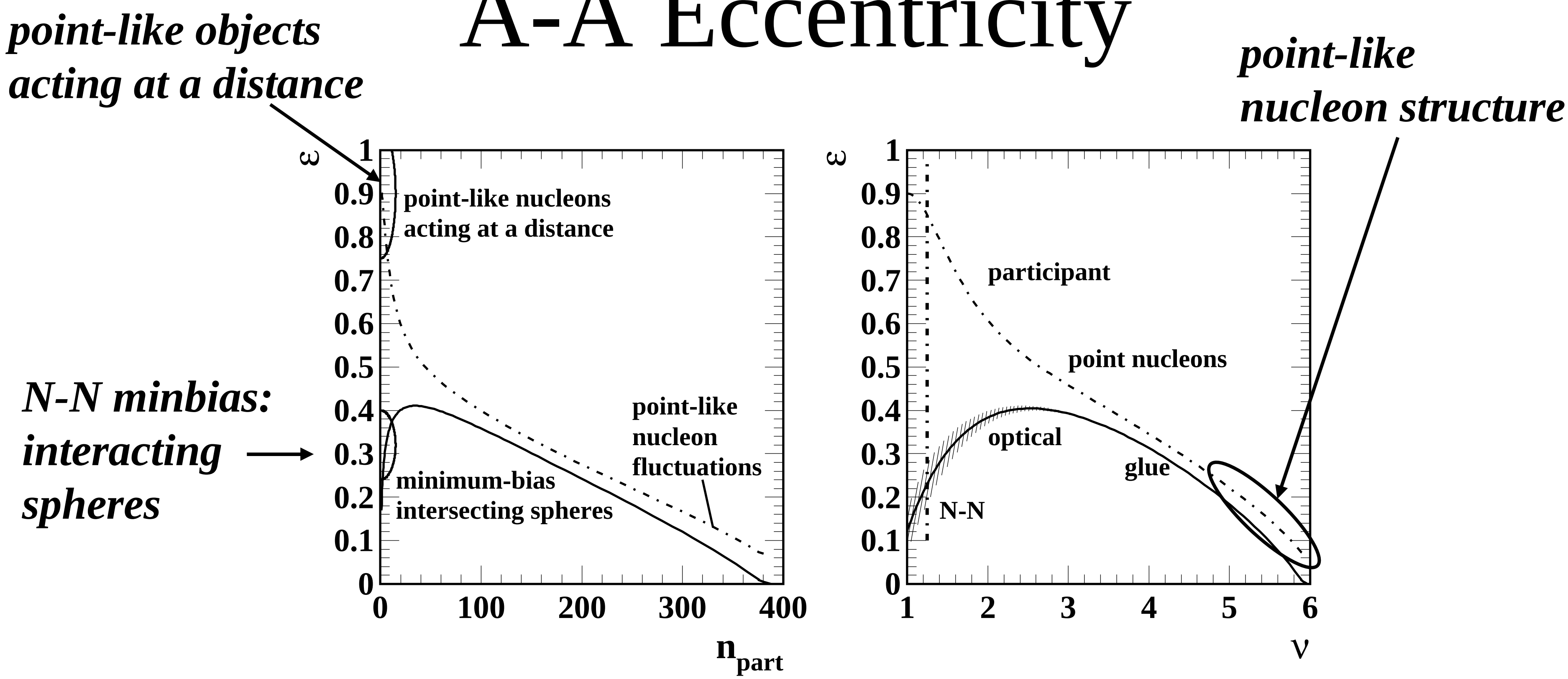
amplitudes follow linear dependence on $\log(\sqrt{s_{NN}}\,/12\ \text{GeV})$

