Preface

Few would argue that quantum and statistical mechanics do not apply to ordinary fluid flows, yet the latter involve such huge numbers of particles that these fundamental theories are rarely useful for solving practical – or even theoretical – fluid problems. Instead, one typically exploits the fact that when the number of particles is large enough, new continuum properties and notions such as fluid particle, fluid temperature and fluid velocity emerge which are governed by the higher-level equations of continuum mechanics and thermodynamics. While the latter can perhaps be obtained from the former, the derivations are not mathematically rigorous and the foundations of continuum mechanics are still actively researched. The continuum laws that emerge are indeed qualitatively new.

In a similar way, one expects new higher-level laws to emerge from the chaos of sufficiently strong hydrodynamic turbulence. While the latter presumably continue to obey the laws of continuum mechanics, their direct application is impractical and one searches for the emergence of new, even higher-level laws. Indeed "developed turbulence as a new macroscopic state of matter" (Manneville, 2010), appearing at high Reynolds number (*Re*), has been considered as a form of matter with properties that cannot simply be reduced to – nor simply deduced from – the governing Navier–Stokes equations. Consequently, it is not surprising that over the years new types of models and new symmetry principles have been developed in order to directly study, model and understand this hypothetical emergent state. Of particular relevance to this book are cascade models and (anisotropic) scale invariance symmetries.

The study of fully developed turbulence remains largely academic and has only had a rather peripheral impact on atmospheric science. This is ironic, since the atmosphere provides an unrivalled strongly nonlinear natural laboratory with the ratio of nonlinear to linear terms – given by the Reynolds number – that is typically $\approx 10^{12}$. Although the atmosphere certainly differs from incompressible hydrodynamics in several important ways, we may nevertheless expect higher-level laws to emerge. Furthermore, it is reasonable to expect that they will share at least some of the features of fully developed turbulence. This was indeed the belief of many of the pioneers of classical turbulence: L. F. Richardson, A. N. Kolmogorov, A. Obukhov, S. Corrsin and R. Bolgiano.

While the pioneers' eponymous laws were in many ways highly successful, when applied to the atmosphere they faced two basic obstacles: the atmosphere's extreme intermittency and its strong stratification, which increases systematically at larger and larger scales. As usual in physics, when one faces such a situation there are two choices. Either one abandons the old law and moves on to something different, or else one generalizes the law so that it is able to fully fit the facts. On several occasions during its development, the law of conservation of energy was faced with such a choice: either treat it as no more than a (sometimes) poor approximation, or else extend the notion of energy beyond mechanical energy to heat energy, to chemical energy, to electrical energy and eventually to mass energy. In this book we follow the latter choice with respect to the classical laws of turbulence: we argue that these obstacles of stratification and intermittency can be overcome with appropriate generalizations. For weather, the key generalizations are from isotropic to anisotropic notions of scale and from smooth, quasi-Gaussian variability to strong, cascade-generated multifractal intermittency. Together, this leads to a model of atmospheric dynamics as a system of coupled anisotropic cascade processes.

An initial application of this model takes us up to the limits of the weather domain: in space to the size of the planet, in time to the lifetime of planetary-sized structures ($\tau_w \approx 10$ days), after which there is a drastic change in the behaviour of all the atmospheric

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fields. It turns out that whereas at shorter time scales fluctuations tend to grow with scale – the weather is perceived as unstable – at longer time scales the average fluctuations tend to decrease, giving on the contrary the impression of stability. It is tempting to identify this new regime with the climate, but we argue that this would be a mistake, that the climate is not just the long-term behaviour of the weather. The reason is that it turns out that the same anisotropic cascade that explains the weather variability can be extended to much lower frequencies. When this is done, there is indeed a "dimensional transition" at τ_w but the model continues to accurately reproduce the lower-frequency variability beyond the transition. It turns out that this is also true of unforced global climate models (GCMs), so that the label "low-frequency weather," or "macroweathers," is appropriate. To paraphrase a popular dictum, "macroweather is what you expect, the weather is what you get." From a stochastic perspective, the climate is unexpected in much the same way as the weather, with analogous consequences for its prediction.

