

## ***Interactive comment on “Reinterpreting aircraft measurements in anisotropic scaling turbulence” by S. Lovejoy et al.***

**S. Lovejoy et al.**

Received and published: 19 June 2009

### **1. Introduction: The phenomenological fallacy**

If the observed break in aircraft spectra of the horizontal wind is spurious, as we concluded in our ACPD paper (LTSH), then there are no longer serious empirical objections to the hypothesis that over wide ranges of scale, atmospheric dynamics follow multiplicative cascades. Indeed, in LTSH we reviewed mounting empirical evidence in favour of the cascade model from satellite radiances, drop sondes and lidar. While not disputing this, Dr. Yano addresses the important issue of how this could possibly be compatible with certain classical views of the atmosphere, in particular with the picture of convection as developed by [Riehl and Malkus, 1958] - which persists today in various phenomenological approaches as the “two-scale hypothesis”, or the “scale separation principle”. He then goes on to give his own proposed resolution of the con-

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Discussion Paper



tradition with the help of certain vague properties claimed for  $1/f$  noises (the existence of “pulse-like” coherent structures). Below, we respond to these points in turn.

Convective models are phenomenological, they are based on the identification of structures and morphologies that are in turn identified with dynamical mechanisms. In the case of convection, Dr. Yano baldly states that “Deep convection is organized into a mesoscale of 100 km in scale.”, yet the largest study to date of the scale by scale statistics of short and long wave radiances [Lovejoy et al., 2009] finds that there is no sign of anything special in the radiances anywhere near the claimed 100 km scale. In as much as this horizontal scale is due to the identification of “deep convection” with 10 km thick cloud, then this would seem to be an example of the “phenomenological fallacy”, i.e. the inference of mechanism from phenomenon [Lovejoy and Schertzer, 2007]. The basic point - made in [Schertzer and Lovejoy, 1985a] and repeated regularly since, has two distinct parts. First, if the scaling is anisotropic, then structures at different scales will generally have (possibly radically) different morphologies in spite of having identical dynamical mechanisms. In anisotropic scaling systems, the identification of phenomena and mechanism is thus demonstrably unwarranted. Figure A1 of LTSH shows a visual example pertinent to our discussion: simulations of clouds in 3D whose statistics respect the anisotropic extensions of the classical Corrsin - Obukhov law of passive scalar advection whose structure/morphologies therefore depend on size. The statistics in the simulation are close to those reported for both lidar studies of aerosol backscatter [Lilley et al., 2008] and cloud radar reflectivity (a surrogate for liquid water content), obtained from CloudSat data [Lovejoy and Schertzer, 2009a].

## 2. Cascades, coherent structures and singularities:

The second part of the answer, directly relevant to much of Dr. Yano’s comment, is that if the underlying dynamics follow a cascade process, then the resulting fields are multifractals characterized by a hierarchy of singularities each distributed over sparse fractal sets with varying dimensions. These singularities are the coherent structures observed in cascade simulations, in the classical numerical simulations and in the data.

[Full Screen / Esc](#)[Printer-friendly Version](#)[Interactive Discussion](#)[Discussion Paper](#)

Indeed, the generic outcome of a multiplicative cascade process is that their statistics obey:

$$\langle (\phi_\lambda)^q \rangle = \lambda^{K(q)}$$

$$Pr(\phi_\lambda > \lambda^\gamma) \sim \lambda^{-c(\gamma)} \quad (1)$$

where  $\phi_\lambda$  is a turbulent flux, (assumed to be the outcome of a multiplicative cascade), the scale ratio  $\lambda = L_{eff}/l$  where  $l$  is the scale of observation (resolution) and  $L_{eff}$  is the outer scale of the cascade (where it “effectively“ starts in order to yield the observed variability at scale ratio  $\lambda$ ),  $\langle . \rangle$  indicates statistical averaging.  $K(q)$  is the moment scaling exponent,  $c(\gamma)$  is the codimension characterizing the sparseness of the singularity of order  $\gamma = \log \phi_\lambda / \log \lambda$ . The statistics can be specified either by the moments ( $K(q)$ ) or probabilities ( $c(\gamma)$ ); the two are related by a Legendre transformation [Parisi and Frisch, 1985] which establishes a one to one relation between  $\gamma$  and  $q$ :  $\gamma = dK(q)/dq$  and  $q = dc(\gamma)/d\gamma$ . The spectra  $E(k)$  of  $\phi_\lambda$  is of the power law form  $E(k) = k^{-\beta}$  ( $k$  is a wavenumber) with exponent  $\beta = 1 - K(2)$ .  $K(2)$  is the scaling exponent for the  $q = 2$  moment; it is the typically not so large “intermittency correction“ that depends on the field, see below (for example in the horizontal wind field in the horizontal it is  $\sim 0.13$ , in the CloudSat cloud reflectivities it has the relatively large values  $0.3 - 0.4$  horizontal, vertical, corresponding to  $\beta \sim 0.5$  i.e. still within Dr. Yano’s flexible definition of a  $1/f$  noise). In Dr. Yano’s loose sense, multiplicative cascades generically generate  $1/f$  noises, although with precise properties (eq. 1). In anisotropic cascades, the singularities / coherent structures / pulses will change their morphologies with scale in a systematic power law manner. In other words, Dr. Yano’s reconciliation of the morphology with the statistics - although essentially correct - would have been much more compelling had it been informed by an up to date understanding of the properties of cascades and multifractals.