Quadrupole Energy Systematics



suggests a common underlying mechanism

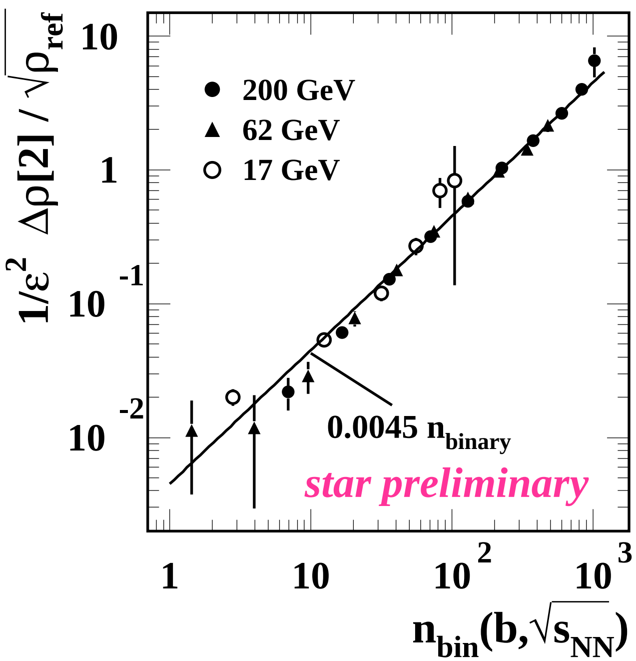
A-A Eccentricity

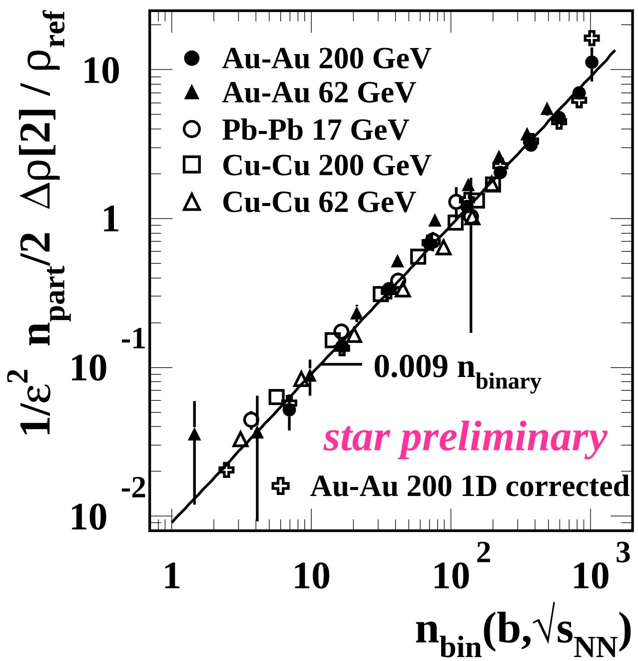


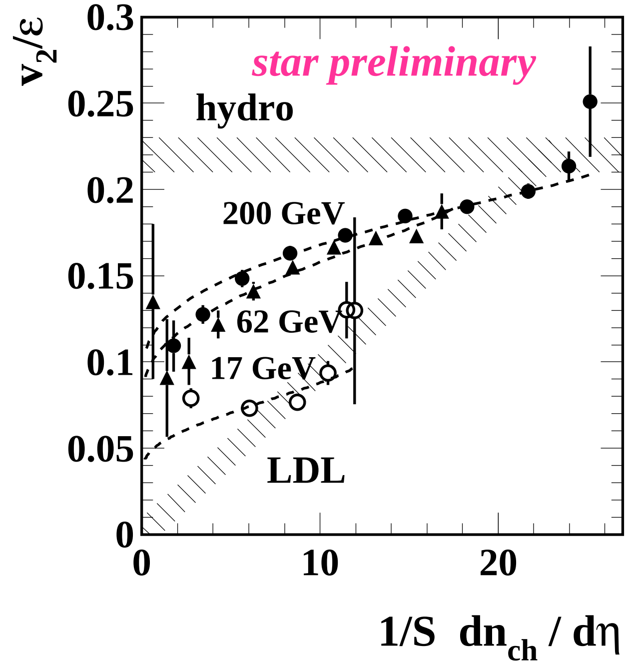
- *Minbias N-N interactions are not point-like objects acting at a distance*
- *The W-S distribution may better describe low-x glue*

we use the optical Glauber eccentricity

Universal Centrality and Energy Trends







universal trends represent all A-A systems for energies above 12 GeV

is this hydro-inspired format relevant to data?

quadrupole represented by initial conditions; no medium properties, EoS, viscosity, hydro

$v_2 \propto \varepsilon$ *does not describe data*

Quadrupole ν and $\sqrt{s_{NN}}$ Dependence

- ***Centrality dependence is universal***
- ***Energy dependence consistent with QCD***
- ***Optical eccentricity represents low-x glue***
- ***Combined trends reveal a universal relation***
- ***Ideal hydro $v_2 \propto \varepsilon$ not observed in data***
- ***What is 'elliptic flow' in N-N collisions?***

