



# What is Climate?

**Issues in the Theoretical Foundations of Climate Science,  
University of Toronto, 15 November, 2018**

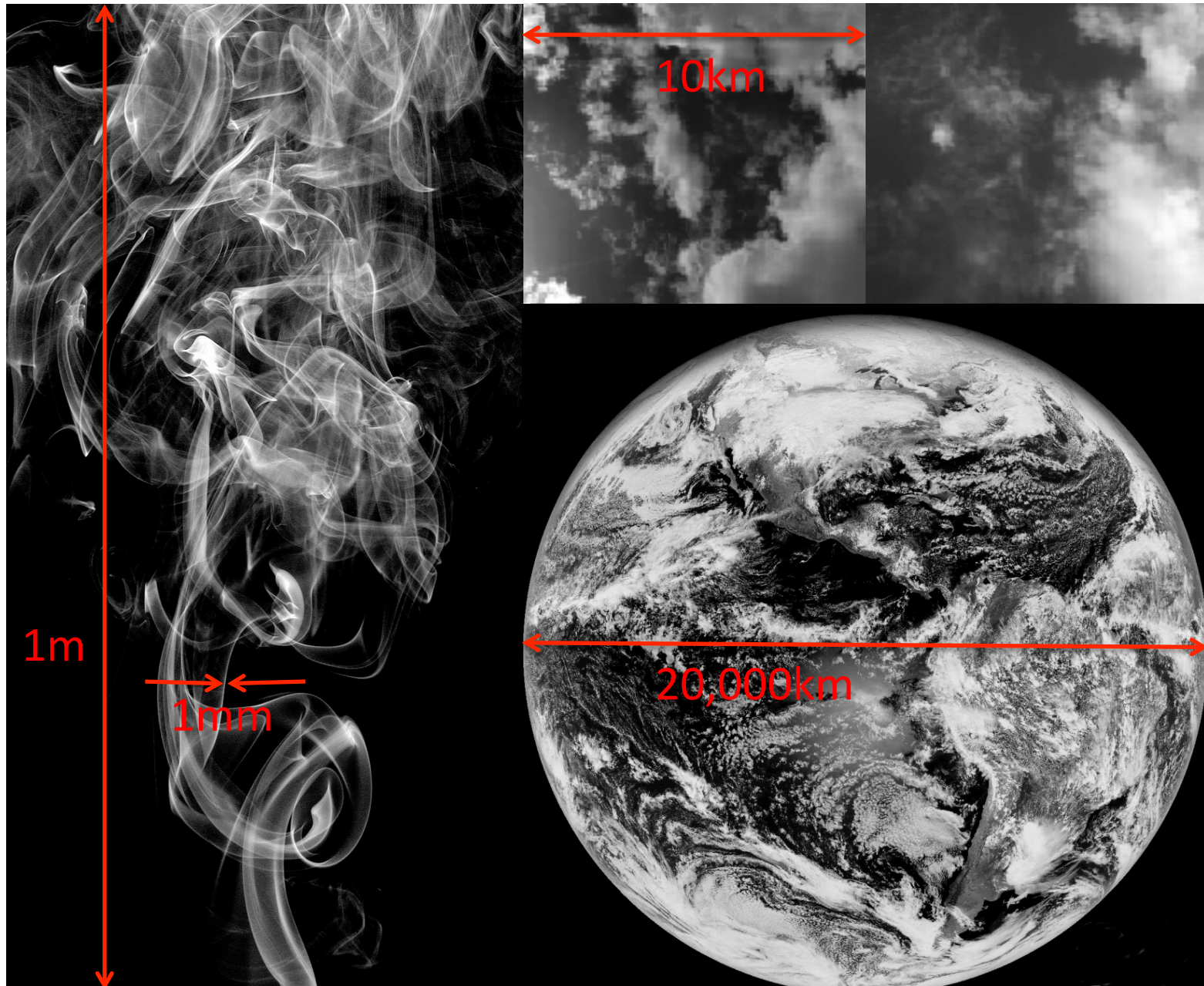
**S. Lovejoy, McGill, Montreal**

## A voyage through scales

Zooming through scales by the  
billion

1mm - 10,000 km

A voyage through scales: Space, 0.1mm – 10,000km

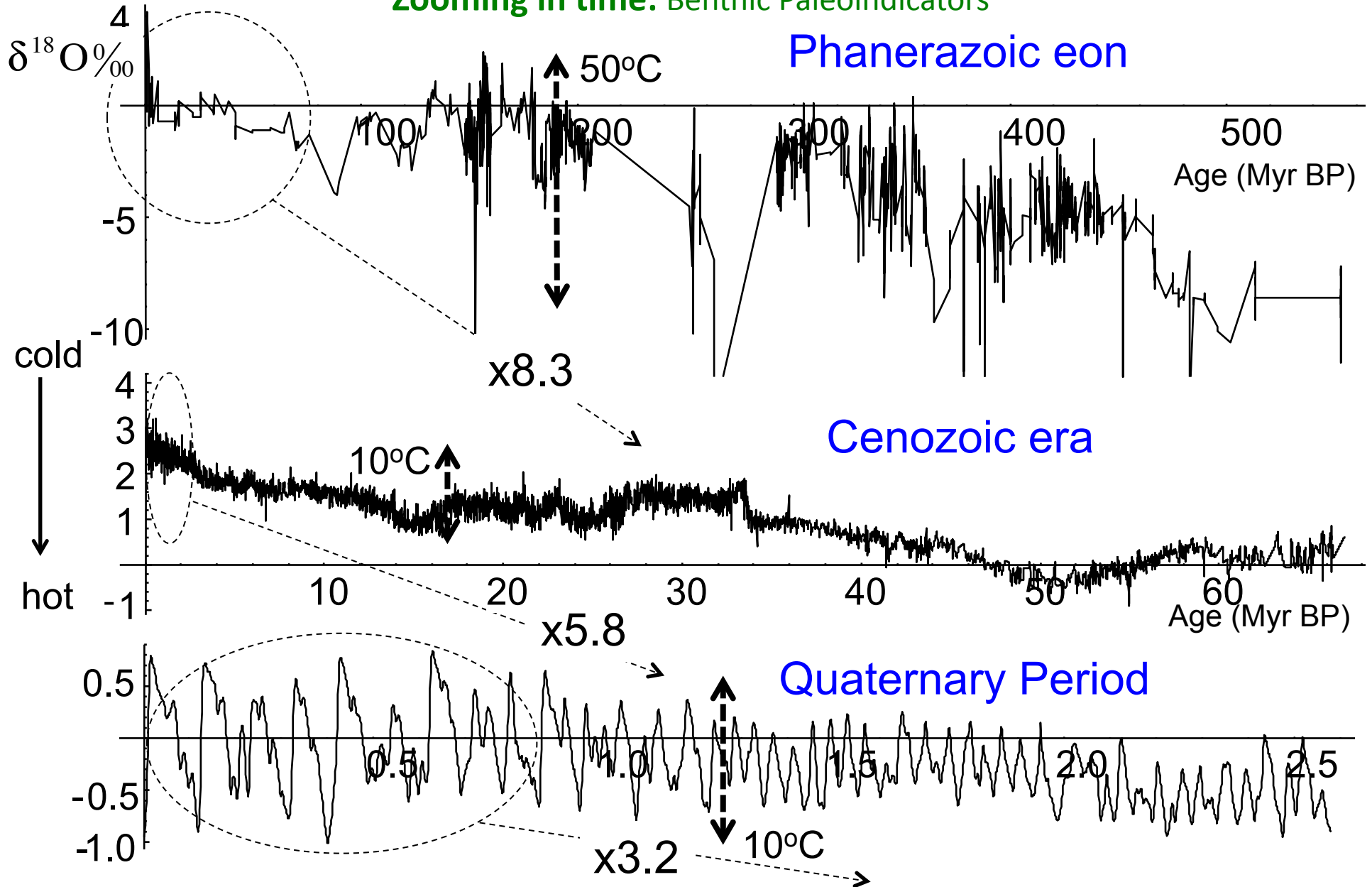


## A voyage through scales

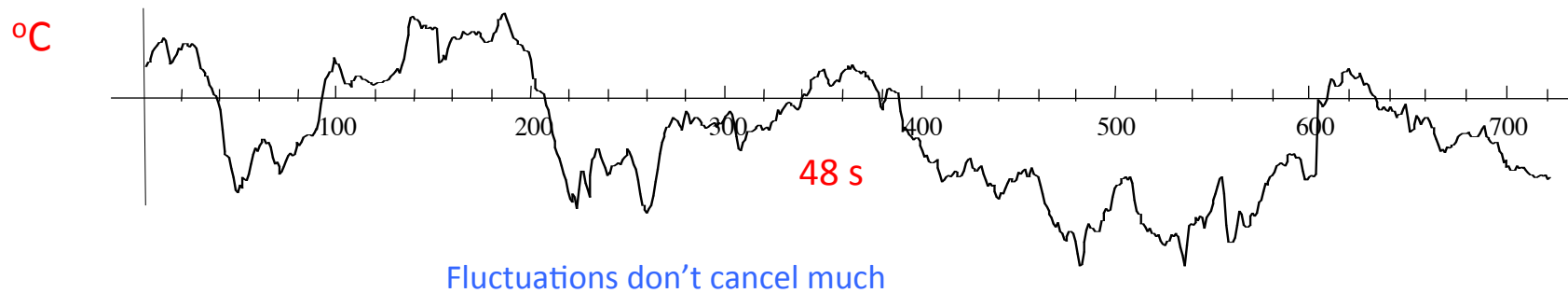
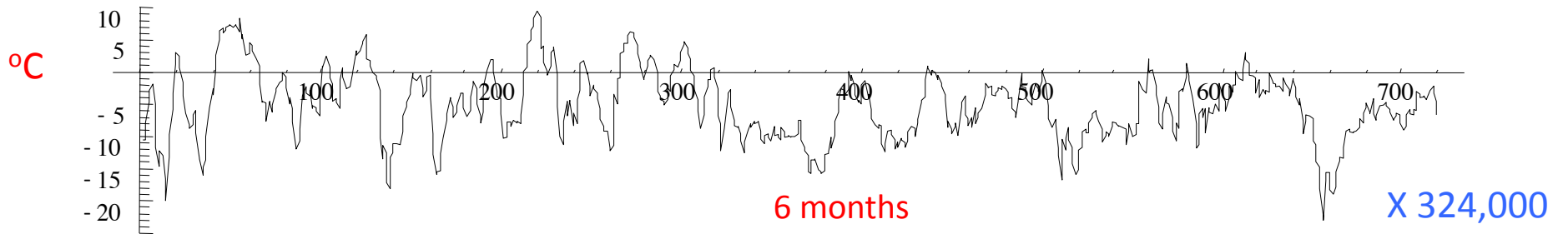
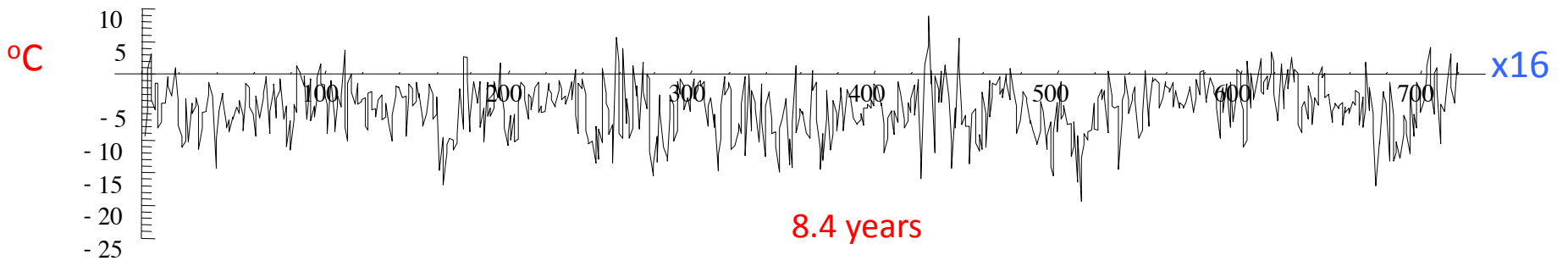
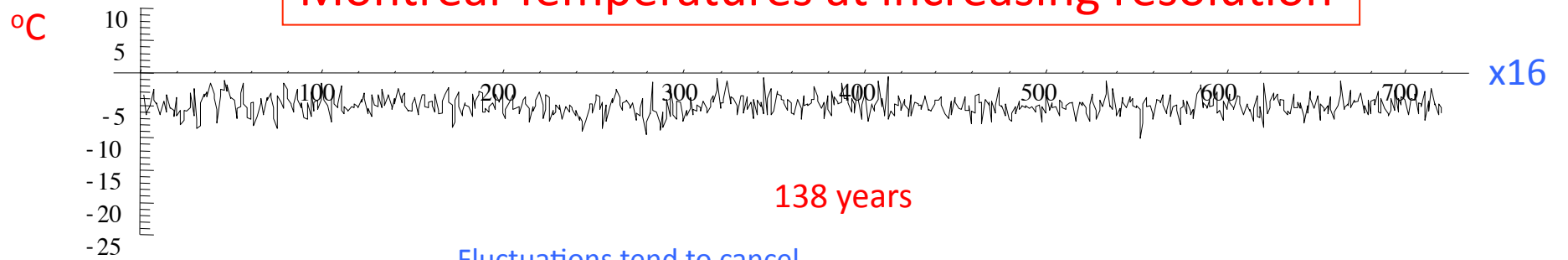
Zooming through scales by the  
billion billion  
milliseconds to half a billion years

