Maxwell's equations

VIII. *A Dynamical Theory of the Electromagnetic Field.* By J. CLERK MAXWELL, *F.R.S.*

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PART I.—INTRODUCTORY.

(1) THE most obvious mechanical phenomenon in electrical and magnetical experiments is the mutual action by which bodies in certain states set each other in motion while still at a sensible distance from each other. The first step, therefore, in reducing these phenomena into scientific form, is to ascertain the magnitude and direction of the force acting between the bodies, and when it is found that this force depends in a certain way upon the relative position of the bodies and on their electric or magnetic condition, it seems at first sight natural to explain the facts by assuming the existence of something either at rest or in motion in each body, constituting its electric or magnetic state, ... distance according to mathematical laws.

For Electromagnetic Momentum		F	G	H
,,	Magnetic Intensity	α	β	γ
,,	Electromotive Force	P	Q	R
,,	Current due to true conduction	p	q	r
,,	Electric Displacement	f	g	h
,,	Total Current (including variation of displacement)	p'	q'	r'
,,	Quantity of free Electricity	e		
,,	Electric Potential	Ψ		

ween these twenty quantities we have found twenty equations, viz.

Three equations of	Magnetic Force	(B)
,,	Electric Currents	(C)
,,	Electromotive Force	(D)
,,	Electric Elasticity	(E)
,,	Electric Resistance	(F)
,,	Total Currents	(A)
One equation of	Free Electricity	(G)
,,	Continuity	(H)

These equations are therefore sufficient to determine all the quantities which occur in them, provided we know the conditions of the problem. In many questions, however, only a few of the equations are required.

The equations

The variations of the electrical displacement must be added to the currents p, q, r to get the total motion of electricity, which we may call p', q', r', so that

$$\left.\begin{aligned} p' &= p + \frac{df}{dt}, \\ q' &= q + \frac{dg}{dt}, \\ r' &= r + \frac{dh}{dt}, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad \text{(A)}$$

Equations of Magnetic Force.

$$\left.\begin{aligned} \mu\alpha &= \frac{dH}{dy} - \frac{dG}{dz}, \\ \mu\beta &= \frac{dF}{dz} - \frac{dH}{dx}, \\ \mu\gamma &= \frac{dG}{dx} - \frac{dF}{dy}. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad \text{(B)}$$

Similarly,

$$\left.\begin{aligned}\frac{d\gamma}{dy}-\frac{d\beta}{dz}&=4\pi p'.\\ \frac{d\alpha}{dz}-\frac{d\gamma}{dx}&=4\pi q',\\ \frac{d\beta}{dx}-\frac{d\alpha}{dy}&=4\pi r'.\end{aligned}\right\} \quad \text{(C)}$$

We may call these the Equations of Currents.

Equations of Electromotive Force.

$$\left.\begin{aligned}P&=\mu\left(\gamma\frac{dy}{dt}-\beta\frac{dz}{dt}\right)-\frac{dF}{dt}-\frac{d\Psi}{dx},\\ Q&=\mu\left(\alpha\frac{dz}{dt}-\gamma\frac{dx}{dt}\right)-\frac{dG}{dt}-\frac{d\Psi}{dy},\\ R&=\mu\left(\beta\frac{dx}{dt}-\alpha\frac{dy}{dt}\right)-\frac{dH}{dt}-\frac{d\Psi}{dz}.\end{aligned}\right\} \quad \text{(D)}$$

Equations of Electric Elasticity,

$$\left.\begin{array}{l} \mathrm{P}=kf, \\ \mathrm{Q}=kg, \\ \mathrm{R}=kh. \end{array}\right\} \quad \ldots \ldots \quad \text{(E)}$$

Equations of Electric Resistance,

$$\left.\begin{array}{l} \mathrm{P}=-\varrho p, \\ \mathrm{Q}=-\varrho q, \\ \mathrm{R}=-\varrho r. \end{array}\right\} \quad \ldots \ldots \quad \text{(F)}$$

Equation of Free Electricity,

$$e+\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}=0. \quad \ldots \ldots \quad \text{(G)}$$

Equation of Continuity,

$$\frac{de}{dt}+\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}=0. \quad \ldots \ldots \quad \text{(H)}$$

Modern form of Maxwell's equations

(Heaviside 1884)

$$\nabla \cdot \underline{E} = \frac{\rho}{\varepsilon_0}$$

Gauss's law

$$\nabla \cdot \underline{B} = 0$$

No magnetic monopoles

$$\nabla \times \underline{B} = \mu_0 \underline{J} + \mu_0 \varepsilon_0 \frac{\partial \underline{E}}{\partial t}$$

Ampere's law

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

Faraday's law