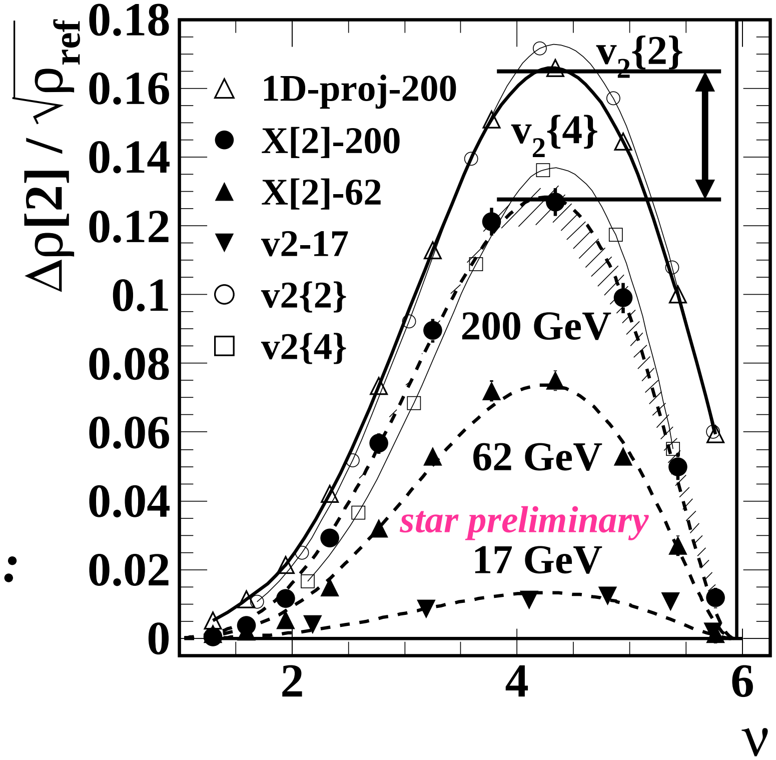
Flow Fluctuations – I: $v_2\{2\} - v_2\{4\}$

nucl-ex/0612021

$$v_2^4\{2\} - v_2^4\{4\} = \overline{v}_2^4 4r^2(1 + r^2/2)$$

$$r^2 \equiv \sigma_{v_2}^2 / \overline{v}_2^2$$

difference attributed to flow fluctuations



flow fluctuations are not required by these data

- ***$v_2\{2\}$ ~ <u>1D projection</u> of 2D autocorrelation: quadrupole + minijets***
- ***$v_2\{4\}$ ~ quadrupole only: no minijets***
- ***$v_2\{2\} - v_2\{4\}$ <u>entirely due to minijets</u>***

Flow Fluctuations – II: Flow Vector

does the 'flow-vector' distribution reveal flow fluctuations?

m = 2 <u>power</u> distribution:

$$q_2 \equiv Q_2 / \sqrt{n}$$

$$\frac{dn}{d\tilde{q}_2^2} \propto \int d\tilde{v}_2 \exp\left\{\frac{-(\tilde{q}_2 - \sqrt{n}\,\tilde{v}_2)^2}{1 + g_2(\nu, n)}\right\} \times \exp\left\{\frac{-(\tilde{v}_2 - \bar{v}_2)^2}{2\sigma_{v_2}^2}\right\}$$

simplify to 1D

assumed flow fluctuations

$$\propto \exp\left\{-\frac{\tilde{q}_2^2}{1 + g_2(\nu, n) + 2n\,\sigma_{v_2}^2}\right\}$$

assume mean v_2 = 0

inferred variance

$$g_2 / 2\pi \approx \Delta\rho[2] / \sqrt{\rho_{ref}}$$ ***minijets only***

measured with 2D autocorrelations

→ Paul Sorensen's talk

- ***change of variance with random track discard assumed to reveal flow fluctuations***
- ***but, g_2 (minijets) varies linearly with n <u>for random discard</u>***
- ***FF – I implies width variation is dominated by minijets***