# A voyage through scales: Time, 0.001s – 4.5 billion years

Zooming in time: Benthic Paleoindicators



# Montreal Temperatures at increasing resolution

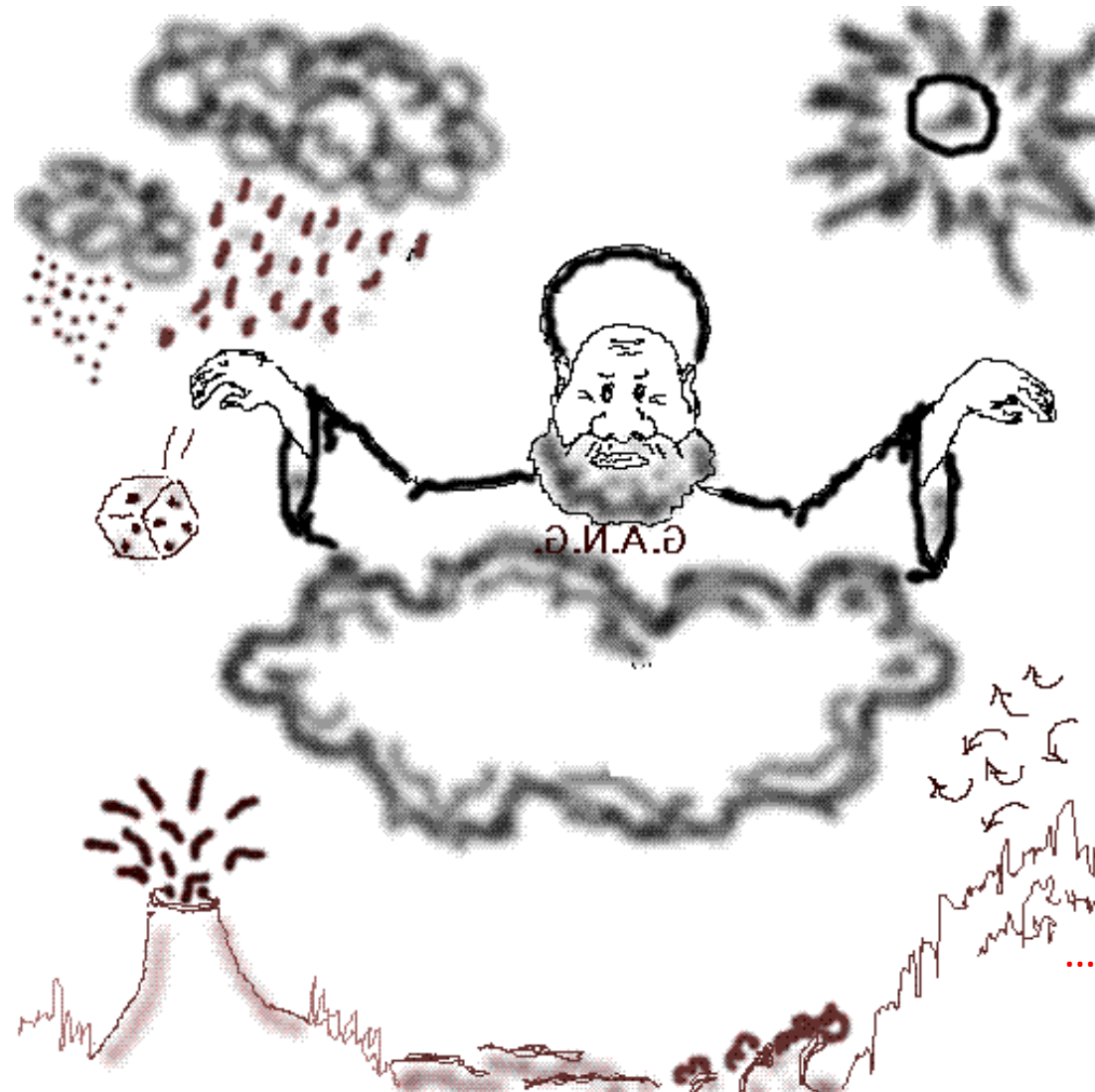


# How to understand this mind-boggling variability? (1)

Deterministic or random?

# Which Chaos?

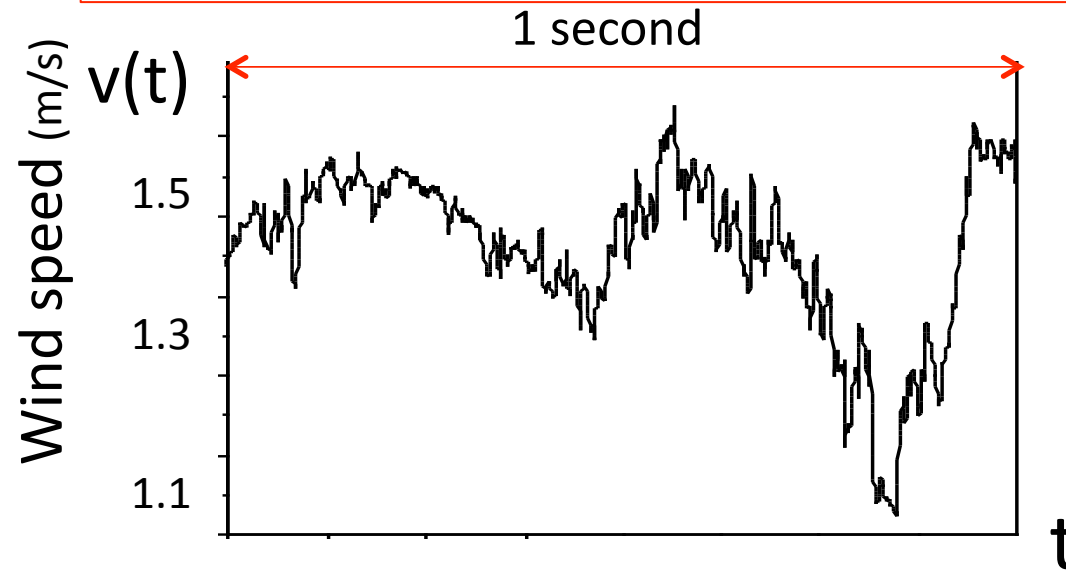
How does God play dice??



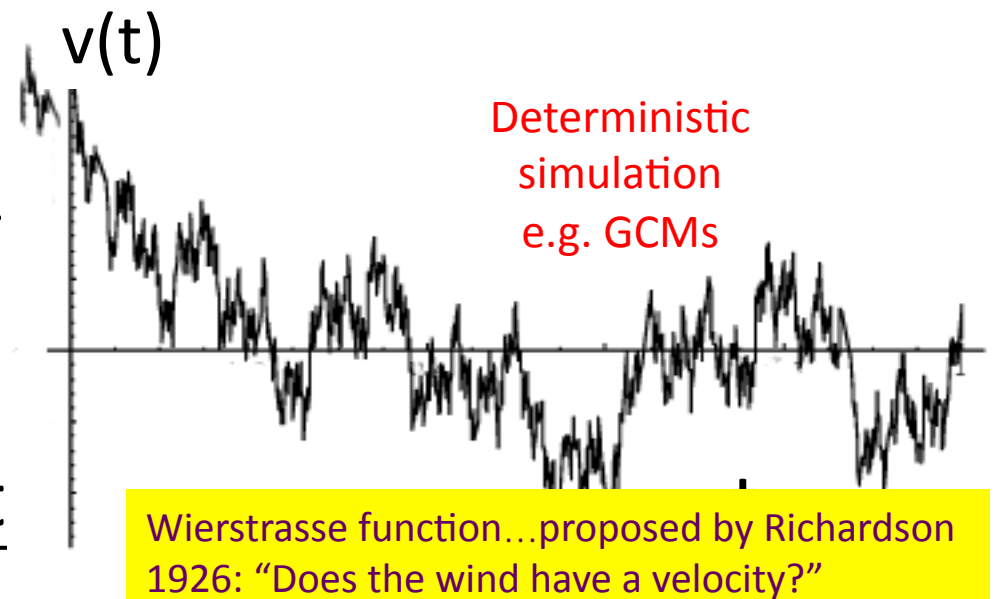
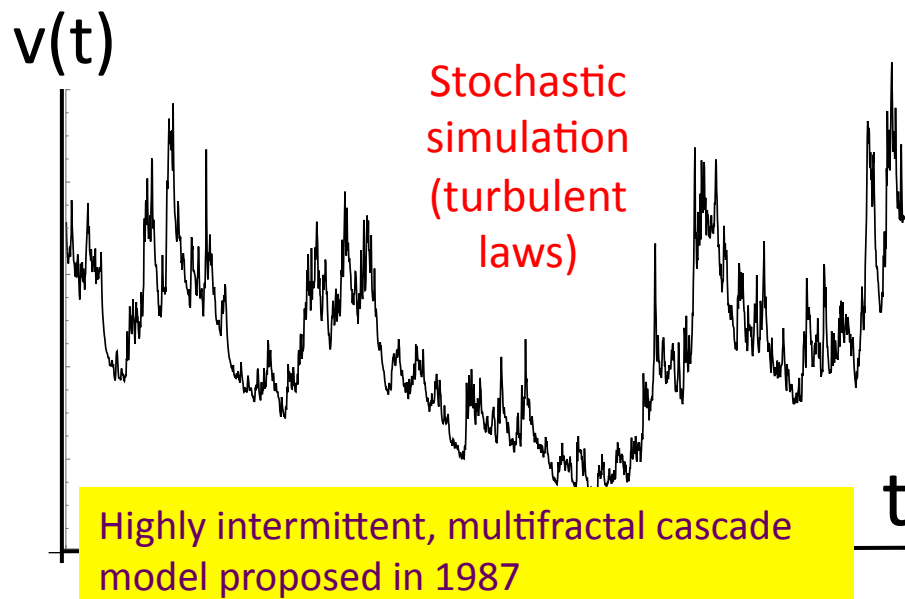
...sorry Einstein!



# Which Chaos? Stochastic or Deterministic?



Data,  
roof of  
physics  
building



# Cosmos versus Chaos through the ages

Chaos-Cosmos (ancient Greeks): first there was chaos... then cosmos...

Scientific ideas about determinism and randomness:

**Determinism:** God supplies the initial conditions (e.g. planets in orbits, Newton, 1670's)  
“...if a sufficiently vast intelligence exists...” Laplace (1749-1827).

**Chance:** Ignorance, subjective

“Chance is nothing” Voltaire: (1694-1778).

**Chance:** Irrelevance of the details

Statistical Mechanics e.g. the bell curve distribution of molecular velocities in a gas  
(Maxwell, Gibbs, Boltzman, 1870-1900).

**Chance:** Objective chance, **Stochastic Chaos** in systems with many degrees of freedom

Quantum Mechanics: Born interpretation of the wave function (1926)

Mathematics: Kolmogorov axiomatized probability theory (1933).

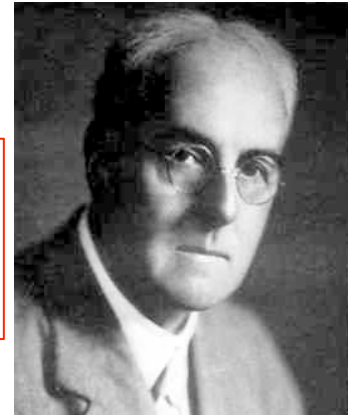
**Determinism:** Random-like **Deterministic Chaos** in systems with few degrees of freedom  
(Lorenz 1963).



1922

# Janus-faced (two strands)

1926



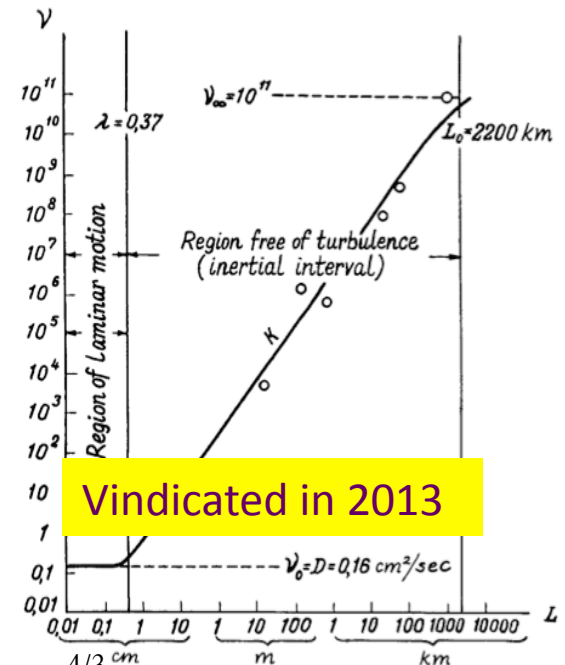
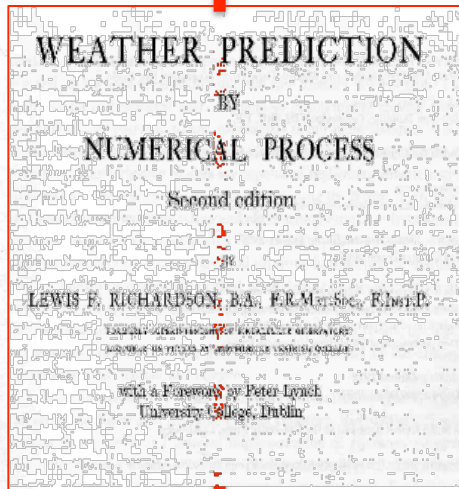
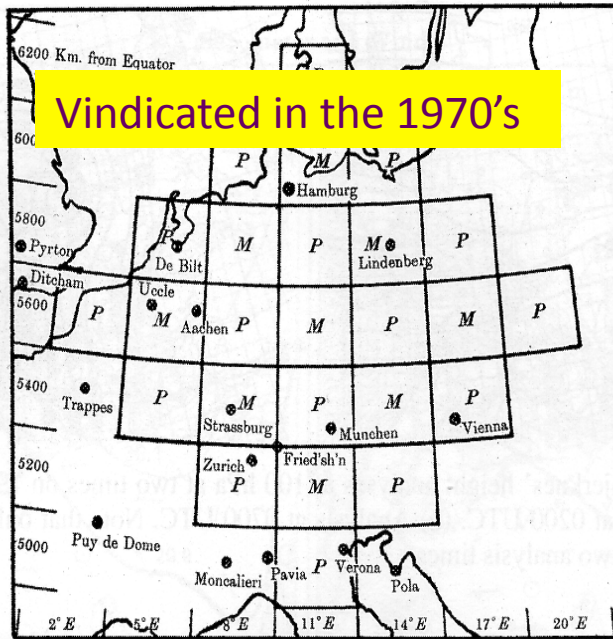
Richardson and  
NWP

Richardson,  
turbulence, scaling

Father of numerical weather prediction

Grandfather of turbulent cascades

Determined the pressure tendency  
at grid point M



$$\nu(L) = KL^{4/3}$$

Effective viscosity

constant

(Redrawn by Monin 1972)

It took six weeks of calculation... and he was wrong  
by a factor of 100!

since  $\nu = L\Delta v$

Kolmogorov's contribution was  $K = \epsilon^{1/3}$

# The Nonlinear Revolution

## 1970 - 1990 - present

### **The Deterministic Chaos Revolution: The Butterfly Effect**

- Tiny perturbations could be amplified
- Random looking phenomena might not be random after all...
- Backlash: an attempt to resurrect Newtonian determinism

### **The Stochastic Chaos alternative: scale symmetries, fractals, multifractals**

- Objective randomness...