In order to find something really new that corresponds to our usual notion of "climate," we have to wait quite a long time – about $\tau_c \approx 10$–30 years – until we find that the mean fluctuations again start to increase with scale – a behaviour which apparently continues to $\tau_{lc} \approx 30$–50 kyr. Yet even this true climate regime – where genuinely new processes and/or forcings are dominant – shares features (including scaling) with the weather/macro weather regime. We show how a single overall weather/macro weather/climate process emerges, and we derive its statistics and estimate its exponents and other parameters.

In these pages, we therefore show how to considerably generalize the classical turbulence laws and to obtain emergent laws of atmospheric dynamics. Empirically, we show that they apply from milliseconds to decades to tens of millennia, from millimetres to the size of the planet. In more detail, we argue (a) that the atmosphere is a strongly nonlinear system with a large number of degrees of freedom, (b) that it nevertheless respects an (anisotropic) scale-invariance symmetry, (c) that this leads to new emergent properties, new dynamical laws. These new laws are statistical, and physically they imply that the variability builds up scale by scale in a cascade-like manner, that it sports many nonclassical statistical characteristics including long-range statistical dependencies and nonclassical extreme values ("heavy"-tailed, algebraic probability distributions). Finally, they can be exploited to understand and to forecast atmospheric fields.

The basic ingredients needed to effect this generalization are multifractals, cascades and generalized scale invariance. The development of these notions was largely motivated by atmospheric applications and arose in the 1980s, a period when nonlinear dynamics was generating excitement in many areas of science. Although a book comprehensively treating multifractals is still lacking, they are *not* the focus here. For our purposes, they are rather the tools needed to generalize the classical turbulent laws to the atmosphere. In the last five years or so, the scope of these applications has dramatically increased, thanks to the existence and accessibility of massive global-scale databases of all kinds. Whereas only ten years ago we were still speculating on the ranges and types of scaling of atmospheric fields, today we can already be confident about a great deal. This confidence is due partly to the qualitative – and in many cases quantitative – agreement between quite different databases over wide ranges of scale, but also to the surmounting of several obstacles in the interpretation of the data (in particular of aircraft data). We therefore place much emphasis on the empirical underpinnings of the new laws. Although over the meteorological scales we extensively analyze the traditional sources of satellite, lidar, drop-sonde and aircraft data, we also investigate at length reanalysis fields that are hybrid products somewhere between the data and the models, as well as the outputs of the models themselves.

Therefore, although many of the ideas in this book have been around since the 1980s, most (perhaps 90%) of the examples are from research performed in only the last five years. It is thanks to these new global datasets that the original 1980s models of 23/9 D anisotropic scaling dynamics, and of three scaling regimes from weather to climate, can be convincingly validated and a new, comprehensive view of atmospheric variability established. Beyond new results on global spatial scales, there are also new results on the space-time variability, including the emergence of waves (Chapter 9). The last two chapters – the research for which was largely undertaken specifically for this book – include the generalization of the emergent weather laws into the low-frequency weather regime (Chapter 10), i.e. between the ≈ 10-day lifetime of planetary structures out to 10–100 years where the true climate regime begins. These longer climate scales are mostly beyond the instrumental range, so in Chapter 11 we analyze various surrogates including multiproxies, paleotemperatures and climate forcings (including solar, volcanic and orbital) as well as GCMs (unforced and forced climate "reconstructions"). We show how the space-time climate variability can be understood by a further extension of the weather/low-frequency weather space-time scaling framework. By quantifying the natural variability as a function of space and time scale, this provides the information necessary to construct statistical tests for assessing anthropogenic influences on the climate. This approach is complementary to the current GCM approach but has the advantage of being largely data rather than model-driven.