### 3. Empirical demonstration of anisotropic cascades with the help of CloudSat:

In the present context, there are two weaknesses with this classical cascade argument.

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Discussion Paper



First, it is somewhat academic since it simply points out that in principle, there need not be a contradiction between convective phenomenology and wide range anisotropic scaling. The second difficulty is more concrete, how to get structures traversing the troposphere in height while being only 100 km or so across. In the language of Generalized Scale Invariance ([Schertzer and Lovejoy, 1985b]; see LTSH) this implies vertical cross-sections being roundish in shape at around 1 - 10 km i.e. with sphero-scales being much larger than the range 0.01 - 1 m observed in passive scalars [Lilley et al., 2008] or in the horizontal wind, see [Lovejoy et al., 2004], [Radkevitch et al., 2008] and LTSH). The point is that if we define a “convective cell” as one that spans most of the vertical extent of the troposphere - and which typically has a comparable horizontal extent (e.g. the 100 km cited by Dr. Yano), then for the argument to work, one would require a cloud liquid water field with a much larger sphero-scale than those which have been observed to date.

What has been missing then is not a theoretical answer in principle - but rather a concrete empirical validation of the 25 year old anisotropic scaling model. Up until recently, in the case of convection, the best empirical demonstration was due to scaling analyses of extensive data sets of one dimensional sections taken in the tropics above and below the tropopause by research aircraft, the NASA ER-2 and WB57F, which have observations of winds, temperature, ozone and most importantly in the present context, total water (Tuck et al., 2003), (Tuck, 2008). However a more complete answer was impossible due to the relative unavailability of high quality cloud vertical cross-section data. Today however, the situation has dramatically changed with the development of orbiting radars on the Tropical Rainfall Monitoring Mission satellite (TRMM, 4.3 km resolution, 3.2 cm wavelength, 1997-present) and CloudSat (1.08 km in the horizontal, 3mm wavelength, 2006 - present). In recent publications [Lovejoy et al., 2008], [Lovejoy and Schertzer, 2009a], the statistical properties of the reflectivities - were already examined in detail showing that although the radars have problems measuring weak effective reflectivity factors ( $Z$ ) that the fluxes  $\phi$  estimated  $Z$  follow the basic prediction of multiplicative cascades - eq. 1 - over the entire range 4.3 km to 20,000 km with

[Full Screen / Esc](#)[Printer-friendly Version](#)[Interactive Discussion](#)[Discussion Paper](#)

accuracies of + -4.6

For precipitation the main limitation of the TRMM radar is its poor measurement of the weak (and zero) rain rate regions. With its much smaller wavelength (3 mm, 94 GHz), the CloudSat radar can detect signals down to  $Z \sim 0.01 \text{mm}^6/\text{m}^3$  and therefore can detect much smaller drops. Since  $Z$  is proportional to the sum of the squared drop volumes it is highly correlated with the sum of the volumes, i.e. the LWC and is thus a good surrogate for convection. Also relevant here is the study of TRMM short and long wave radiances from the VIRS instrument [Lovejoy et al., 2009] that found that the predictions of multiplicative cascades were obeyed to within + -0.5

A particularly relevant finding of [Lovejoy and Schertzer, 2009a] was the direct empirical estimate of the relation between the horizontal and vertical extents of cloud structures. To explain how this can be empirically determined, recall the familiar space-time (“Stommel”) diagrams - those conceptual aides found in most introductory textbooks displaying the lifetimes of various meteorological structures as functions of their size (from dust devils to meso-scale convective complexes to planetary waves). It is possible to use the mean absolute fluctuations in horizontal ( $\Delta x$ ) and for time lags ( $\Delta t$ ) to empirically determine such diagrammes. The idea is to objectively define a “typical structure” by a given mean absolute fluctuation  $\langle \Delta Z \rangle$ ; this is discussed in [Lovejoy and Schertzer, 2009b]; here we apply the same idea but to space (horizontal) and to space (vertical,  $\Delta z$ ) obtaining one-to-one relations between horizontal and vertical scales. If for a given vector displacement  $(\Delta x, \Delta z)$  in the vertical plane, we determine the mean absolute reflectivity fluctuation  $\langle \Delta Z(\Delta x, \Delta z) \rangle$ , then for a given  $\Delta z$ , the corresponding  $\Delta z(\Delta x)$  may be determined from the implicit equation:  $\langle \Delta Z(\Delta x(\Delta z), 0) \rangle = \langle \Delta Z(0, \Delta z) \rangle$ .

Fig. A2 of LTSH shows the result on 16 CloudSat orbits, both the ensemble averaged relations. It is concluded that the scaling is almost identical to those of aerosols [Lilley et al., 2004] but with (roughly) isotropic scales (the “sphero-scales” =  $l_s$ , see the intersection with the bisectrix) about 1000 times larger i.e about 50 m rather than 50 cm

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Discussion Paper



(although with huge variability, one standard deviation is about a factor 10 in  $l_g$ ).