# Two revolutions: unity lost

Up until 1970's weather and climate science were a pragmatic combination of both deterministic and statistical approaches.

## The Numerical revolution: NWP, GCMs

### Milestones:

Initialization

Ensemble forecasting

4D var (data assimilation)

Extension to climate

Earth System Models

Today: GCMs increasingly answer all questions

-Simulation replaces understanding

-Science reduced to engineering

-Theory/data connection broken

## The Nonlinear revolution

### Milestones:

Irrelevance of details, Stochastic chaos

Objective randomness

Scaling symmetries

Fractals, multifractals

Anisotropic scaling

Empirical vindication of Richardson

*Understanding*

Separate nonlinear processes divisions

EGU (1989), AGU (!997)

## 2010's: Unity refound?

GCMs respect scaling laws... and control runs can be stochastically forecast, and scaling yields better climate projections

The neglected strand of atmospheric science:

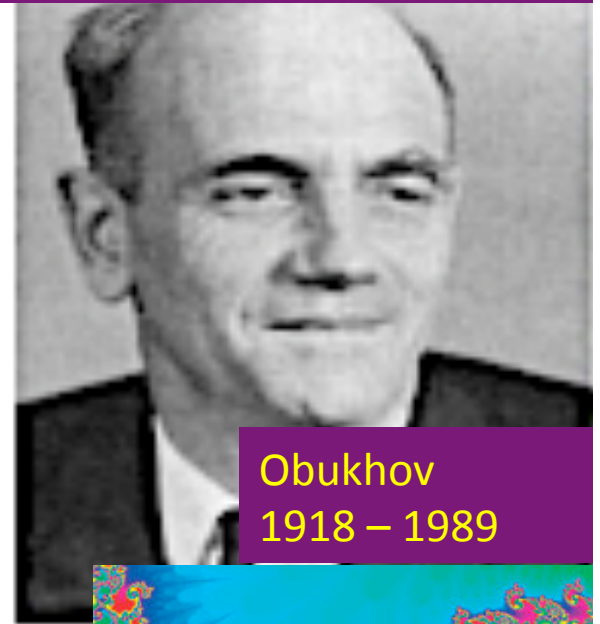
# Pioneers of turbulence



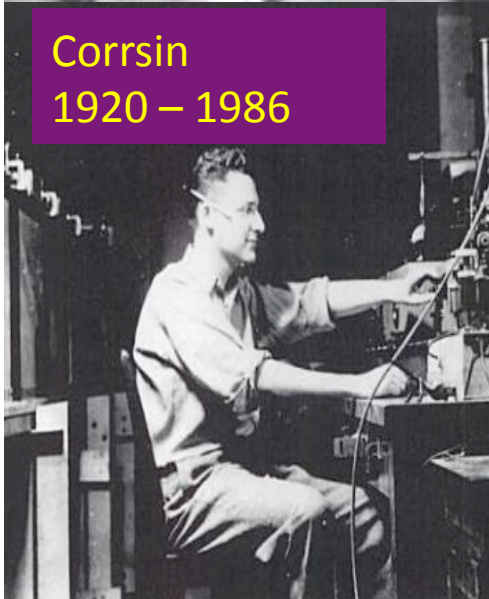
Richardson  
1881 - 1953



Kolmogorov  
1903 - 1987



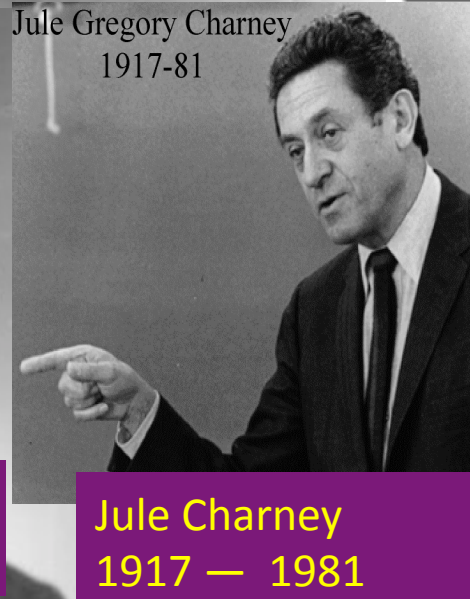
Obukhov  
1918 - 1989



Corrsin  
1920 - 1986

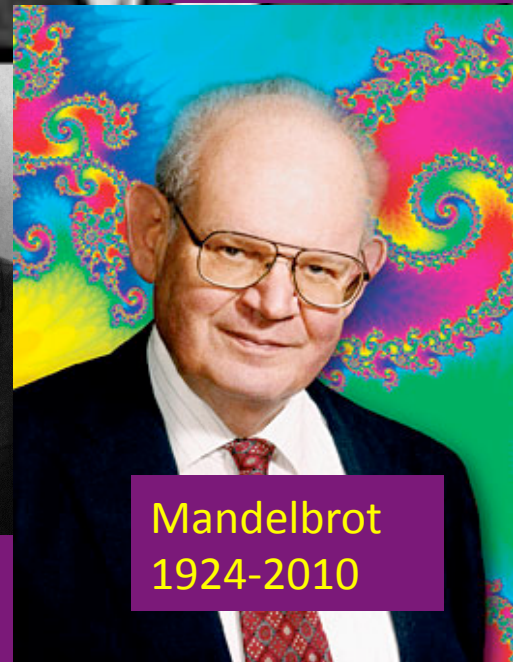


Ralph Bolgiano, Jr.  
1922 - 2002



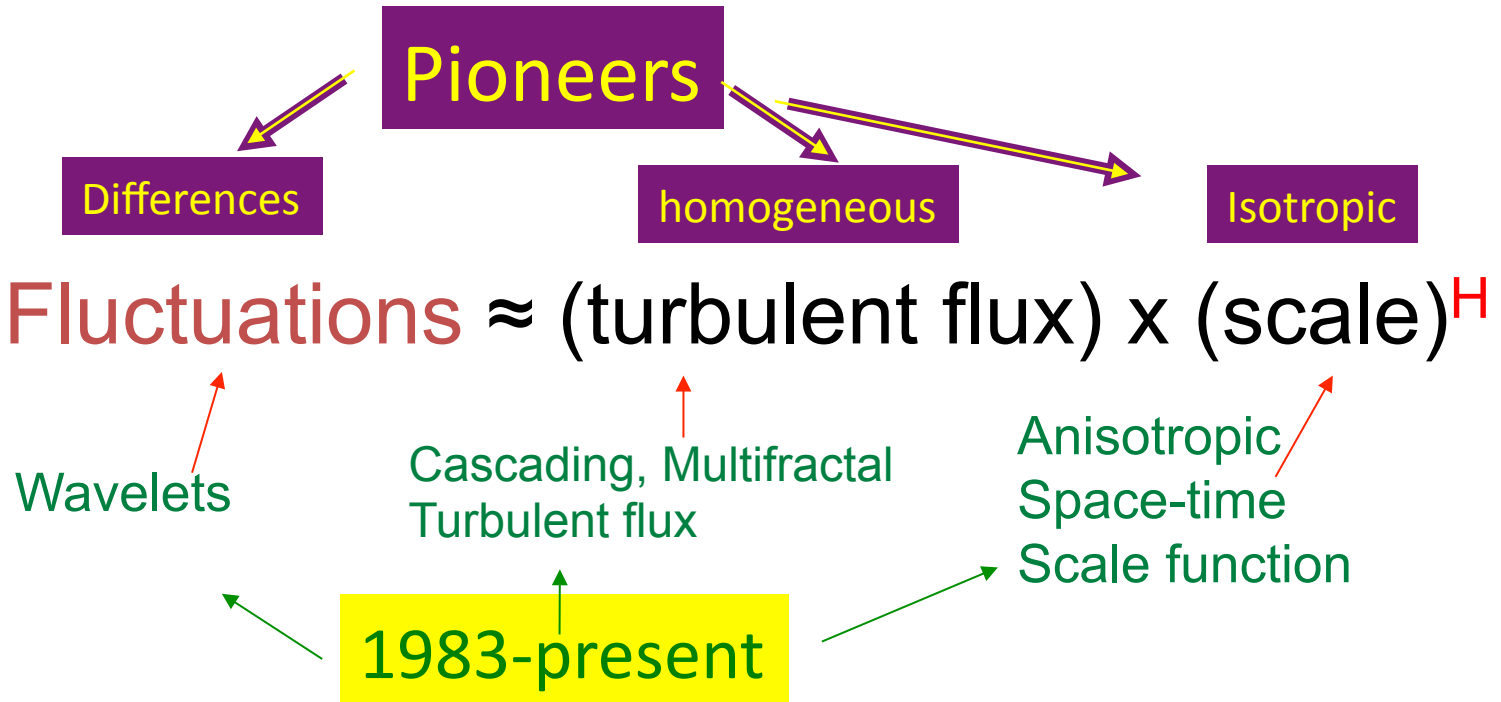
Jule Gregory Charney  
1917-81

Jule Charney  
1917 - 1981



Mandelbrot  
1924-2010

# Laws of Atmospheric Turbulence



Fourier domain:

$$\left( \frac{\text{Variance}_{\text{observables}}}{\text{wavenumber}} \right) = \left( \frac{\text{Variance}_{\text{flux}}}{\text{wavenumber}} \right) (\text{wavenumber})^{-2H} = (\text{wavenumber})^{-\beta}$$

Space:  $E(k) \approx k^{-\beta}$

Time:  $E(\omega) \approx \omega^{-\beta}$

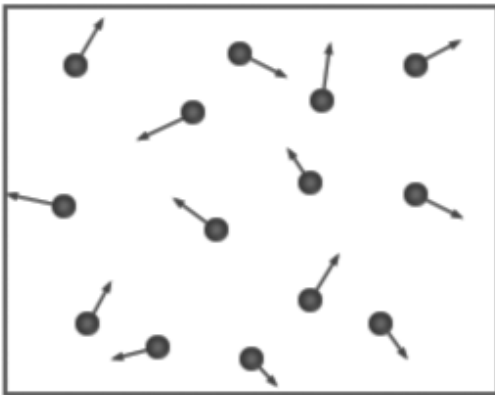
# How to understand this mind-boggling variability? (2)

High level or low level laws?

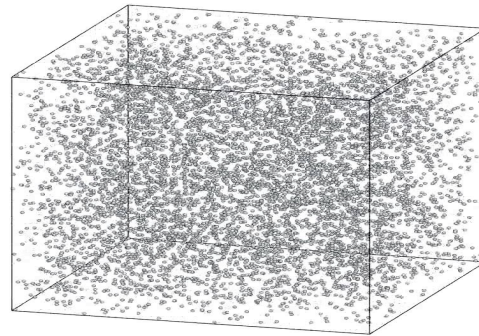


# Emergent laws: Which level?

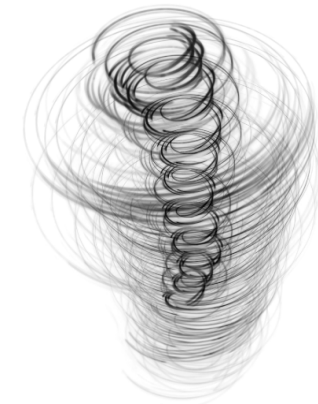
Mechanics of a  
few particles



Statistical  
Mechanics:  
many particles



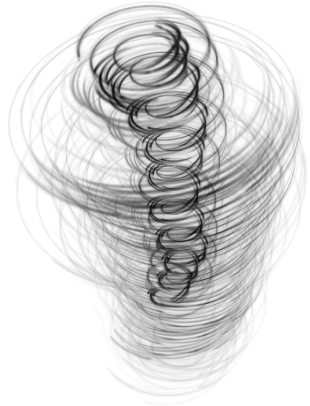
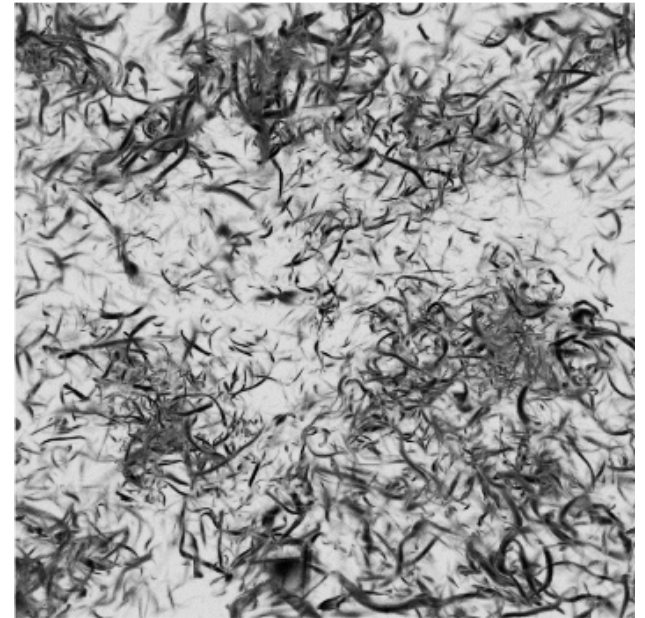
Collective  
behaviour of  
Thermodynamics,  
continuum  
mechanics, GCMs



Irrelevance of most of  
the details, collective  
behaviour of many,  
many components

# The hierarchy continues

Collective behaviour of many vortices: **Turbulent laws**



Continuum mechanics of a **single vortex**

Irrelevance of most of the details, collective behaviour of many, many components

“spaghetti” picture

Continuum mechanics Of several vortices



# How to understand this mind-boggling variability? (3)

What about the “details”?

Do we (deterministically, mechanistically, *numerically*) account for as many details as possible?

Or

Are most details *irrelevant* and we just need their statistics ?

Scalebound or scaling?