Flow Fluctuations – III: Eccentricity

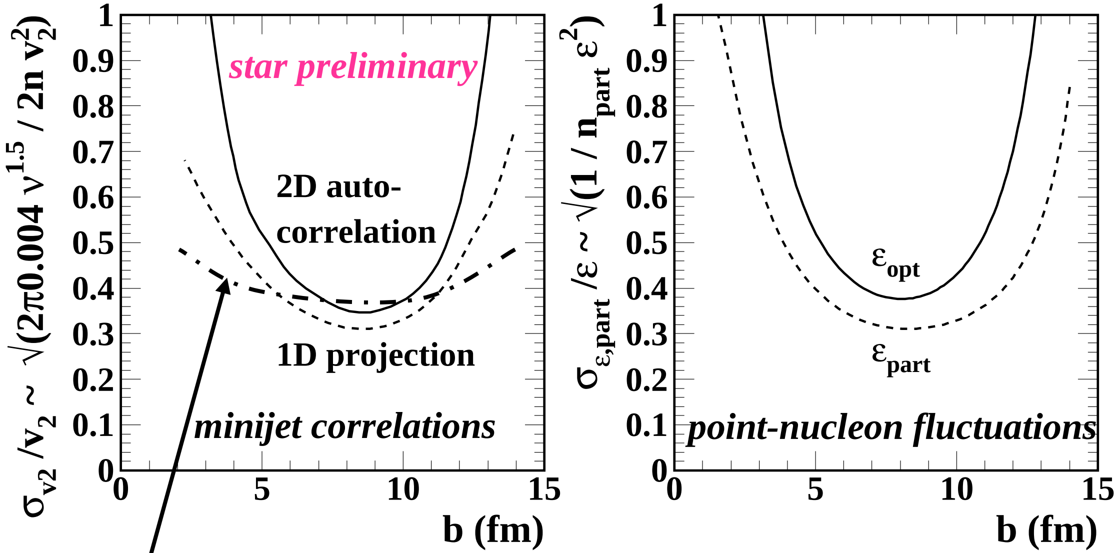
for 200 GeV minijets fitted with cos(2ϕ_Δ):

$$g_2 / 2\pi = 0.004\nu^{1.5}$$

(to a few percent)

we also observe:

$$\Delta\rho[2]/\sqrt{\rho_{ref}} \approx 0.0045\, n_{bin}\, \varepsilon^2$$



compare nucl-ex/0612021

mis-taking $"\sigma^2_{v_2}" \approx g_2/2n$ ***implies***

$$\frac{"\sigma^2_{v_2}"}{v_2^2} = \frac{2\pi\, 0.004\, \nu^{1.5}}{2n\, v_2^2} = \frac{0.004\, \nu^{1.5}}{2\Delta\rho[2]/\sqrt{\rho_{ref}}}$$

$$\sim \frac{\nu^{1.5}}{2n_{bin}\, \varepsilon^2} \sim \nu^{0.5} \frac{1}{n_{part}\, \varepsilon^2}$$

but $1/n_{part} \sim \sigma^2_{\varepsilon,part}$

thus, $\dfrac{"\sigma^2_{v_2}"}{v_2^2} \sim \dfrac{\sigma^2_{\varepsilon,part}}{\varepsilon^2}$

- ***seems to confirm $v_2 \propto \varepsilon$***
- ***true fluctuations may be small***

What Do We Learn From $v_2(p_t)$?

Cooper-Frye Formalism:

$$\rho(m_t) \equiv dn/dm_t^2 \propto \exp(-[m_t - m_0]/T)$$

$$= \exp(-m_0[\cosh(y_t) - 1]/T)$$ ***black-body radiation***

$$\rho(m_t, \beta_t) \to \exp(-[p_\mu u^\mu - m_0]/T)$$ ***Cooper-Frye expression***

$$= \exp(-[\gamma_t\{m_t - \beta_t p_t\} - m_0]/T)$$

$$= \exp(-m_0[\gamma_t\{\cosh(y_t) - \beta_t \sinh(y_t)\} - 1]/T)$$

$$= \exp(-m_0[\cosh(y_t - \Delta y_t) - 1]/T)$$

$$\tanh(\Delta y_t) = \beta_t$$ ***source velocity***

boost

black-body radiation from a boosted source

relativistic transformations simple on y_t

Quadrupole y_t Dependence and Boost

$$\beta_t(\phi) = \beta_t[0] + \beta_t[2]\cos(2[\phi - \Psi_r])$$

$$\rho[2] = \exp\left\{-\left[\gamma_t\left(m_t - \beta_t(\phi)\,p_t\right) - m_0\right]/T\right\}$$

quadrupole component

$$\frac{2}{2\pi}\int d\phi\,\rho[2](m_t;\beta_t(\phi))\cdot\cos(2[\phi - \Psi_r])$$

$$\rightarrow \frac{V_2}{2\pi} \approx \rho[2]_0(m_t;\beta_t[0])\bullet\frac{\gamma_t\beta_t[2]p_t}{T}$$