Although long in gestation, this book comes at a critical moment for atmospheric science. While ever bigger computers, ever higher resolution devices and ever larger quantities of data have resulted in our present golden age, it has come at a price: they have gobbled up most of our resources. Sometimes, it seems that there are only barely enough left over to support a narrow focus on applications to numerical weather and climate modelling. One can easily get the impression that a basic understanding of the atmosphere's variability in space and in time is a luxury that we cannot afford. Yet today's continuing lack of consensus about these questions is increasingly hampering the development of the numerical models themselves. For example, without knowledge of the effective dimension of atmospheric motions it will not be possible to place the currently ad hoc "stochastic parametrizations" in modern Ensemble Forecasting Systems on a solid theoretical basis. As we argue here, this new synthesis – which is remarkably simple – provides a compelling and consistent picture of atmospheric variability and dynamics from weather through climate scales and suggests numerous ways forward, including the possibility of direct stochastic forecasting (Chapter 9).

From the above, the reader may correctly infer that this book is squarely oriented towards practising atmospheric scientists (especially meteorologists and climatologists) and that it includes a (hopefully) accessible exposition of the necessary nonlinear tools. Occasionally, when a topic is a bit too technical but nevertheless important either for applications or for the theory, details are given in appendices. Similarly, advanced or optional sections are indicated by asterixes. In addition, at the end of each chapter, under the rubric *Summary of emergent laws in Chapter . . .*, we give a succinct summary of the developments in the chapter that are important for developing the main theory. These summaries will allow readers to skip details that are unimportant while maintaining the basic thread of the argument. Finally, to highlight them, the more important formulae have been placed in boxes. Let the reader be warned, however, that this is neither a textbook nor a conventional monograph. It is rather a systematic presentation of arguments and evidence for a new framework for understanding atmospheric dynamics.

In order to make the material as accessible as possible, the basic philosophy has been to first present empirical analyses demonstrating the existence of wide-range scaling: an overview in Chapter 1, the horizontal wind in Chapter 2, the state variables and radiances in the horizontal in Chapter 4, in the vertical in Chapter 6, and in time in Chapters 8, 10 and 11. The analyses proceed from the (familiar) Fourier (power) spectra applicable to essentially any field, to trace moments in Chapter 3 needed to analyse cascades, followed by further related analysis techniques (generalized structure functions, wavelets, the probability distribution multiple scaling technique etc.) in Chapter 5. For readers primarily interested in the longer time scales, Chapters 10 and 11 are to some degree independent of the preceding, making only light use of the formalism and relying extensively on the use of Haar fluctuations (wavelets). However, this underexploited technique is actually quite straightforward – even intuitive – and allows systematic comparisons to be made of different types of data and over different and large scale ranges. It gives a far clearer picture of the macro weather and climate variability than is otherwise possible, so that any effort expended to understand this analysis technique will be rewarded.

Following the empirical motivation, the theory is introduced gradually and as needed: first the basic elements of turbulence theory (Chapter 2), then elementary (discrete in scale) cascades (moment statistics, Chapter 3), with the more general treatment of multifractals including probabilities and continuous in scale simulations reserved for Chapter 5. In Chapter 6 we go beyond isotropy, by introducing generalized scale invariance, but only in the simplest self-affine form needed to handle scaling different in two orthogonal

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directions: atmospheric stratification. Only in Chapter 7 do we treat the more general case needed for cloud and other morphologies whose anisotropies vary both with scale and with position. Going beyond space to space-time involves extra complications, if only because causality must be taken into account, and this is why its introduction is delayed until Chapter 8, where we give both an empirical overview and the basic theory needed to understand the space-time scaling in the weather regime. In Chapter 9 we extend this to an explicit treatment of causality, to turbulence-driven waves as an emergent scaling process, to predictability and (stochastic) forecasting. In Chapter 10 we extend the space-time weather model into the macro weather regime, showing that it not only predicts the observed sharp "dimensional transition" between weather and macro weather at about 10 days (the lifetime of planetary structures), but that it does remarkably well up to scales of decades and centuries. At scales below the transition in the weather regime, fluctuations generally grow with increasing scale, but at larger scales, in the low-frequency weather regime, on the contrary they diminish with scale – the atmosphere appears "stable." However, this is not the full story. In Chapter 11 we show – with the help of instrumental, multiproxy and paleodata – how the low-frequency weather regime eventually gives way to a new climate regime where fluctuations once again grow with scale, and attempt to address the question as to whether or not GCMs predict the climate or merely macro weather.

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