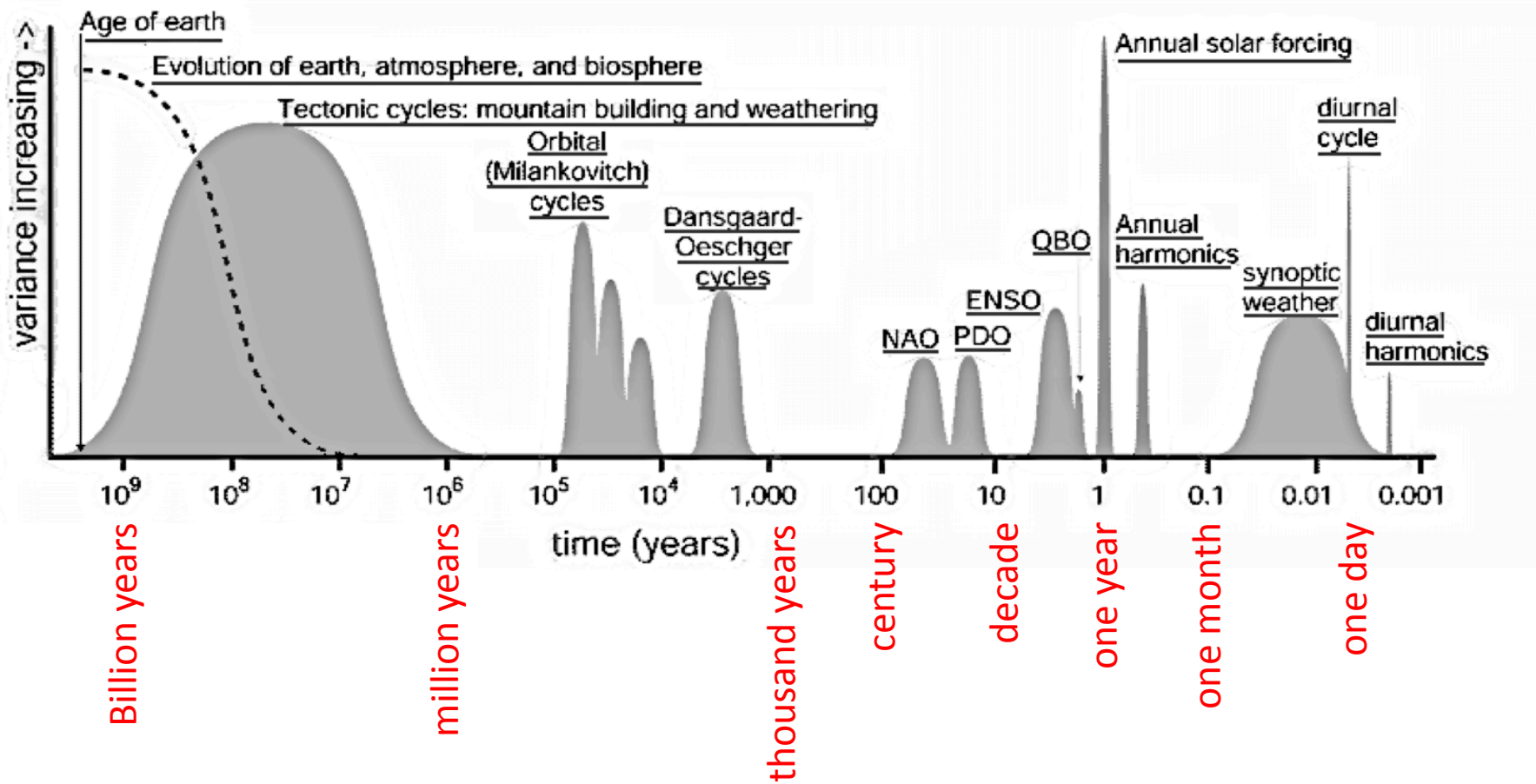
Supercomputers... or laptops?

# **From Van Leeuwenhoek to Mandelbrot**

**Scalebound thinking and the missing quadrillion**

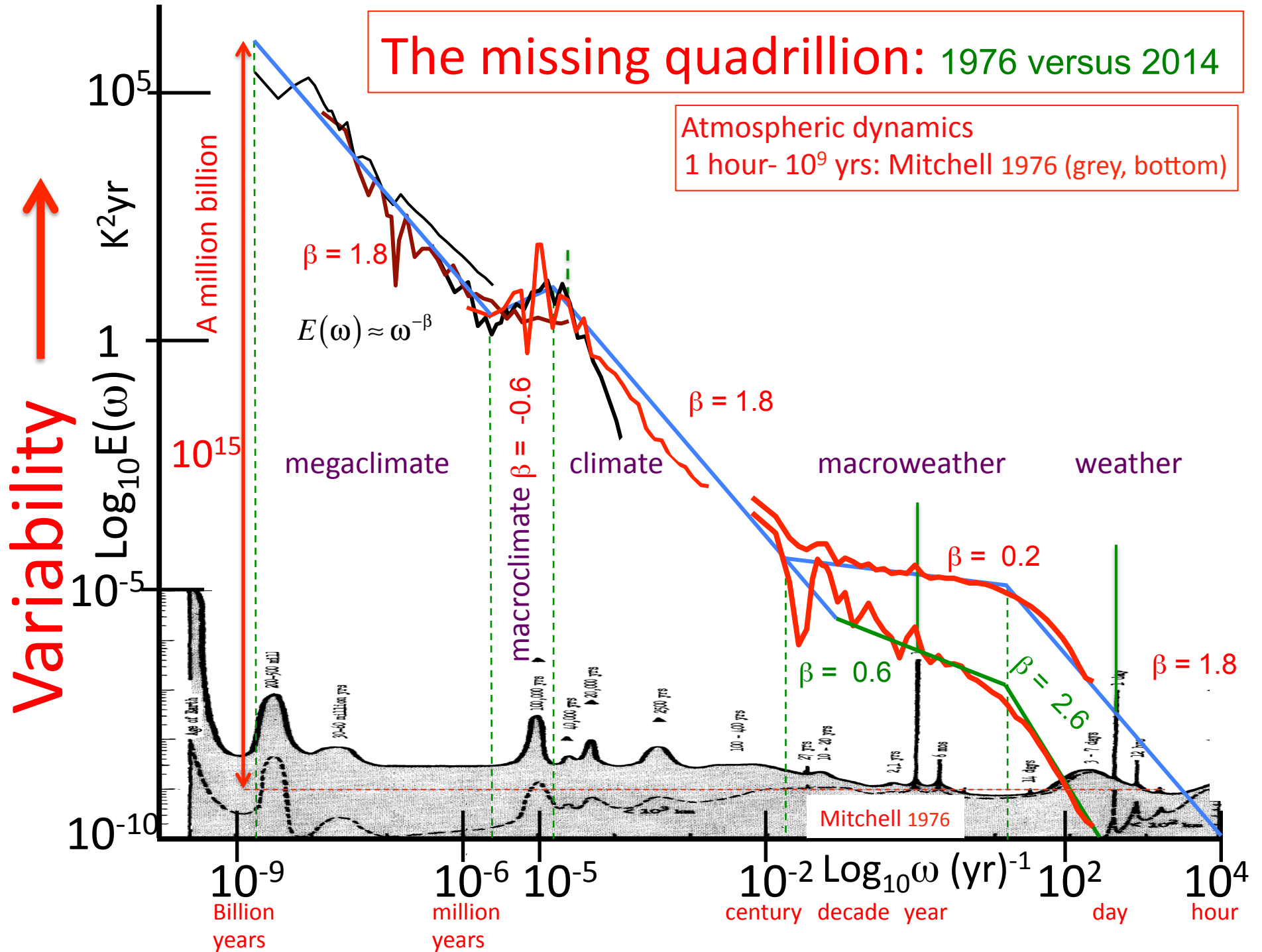


# Scalebound "Powers of ten" view

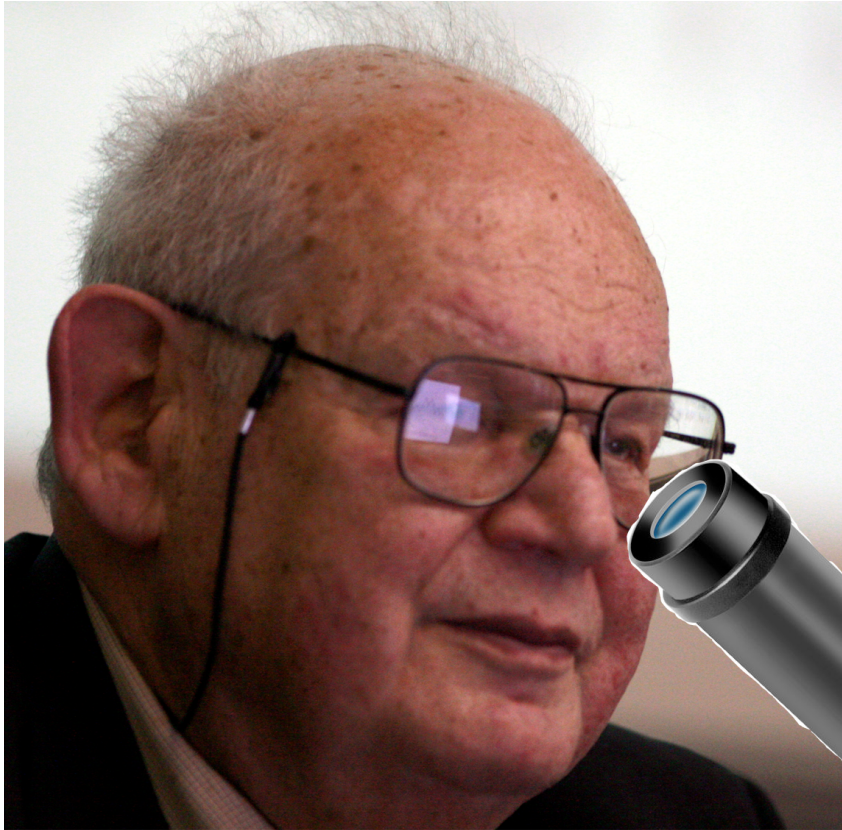


# The missing quadrillion: 1976 versus 2014

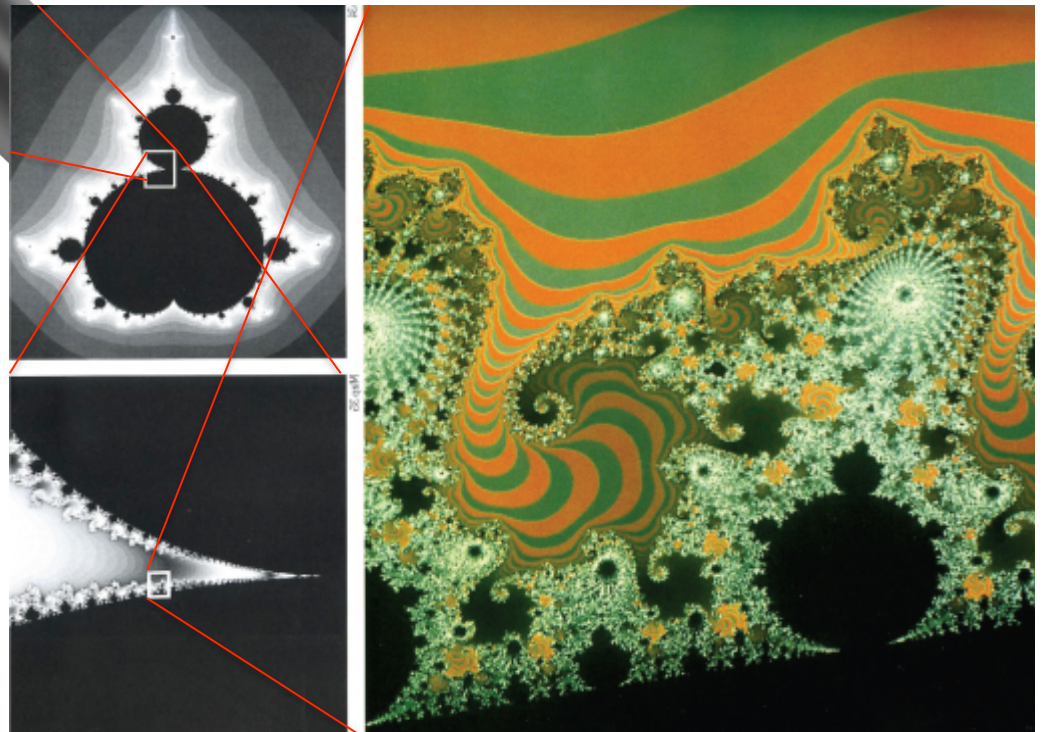
Atmospheric dynamics  
1 hour- 10<sup>9</sup> yrs: Mitchell 1976 (grey, bottom)