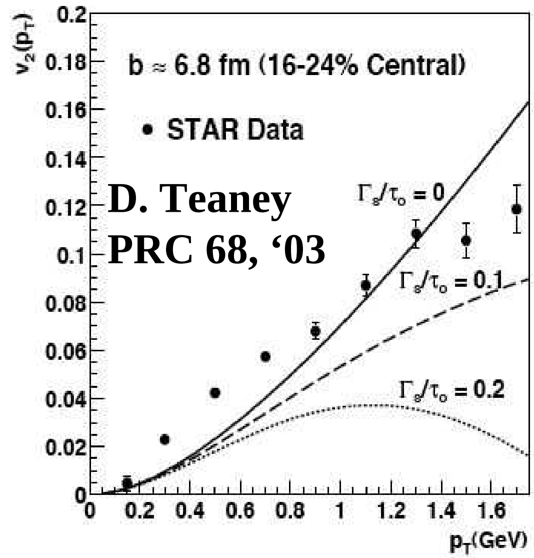
Fourier amplitude

$$\frac{v_2}{p_t} \approx \frac{\rho[2]_0(m_t;\beta_t[0])}{\rho[2]_0(m_t;\beta_t[0]) + \rho_{nonflow}}\bullet\frac{\gamma_t\beta_t[2]}{T}$$

$\rho_{\mathbf{total}}$

$$\frac{V_2}{2\pi p_t} \approx \rho_{total}\frac{v_2}{p_t} = \rho[2]_0(m_t;\beta_t[0])\bullet\frac{\gamma_t\beta_t[2]}{T}$$

boosted M-B spectrum



Quadrupole y_t Dependence and Boost

minimum-bias event sample

$$\beta_t(\phi) = \beta_t[0] + \beta_t[2]\cos(2[\phi - \Psi_r])$$

$$\rho[2] = \exp\left\{-\left[\gamma_t\left(m_t - \beta_t(\phi)\, p_t\right) - m_0\right]/T\right\}$$

quadrupole component

$$\frac{2}{2\pi}\int d\phi\, \rho[2](m_t;\beta_t(\phi))\cdot\cos(2[\phi - \Psi_r])$$

$$\rightarrow \frac{V_2}{2\pi} \approx \rho[2]_0(m_t;\beta_t[0]) \bullet \frac{\gamma_t \beta_t[2] p_t}{T}$$