#### 4. Conclusions:

In LTSH, we showed that repeatedly observed breaks in the scaling of the key horizontal wind field were spurious consequences of anisotropic, scaling turbulence perturbing aircraft trajectories and biasing the statistics. This finding removed the last major empirical obstacle to the demonstration of wide range atmospheric scaling. But even if this evidence is compelling, theoretical objections remain. In his comment, Dr. Yano articulated a widespread view: that the growing body of statistical evidence that the various atmospheric fields have wide range scaling properties contradicts longstanding meteorological phenomenologies and empirical morphologies. He concentrated his argument on the important example of deep convection. In our response, we reiterated the now classical (anisotropic) cascade argument reconciling - in principle - the structures, morphologies and the statistics: that the structures are simply cascade generated singularities and that the anisotropy allows for rich morphologies which change with scale. This is basically a more precise formulation of Dr. Yano's unnecessarily vague and outdated "1/f noise" and "pulse-like" morphologies argument. However in order to go beyond this explanation in principle and to concretely overcome misunderstandings, we cited empirical analyses of state-of-the-art cloud reflectivity data from TRMM and CloudSat which showed that the anisotropic multiplicative cascade framework was accurately obeyed over most of the relevant range of scales in both horizontal and vertical. Using the statistics of the turbulent fluxes estimated from the reflectivities, we were able to directly calculate the relation between horizontal and vertical scales - including an estimate of its variability, and directly confirmed that due to the occasional existence of large sphero-scales - deep convection ( $\sim 10$  km thick) can readily correspond to structures ( $\sim 100$  km in the horizontal. Most importantly, rather than being in contradiction with deep convection phenomenology, this correspondence was shown to be precisely a consequence of the anisotropic scaling cascades!

Acknowledgements: We thank Chris Mills for help with the CloudSat analysis.

S2597

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Discussion Paper



## References:

Lilley, M., Lovejoy, S., Strawbridge, K., and Schertzer, D., 23/9 dimensional anisotropic scaling of passive admixtures using lidar aerosol data, *Phys. Rev. E*, 70, 036307-036301-036307, 2004.

Lilley, M., Lovejoy, S., Schertzer, D., Strawbridge, K. B., and Radkevitch, A., Scaling turbulent atmospheric stratification, Part II: empirical study of the the stratification of the intermittency, *Quart. J. Roy. Meteor. Soc.*, DOI: 10.1002/qj.1202, 2008.

Lovejoy, S., Schertzer, D., and Tuck, A. F., Fractal aircraft trajectories and nonclassical turbulent exponents, *Physical Review E*, 70, 036306-036301-036305, 2004.

Lovejoy, S., and Schertzer, D., Scale, scaling and multifractals in geophysics: twenty years on, in *Nonlinear dynamics in geophysics*, edited by J. E. A.A. Tsonis, Elsevier, 2007.

Lovejoy, S., Schertzer, D., and Allaire, V., The remarkable wide range scaling of TRMM precipitation, *Atmos. Res.*, 10.1016/j.atmosres.2008.02.016, 2008.

Lovejoy, S., and Schertzer, D., Reconciling convection and scaling with the help of CloudSat, *Q. J. Roy. Meteor. Soc.*, (in preparation), 2009a.

Lovejoy, S., and Schertzer, D., Towards a new synthesis for atmospheric dynamics: space-time cascades, *Atmos. Res.*, submitted, 2009b.

Lovejoy, S., Schertzer, D., Allaire, V., Bourgeois, T., King, S., Pinel, J., and Stolle, J., Atmospheric complexity or scale by scale simplicity? , *Geophys. Resear. Lett.*, 36, L01801, doi:10.1029/2008GL035863, 2009.

Parisi, G., and Frisch, U., A multifractal model of intermittency, in *Turbulence and predictability in geophysical fluid dynamics and climate dynamics*, edited by M. Ghil, et al., pp. 84-88, North Holland, Amsterdam, 1985.

Radkevitch, A., Lovejoy, S., Strawbridge, K. B., Schertzer, D., and Lilley, M., Scaling

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Discussion Paper



turbulent atmospheric stratification, Part III: empirical study of Space-time stratification of passive scalars using lidar data, Quart. J. Roy. Meteor. Soc., DOI: 10.1002/qj.1203, 2008.

Riehl, H., and Malkus, J. S., ON the heat balance in the equatorial trough zone, Geophysica (Helsinki), 6, 503-538, 1958.

Schertzer, D., and Lovejoy, S., The dimension and intermittency of atmospheric dynamics, in Turbulent Shear Flow 4, edited by B. Launder, pp. 7-33, Springer-Verlag, 1985a.

Schertzer, D., and Lovejoy, S., Generalised scale invariance in turbulent phenomena, Physico-Chemical Hydrodynamics Journal, 6, 623-635, 1985b.

---

[Interactive comment on Atmos. Chem. Phys. Discuss., 9, 3871, 2009.](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)

[Discussion Paper](#)