# The Scaling view



Mandelbrot (1924-2010)  
zooming into the  
Mandelbrot set



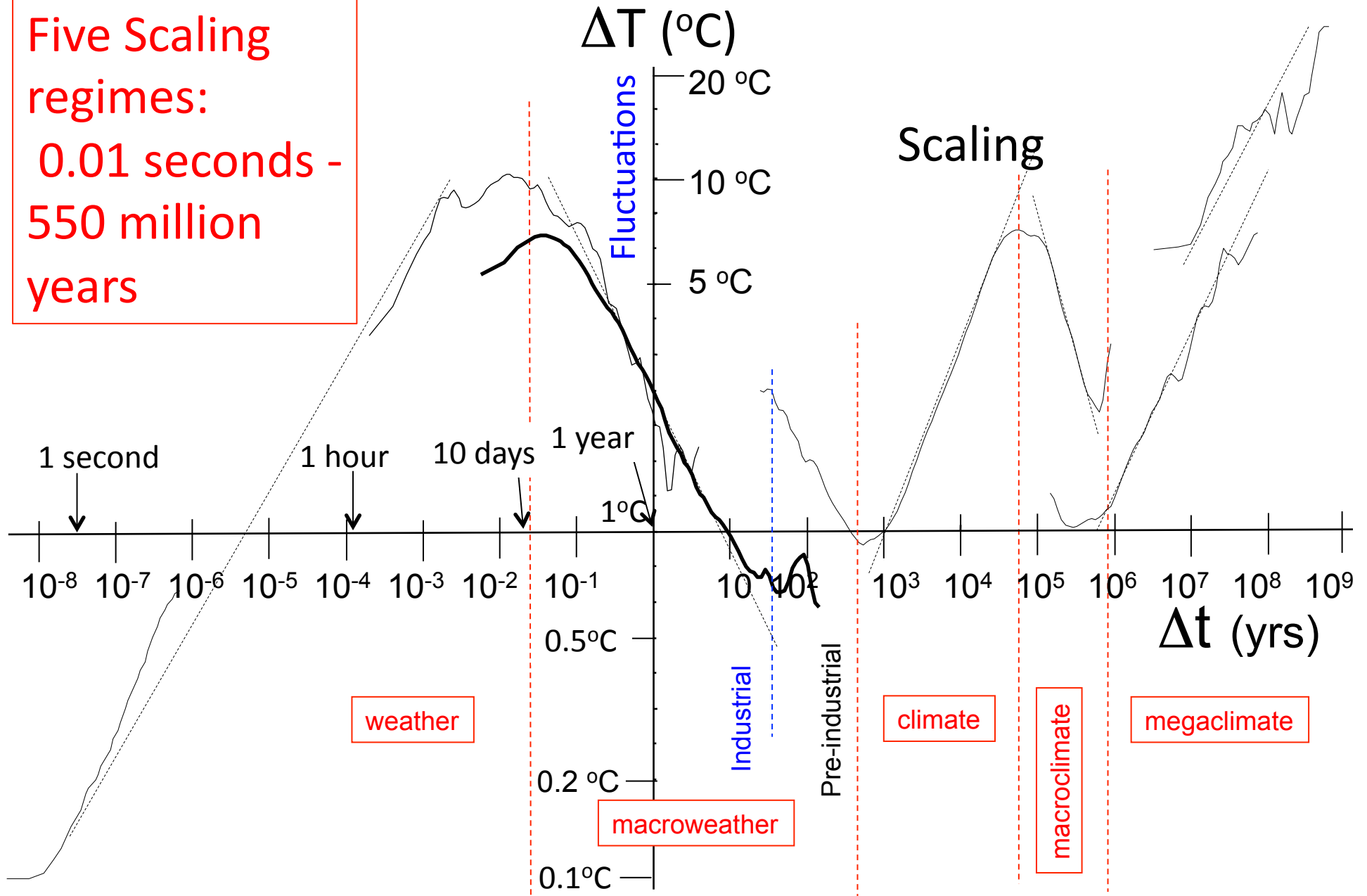


# Classifying atmospheric variability using Scale Invariance

- What is the weather?  
**Macroweather?**
- What is the Climate?

New simple technique (re)discovered in 2012: Fluctuation analysis

Five Scaling regimes:  
0.01 seconds -  
550 million  
years



$$\Delta T \approx \Delta t^H$$

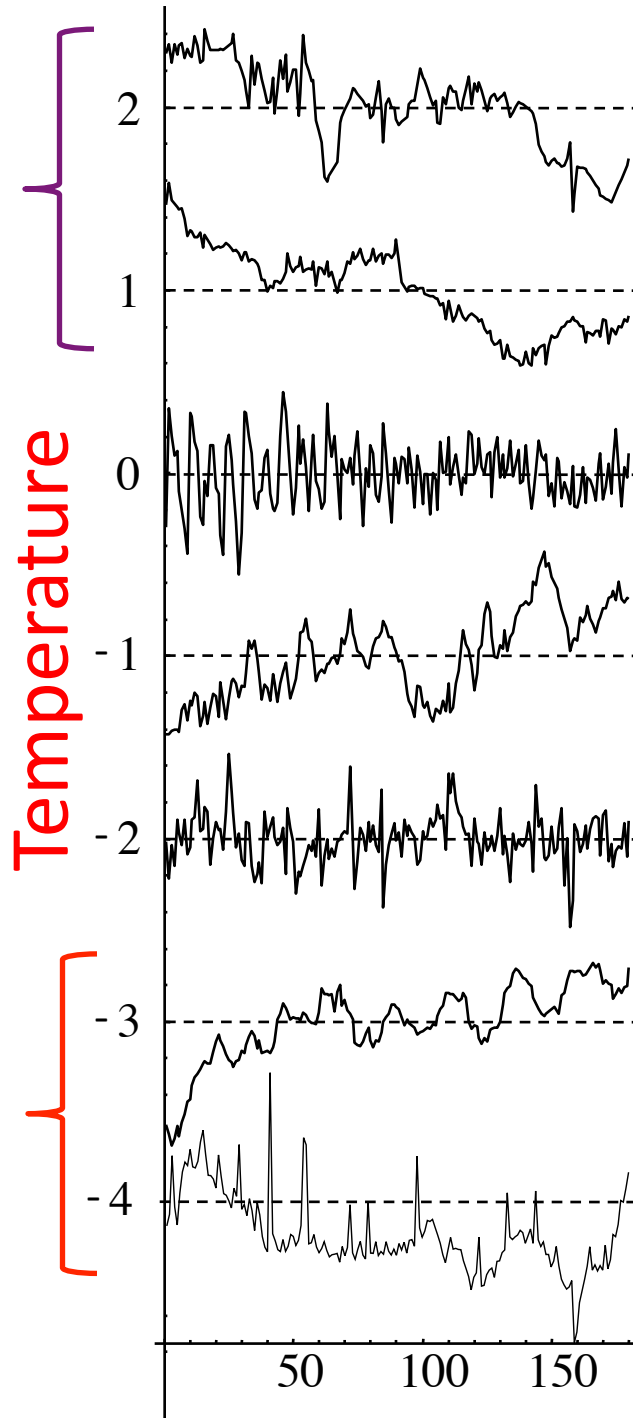
$H \approx 0.4$   
growing

$H \approx -0.8$   
decreasing

$H \approx 0.4$   
growing

$H \approx -0.4$   
decreasing

$H \approx 0.4$   
growing



**Megaclimate**  
Veizer: 290 Mys - 511 Myrs BP (1.23 Myr)

**Megaclimate**  
Zachos: 0-67 Myrs (370 kyr)

**Macroclimate**  
Huybers: 0-2.56 Myrs (14 kyr)

**Climate**  
Epica: 25-97 BP kyr (400 yrs)

**Macroweather**  
Berkeley: 1880-1895 AD (1 month)

**Weather**  
Lander Wy.: July 4-July 11, 2005 (1 hour)

**Weather**  
Thermistor, Montreal (0.017s)

**t**

# How does scaling help?

Scaling, scale invariance:

$$\text{Typical Fluctuation} \approx (\text{scale})^H$$

$H > 0$ : Fluctuations grow with scale, unstable

$H < 0$ : Fluctuations decrease with scale, stable

~~“The climate is what you expect, the weather is what you get”~~

## Expect Macroweather!

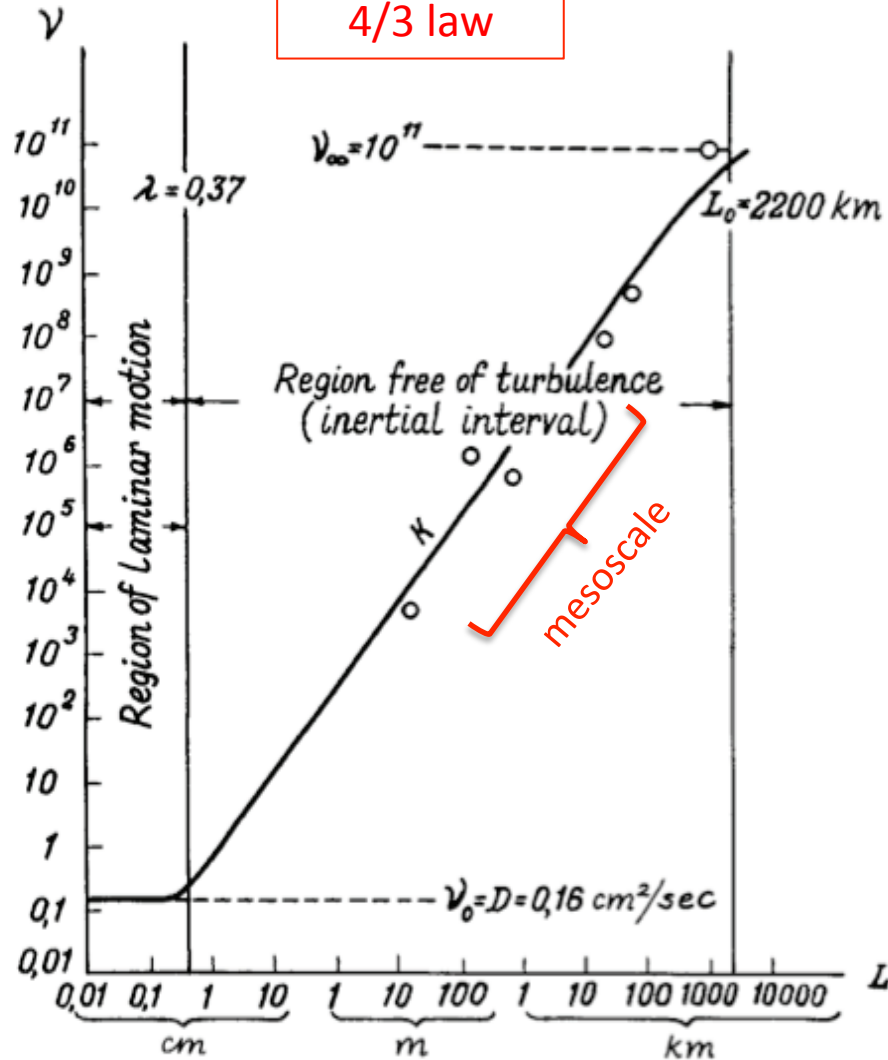
Weather:  $H > 0$ , macroweather,  $H < 0$ , climate,  $H > 0$

# An overview of atmospheric turbulence

How is it that in 2018 there is no consensus on the large scale statistical properties of the atmosphere?

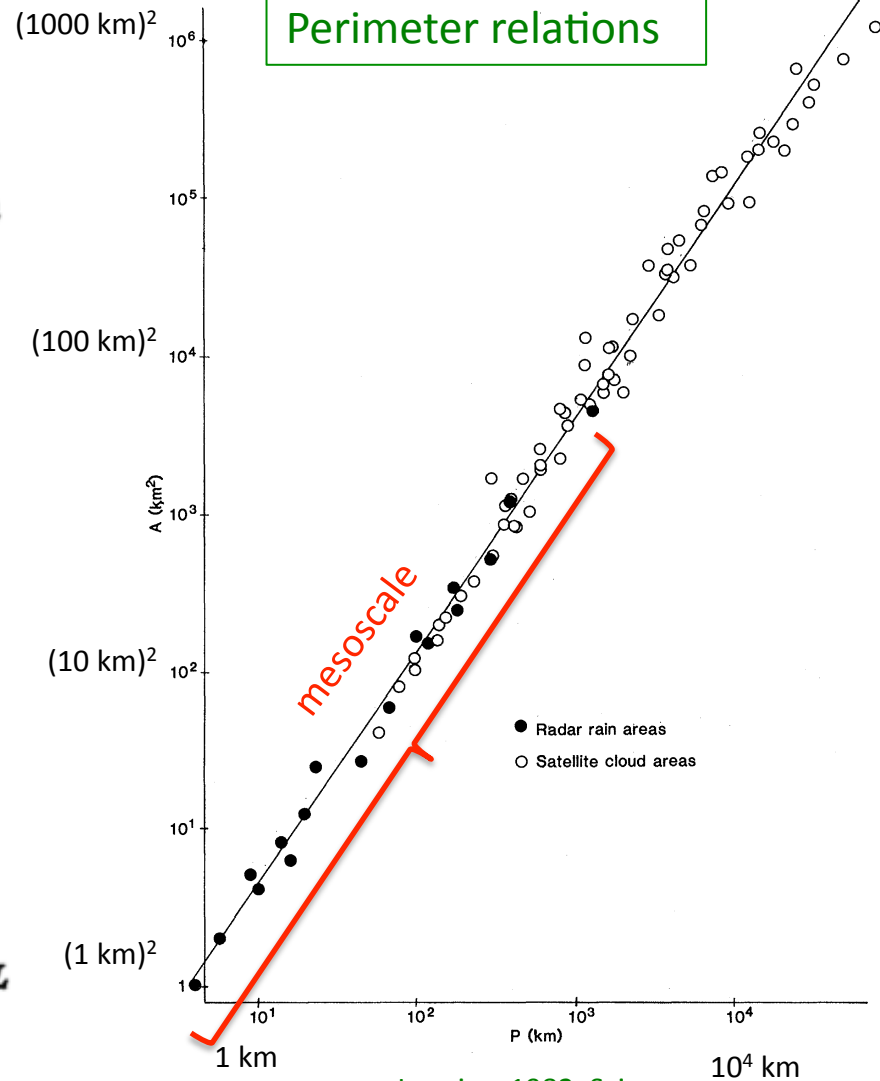
# Early indications of wide range scaling

