

## Electricity and Magnetism, PHYS 340 Problem set 6

1. Consider a volume current in a slab infinite in the  $x, y$  directions and that varies in the  $z$  direction and points in the  $x$  direction:

$$\underline{J}(x, y, z) = \begin{cases} \hat{x} \left( \frac{j_0 |z|}{a} \right); & -a \leq z \leq a \\ 0 & \text{otherwise} \end{cases}$$

- a) What is  $\underline{B}$  inside the slab, and above and below it?  
b) Sketch a plot of  $B_y(z)$ .
- 2.** Problem 6.1 (6.1).  
**6.** Problem 6.7 (6.7).  
**7.** Problem 6.9 (6.9).  
**8.** Problem 6.15 (6.15). *Optional (Laplacian in spherical geometry).*  
**9.** Problem 6.17 (6.17).  
**10.** Problem 7.8 (7.8).  
**11.** Problem 7.15 (7.15).  
**12.** A magnetohydrodynamic (MHD) generator is a device that has been proposed for generating power from flow of ionized plasma, e.g., in nuclear fusion reactors. The plasma flows in the  $z$  direction through a rectangular pipe, whose cross section is parallel to the  $x$ - $y$  plane, and there is a magnetic field  $\underline{B} = B\hat{x}$  in the plasma ( $\hat{x}$  is the unit vector in the  $x$  direction). The  $x, y, z$  dimensions of the pipe are  $w, h, l$  respectively. The walls at  $x = \pm w/2$  are insulating, and the walls at  $y = \pm h/2$  are conducting.
- a) Show that the potential between the conducting walls is  $V = vBh$ , where  $v$  is the fluid velocity.  
b) Suppose the conducting walls are connected by a wire with resistance  $R$ . Determine the current in the wire, if  $\rho$  is the resistivity of the plasma.  
(Hint: There are currents in series in the wire and in the plasma.)

### Supplementary problems F:

- F1.** Problem 5.24 (5.23).  
**F2.** Problem 5.41 (5.39).  
**F3.** Problem 6.8 (6.8).  
**F4.** Problem 6.25 (6.23).  
**F5.** Problem (7.16).  
**F6.** Problem 7.18 (7.18).

(Numbers from Griffiths book, 4<sup>th</sup> edition; in parentheses, to the 3<sup>rd</sup> edition)