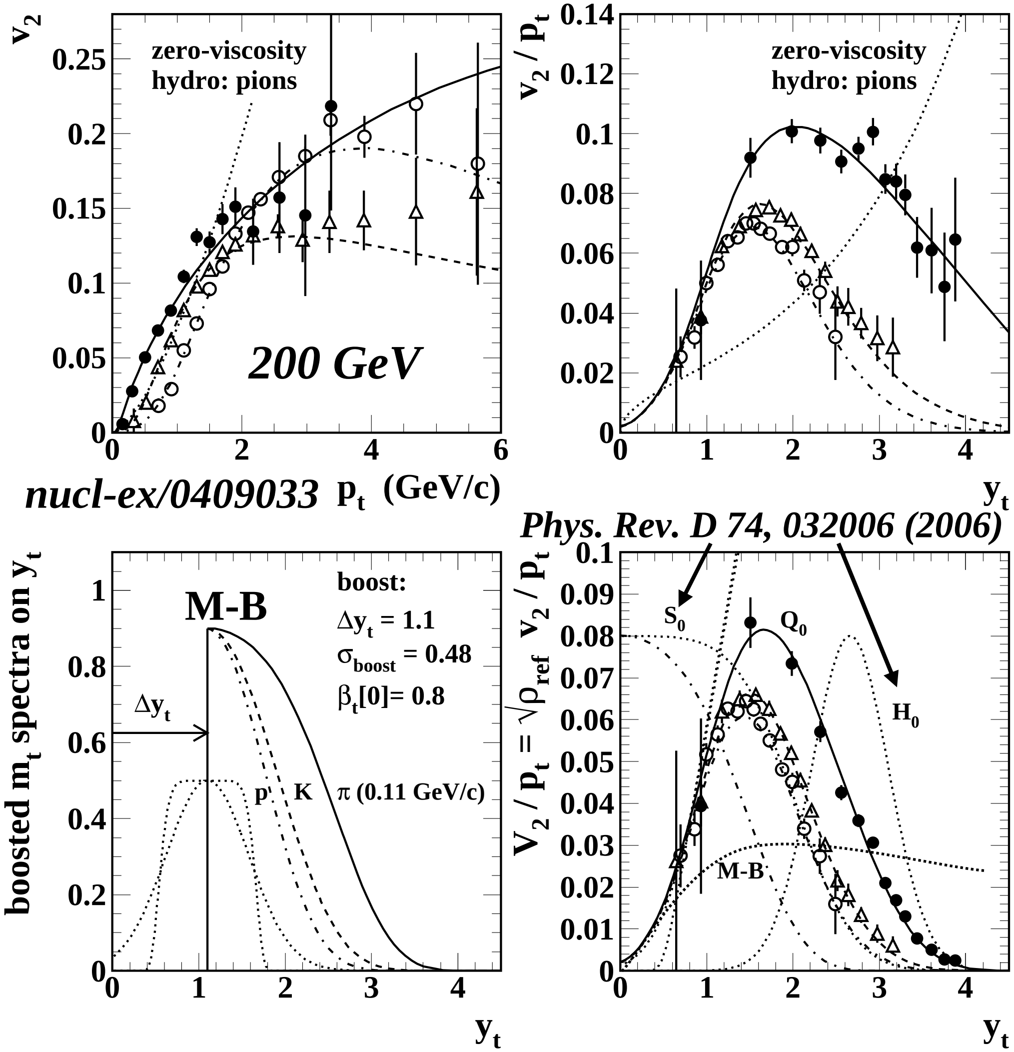
Fourier amplitude

$$\frac{v_2}{p_t} \approx \frac{\rho[2]_0(m_t;\beta_t[0])}{\rho[2]_0(m_t;\beta_t[0]) + \rho_{nonflow}} \bullet \frac{\gamma_t \beta_t[2]}{T}$$

$\boldsymbol{\rho_{\mathbf{total}}}$

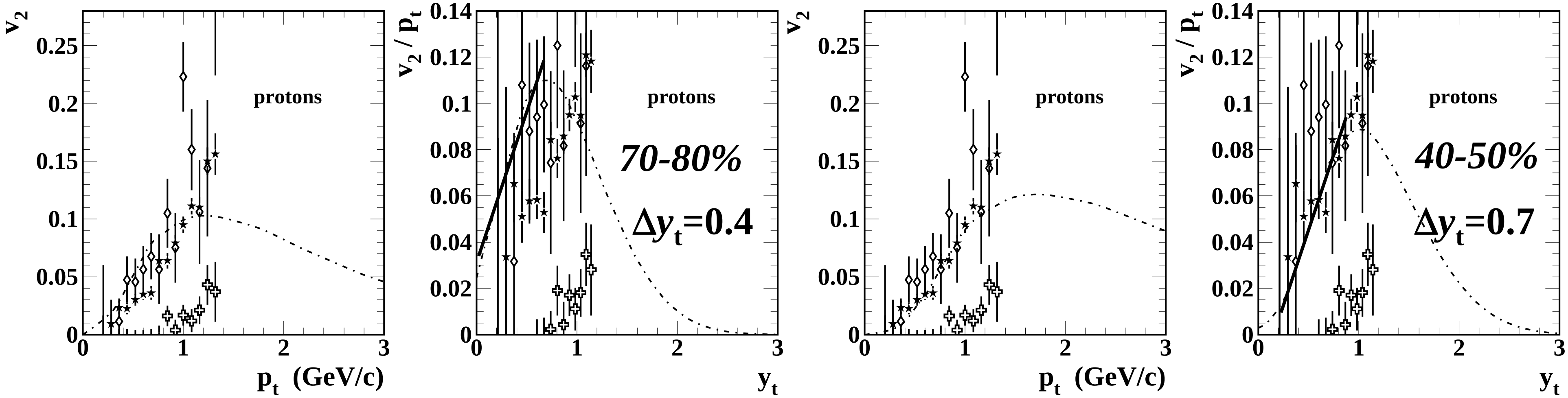
$$\frac{V_2}{2\pi\, p_t} \approx \rho_{total}\, \frac{v_2}{p_t} = \rho[2]_0(m_t;\beta_t[0]) \bullet \frac{\gamma_t \beta_t[2]}{T}$$