Richardson's  
4/3 law



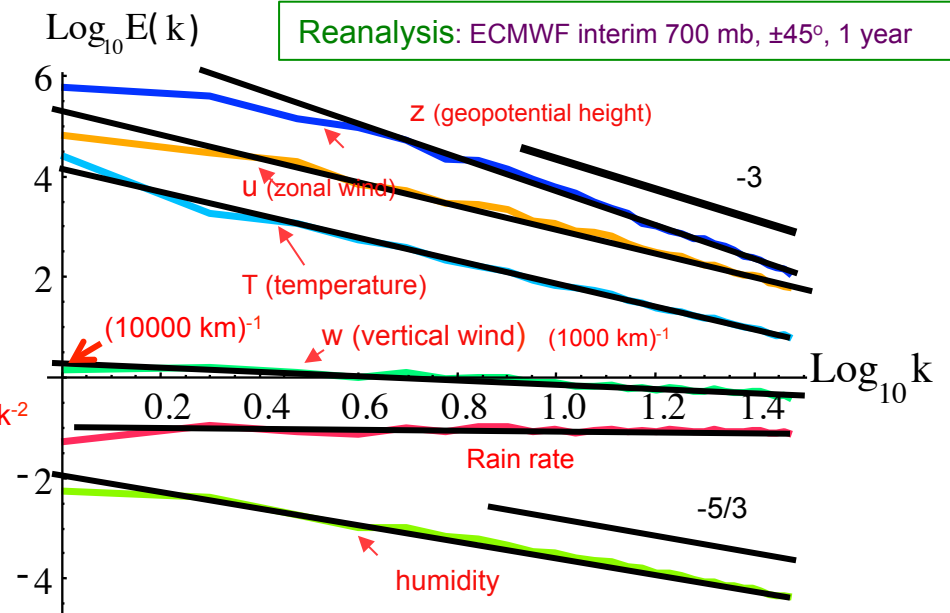
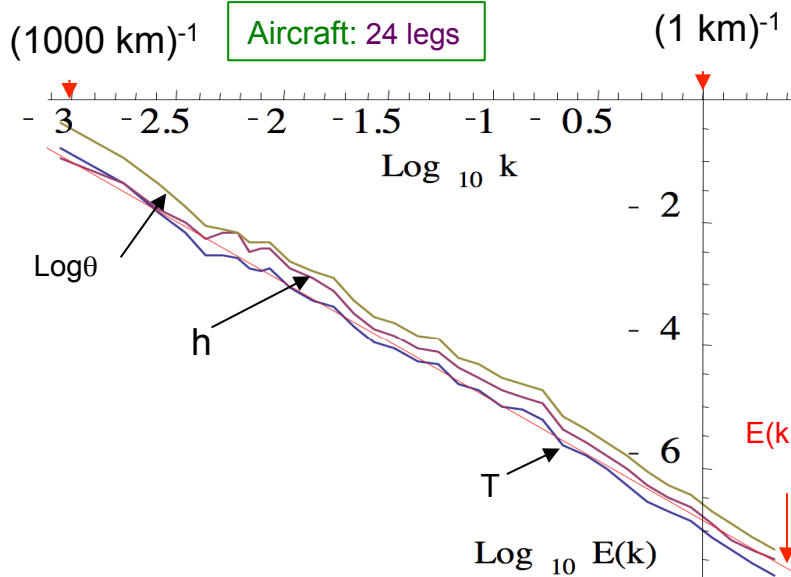
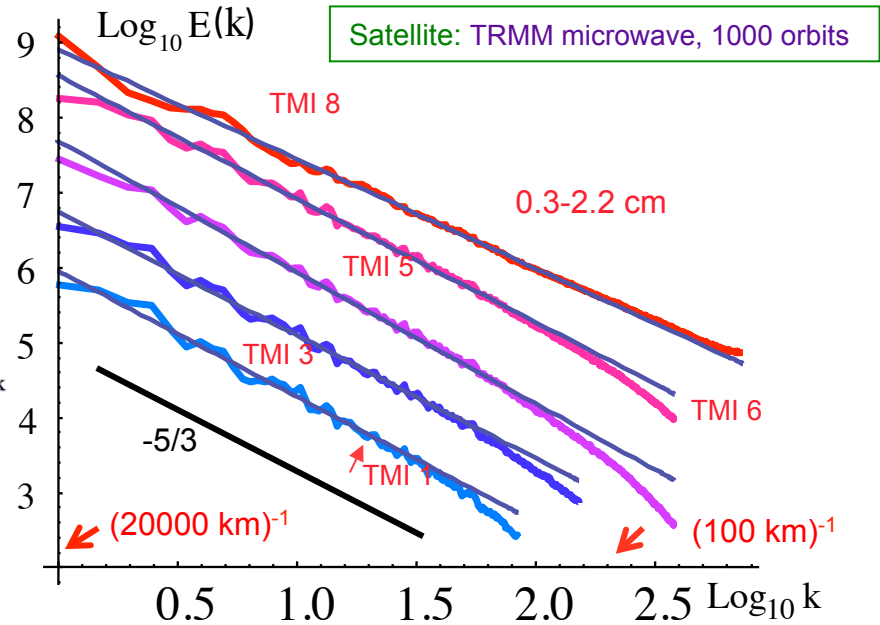
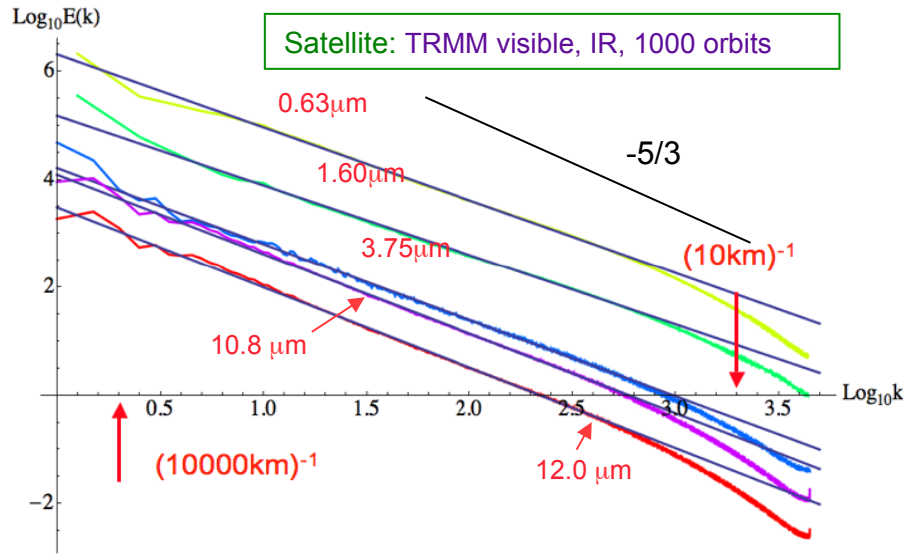
Richardson 1926, redrawn by Monin 1972

Cloud and rain Area-  
Perimeter relations



Lovejoy, 1982, Science

# Today: Planetary scale Horizontal Scaling $E(k) = k^{-\beta}$



# Origin of scaling...

Equations of motion



Anisotropic (stratified) zoom scale ratio  $\lambda$



$\lambda^H$  (Equations of motion)

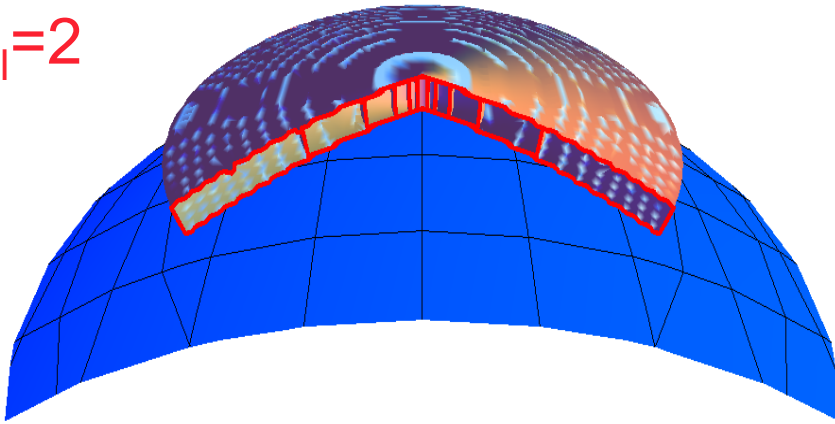
Generalized Scale Invariance:

Scale is an emergent quantity determined by the turbulent dynamics....

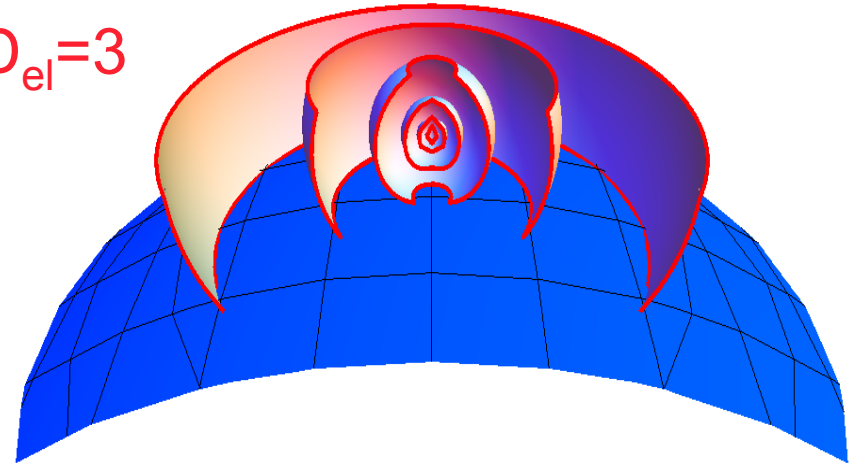


# Anisotropic Scaling (Generalized Scale Invariance) (Schertzer and Lovejoy 1985)

$D_{el}=2$

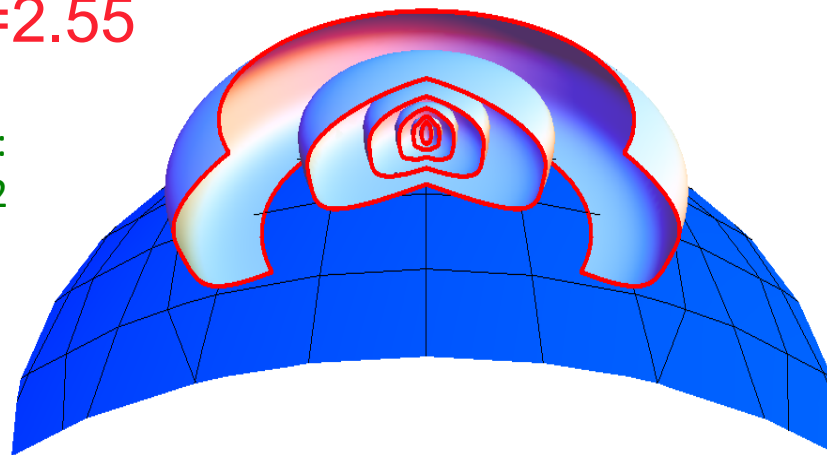


$D_{el}=3$



$D_{el}=23/9=2.55$

empirical:  
 $2.57 \pm 0.02$



**The 23/9D model:**

$$\underbrace{\Delta v(\Delta x) = \varepsilon^{1/3} \Delta x^{1/3}}_{\text{Kolmogorov}}; \quad \overbrace{\Delta v(\Delta z) = \phi^{1/5} \Delta z^{3/5}}^{\text{Bolgiano-Obukhov}}$$

$\text{Volume} \approx L \cdot L \cdot L^{H_z} \approx L^{D_{el}} \quad D_{el} = 2 + H_z = 23/9$

$H_z = (1/3)/(3/5) = 5/9$

# Anisotropic, Stratified Scaling

Stochastic

5km



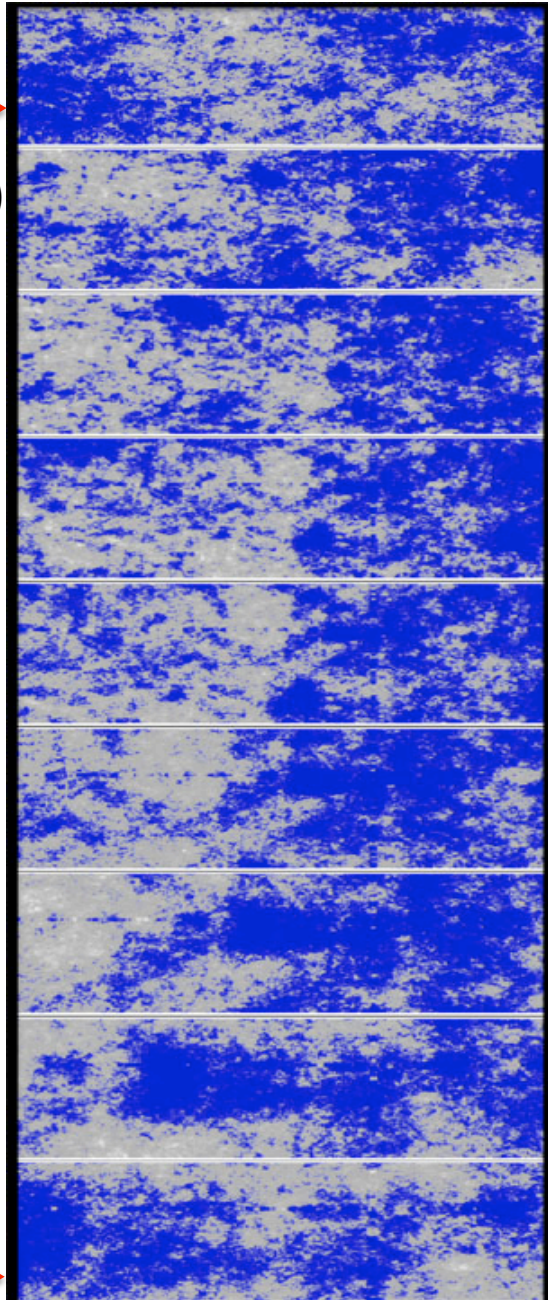
Blow up X 2.9 (each)



**Isotropic**

Total: X5000

1 m



# Stochastic Fractional Energy Balance Equation

FEBE

$$\tau^H \frac{d^H T}{dt^H} + T = \lambda \mathfrak{S}(t) \quad \leftarrow \text{Forcing (stochastic)}$$