boosted M-B spectrum

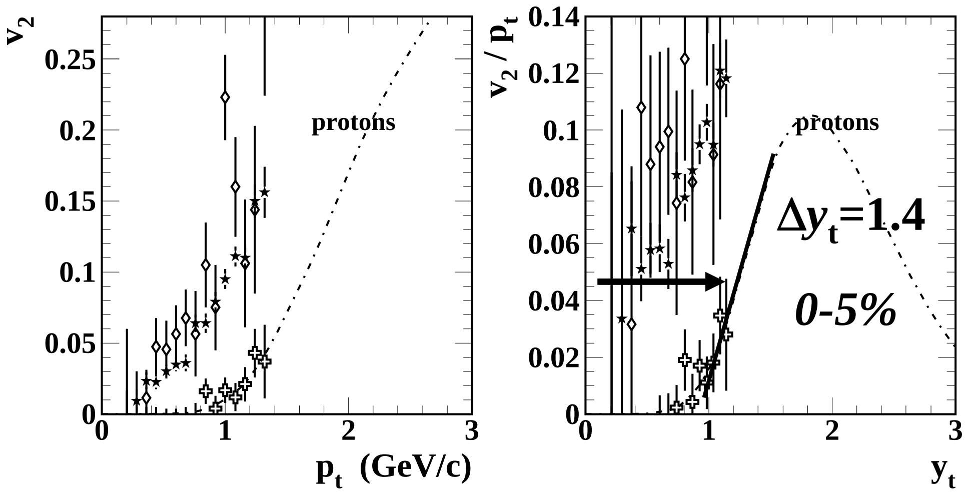


boosted source simply described

Boost Centrality Dependence



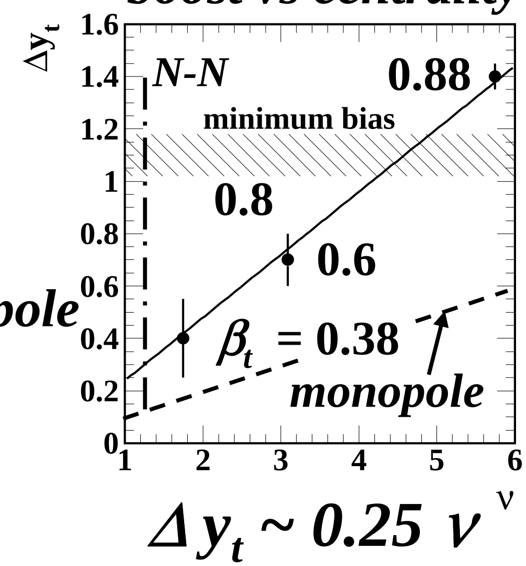
Phys. Rev. C 72, 014904 (2005)



a <u>small fraction</u> of emitted hadrons carries the quadrupole ~ 5-10%?

$\sigma_{boost} \leq 0.3$

boost vs centrality



$\Delta y_t \sim 0.25\ \nu$

quadrupole hadrons emerge from a rapidly expanding cylinder

Quadrupole y_t Dependence

- ***Hydro predictions apply to the numerator of v_2, not the denominator***
- ***Can hydro predict boosts for different multipoles?***
- ***Is a hydro description required? … permitted?***
- ***Is ‘elliptic flow’ actually glue-glue scattering?***

Summary

- ***2D angular autocorrelations separate 'flow' (quadrupole) from 'nonflow' (minijets)***
- ***Quadrupole trends depend only on initial parton conditions, not on system evolution or EoS***
- ***Accurate data inconsistent with hydro expectation $v_2 \propto \varepsilon$***
- ***Optical Glauber eccentricity better models low-x glue***
- ***Claimed 'flow fluctuations' are a manifestation of minijets***
- ***Quadrupole y_t dependence reveals a boost distribution***
- ***Radial boost and quadrupole are present in N-N collisions***
- ***'Elliptic flow' may represent a novel manifestation of QCD***