Storage  $\uparrow$  Climate sensitivity  $\uparrow$

$$\mathfrak{S}(t) = \langle \mathfrak{S}(t) \rangle + \gamma(t)$$

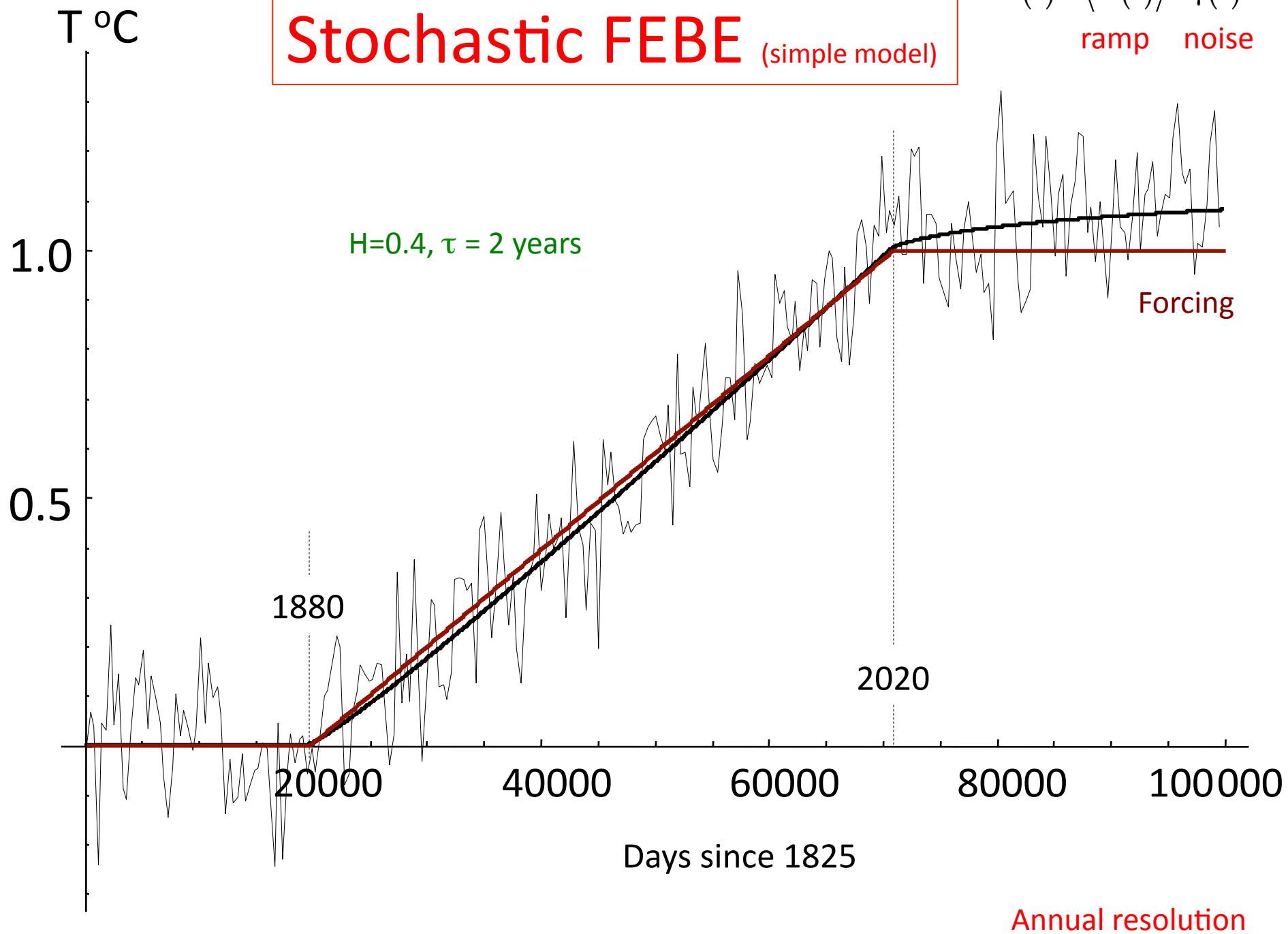
Assumptions:

- a) linearity of response (forcing  $\approx$  1% of long term mean)
- b) Scaling of storage mechanisms

# Stochastic FEBE (simple model)

$$\mathfrak{S}(t) = \langle \mathfrak{S}(t) \rangle + \gamma(t)$$

ramp noise



# Stochastic Unification of externally forced and internal variability

The forcing:

$$\mathfrak{S}(t) = \langle \mathfrak{S}(t) \rangle + \mathfrak{S}_i(t)$$

Ensemble average (deterministic e.g. anthropogenic) →  $\langle \mathfrak{S}(t) \rangle$   
 Random deviation due to “innovations” →  $\mathfrak{S}_i(t)$

$$\langle \mathfrak{S}(t) \rangle = F(t) \quad \text{External forcing}$$

$$\mathfrak{S}_i(t) = \sigma \gamma(t) \quad \text{Internal forcing: “innovations” unbalanced internal heat sources}$$

Amplitude of the innovations →  $\sigma$   
 Stochastic innovations (mean= 0) →  $\gamma(t)$

Temperature response:

$$T(t) = \langle T(t) \rangle + T_i(t)$$

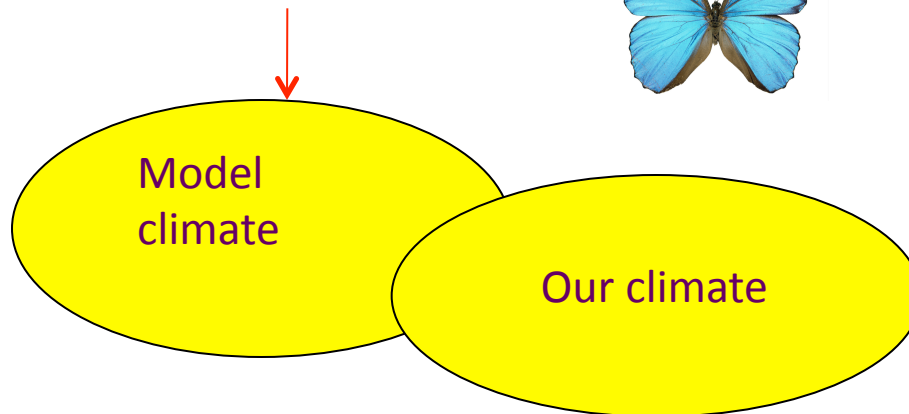
Forced response to external forcing →  $\langle T(t) \rangle$   
 Internal variability (Temperature anomalies) →  $T_i(t)$

## Clarification of internal versus externally forced variability

Externally forced variability:  $\langle T(t) \rangle$       Internal:  $T_i(t) = T(t) - \langle T(t) \rangle$

# Forecasts and projections should be based on real world climates

Weather systems (<10 days) generated by GCMs  
= random weather noise (statistics)...  
but not fully realistic



Scaling models can use data to force convergence to the real climate.

# Using scaling for long range (macroweather) forecasts



Stochastic Seasonal to Interannual Prediction System

Based on high frequency FEBE response:

$$G_{high}(t) \propto t^{H-1}$$



# StocSIPS

# StocSIPS

Stochastic Seasonal to Inter-annual Prediction System

Stochastic Seasonal to Interannual Prediction System

Home

Forecasts

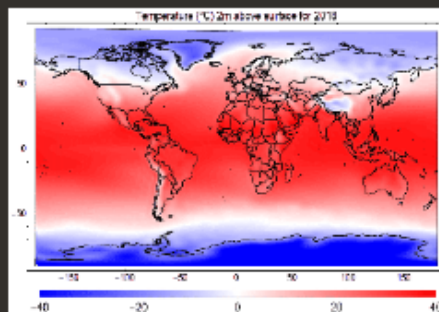
CanStoc

Hindcasts

Verification

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Contact Us

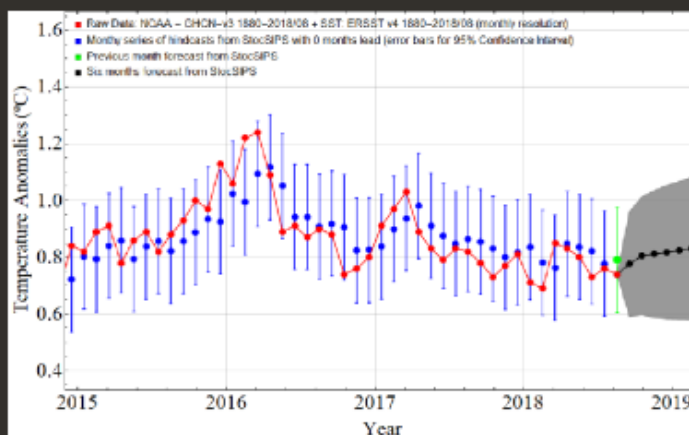


## A different way of forecasting 1

The Stochastic Seasonal and Interannual Prediction System (StocSIPS) is a revolutionary new technique for forecasting the state of the atmosphere from several weeks to decades. The core StocSIPS technology is the ScaLInG Macroweather Model (SLIMM) forecast module. The science behind StocSIPS is the discovery that the atmosphere has a truly elephantine memory. This memory is exploited by SLIMM that extracts information from many years of past data.



## Global temperature forecast and hindcasts for monthly, seasonal and annual resolutions



### Monthly resolution (forecast)

**Reference:** Global temperature anomalies at monthly resolution (shown in red in the figure) updated until August, 2018 with respect to the 20<sup>th</sup> Century (1901-2000) average. Global Surface Temperature Anomalies taken from <http://www.ncdc.noaa.gov/monitoring-references/faq/anomalies.php>.

**Hindcasts:** The series of hindcasts with zero months lead time since 2015 produced with StocSIPS is shown in blue in the figure with the corresponding 95% confidence interval (green for the last month).

**Forecast:** The median forecast until June, 2018 is shown in black in the figure (grey for the corresponding 95% confidence interval).



Regional temperature forecasts at different temporal resolutions and lead times (main Forecast page here)



# Stochastic Seasonal and Interannual Prediction System (StocSIPS)

Lovejoy, Del Rio Amador, Hebert, 2015

Fractional Gaussian noise = fGn (scaling, smoothed white noise)

$$T(t) = \sigma_\gamma \int_{-\infty}^t (t-t')^{-(1/2-H)} \gamma(t') dt'$$

Gaussian noise

- Power law correlation. Vast memory that can be exploited.
- Predictor for  $-1/2 < H < 0$  based on past data.

$$\hat{T}(n+k | n) = \sum_{j=0}^p G_H(j) T(n-j)$$

kernel

predictor

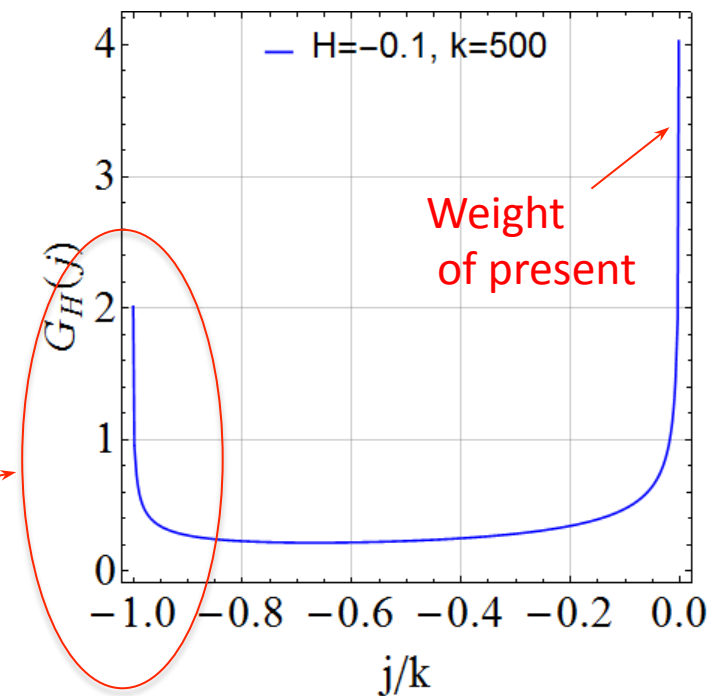
data

Weight of the distant past

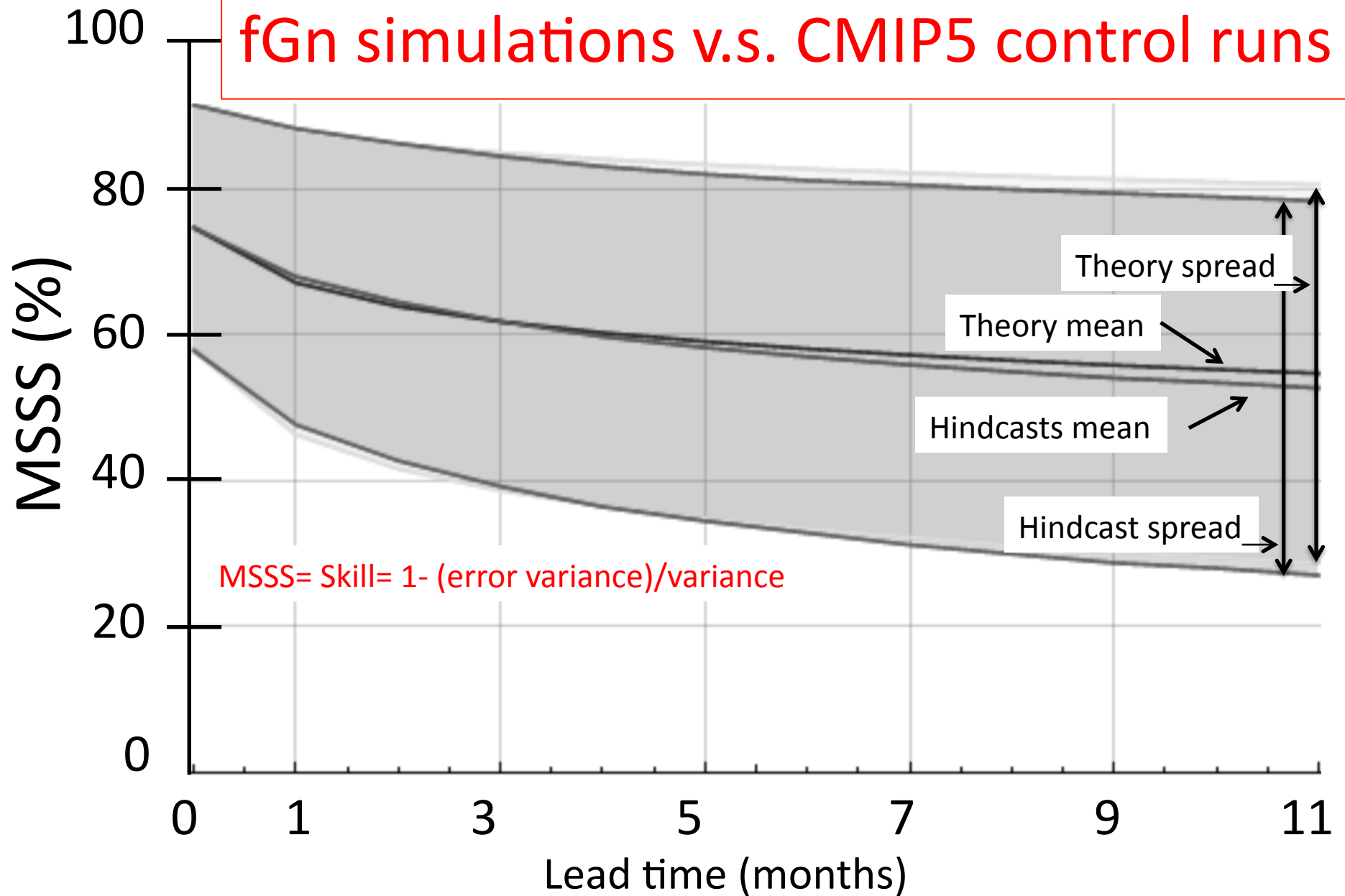
**“The ‘closest witnesses’ to the unobserved past have special weight”**

Gripenberg and Norris 1996

Kernel for  $H = -0.1$

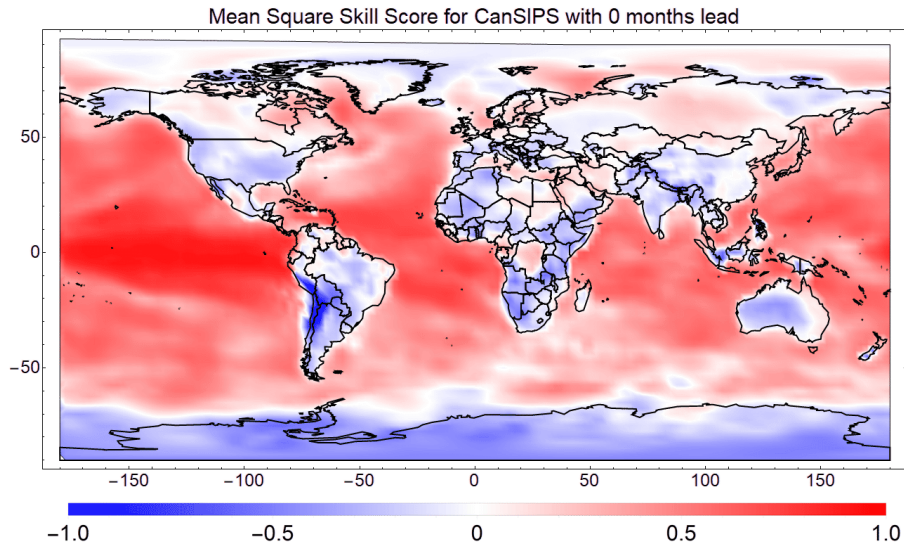


# Hindcasting: fGn simulations v.s. CMIP5 control runs



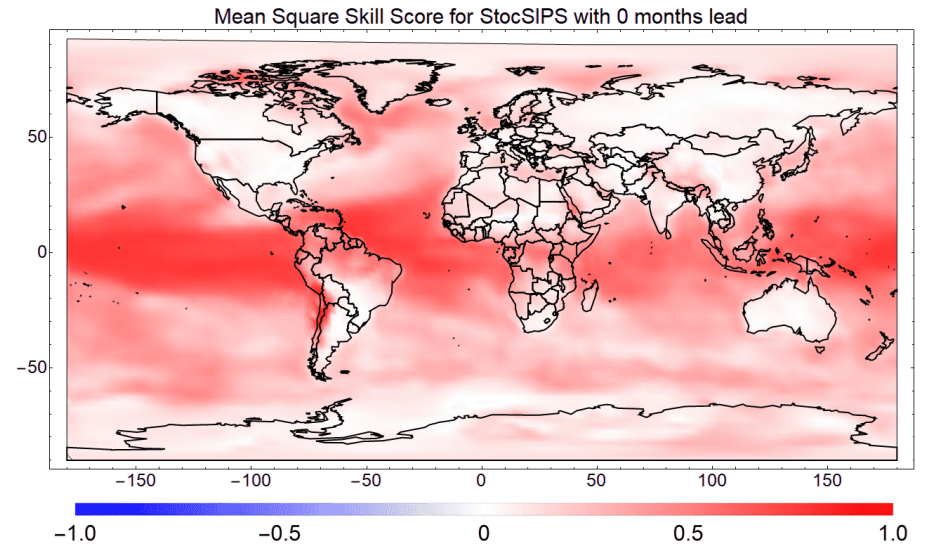
# 0-months lead Skill (hindcasts 1980-2010)

## CanSIPS (GCM)

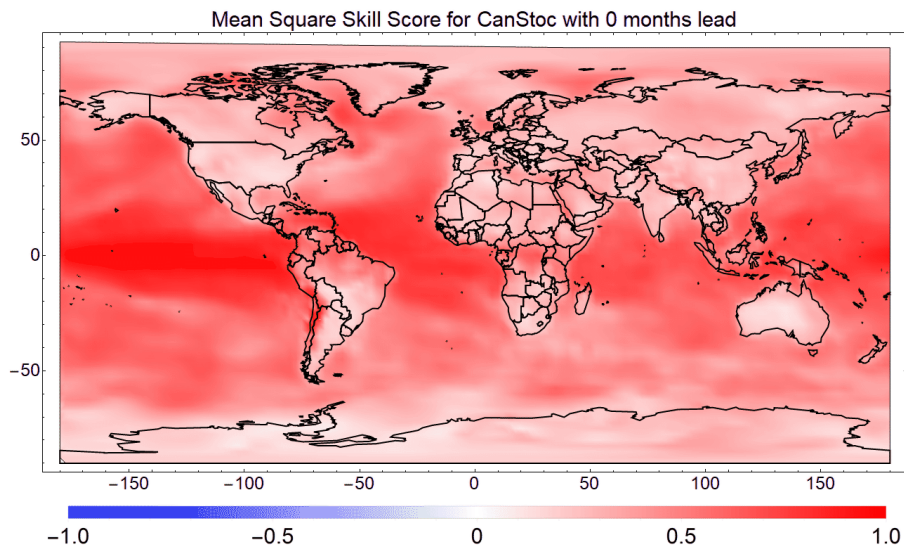


## StocSIPS

Red: high skill, blue, low skill

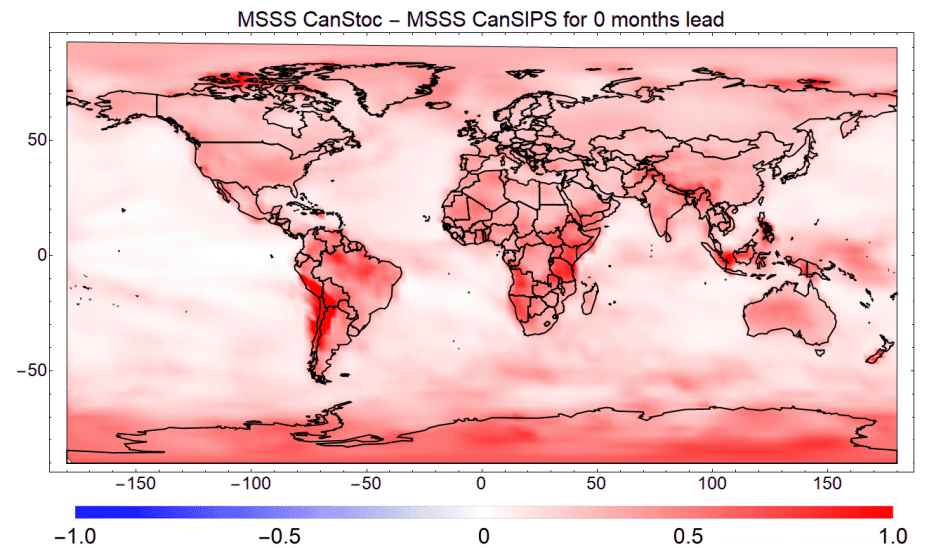


## CanStoc (hybrid)



## CanStoc - CanSIPS

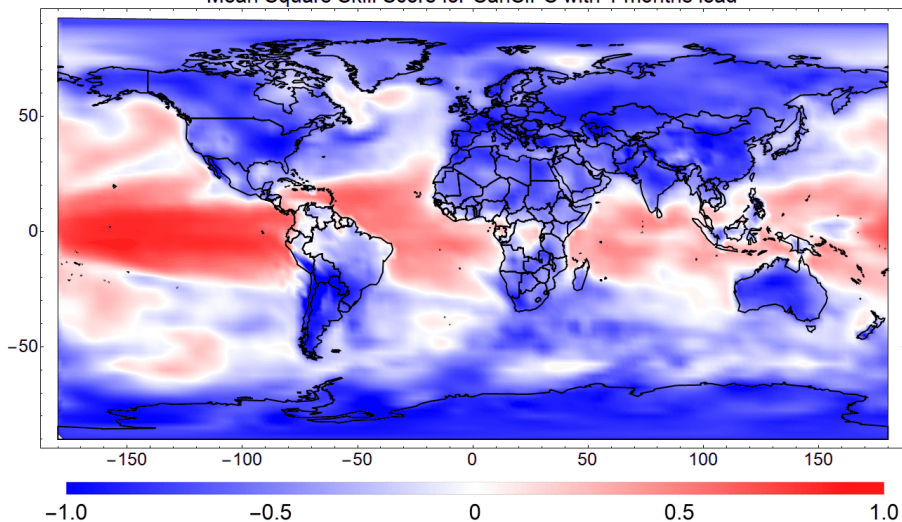
Red: CanStoc higher skill than CanSIPS



# 1-month lead (hindcasts 1980-2010)

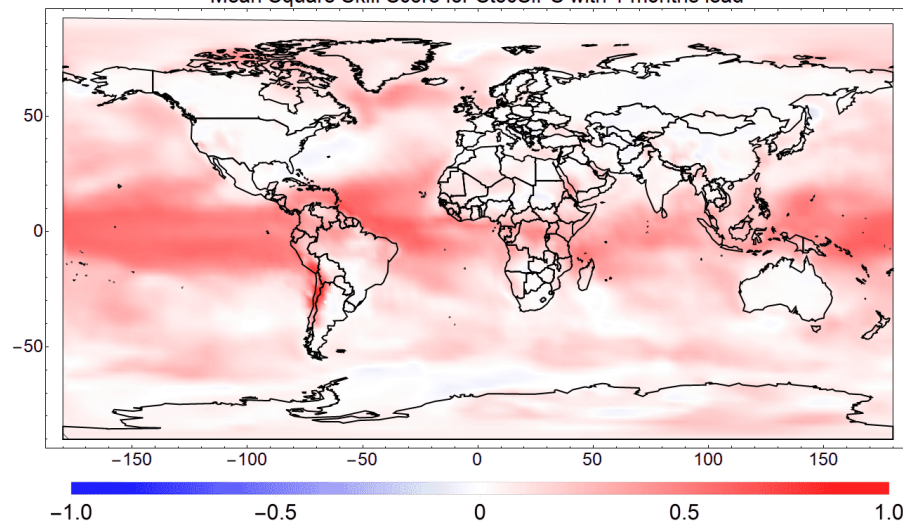
## CanSIPS

Mean Square Skill Score for CanSIPS with 1 months lead



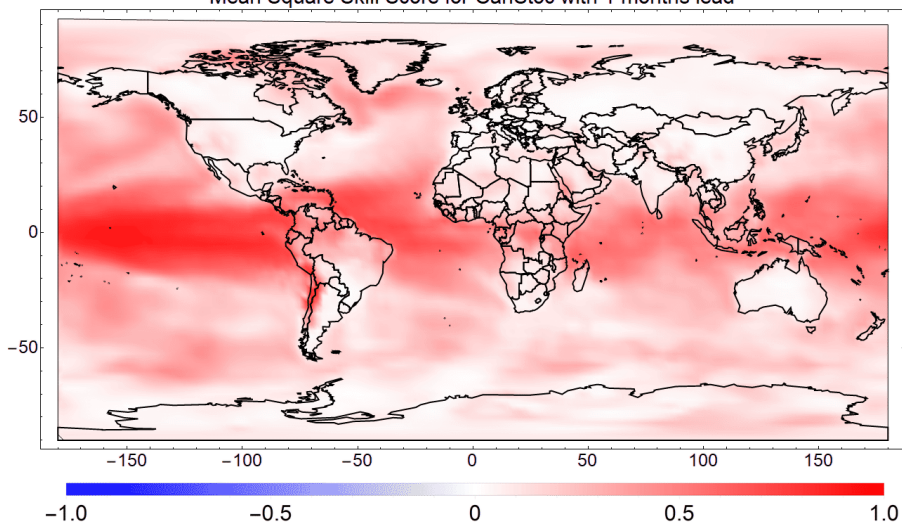
## StocSIPS

Mean Square Skill Score for StocSIPS with 1 months lead



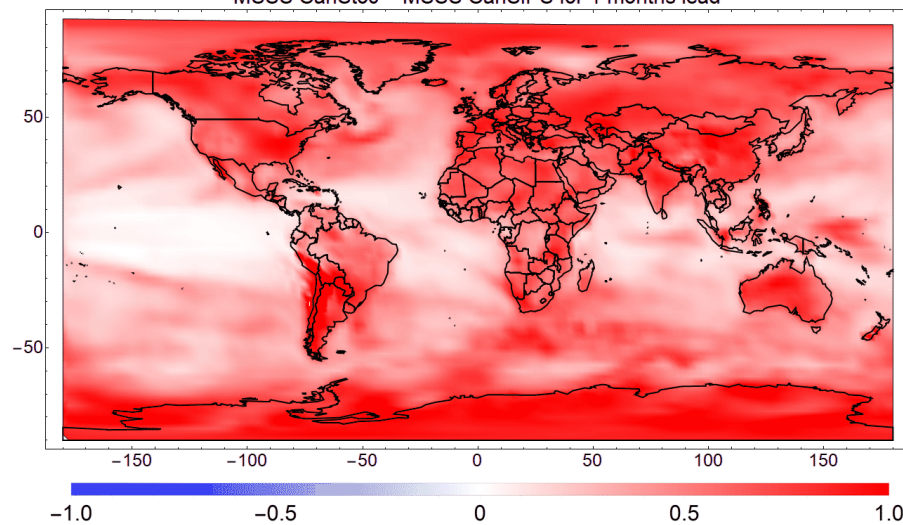
## CanStoc

Mean Square Skill Score for CanStoc with 1 months lead



## CanStoc - CanSIPS

MSSS CanStoc - MSSS CanSIPS for 1 months lead



# Using scaling for projections

- Key assumption: linearity of the response
- Based on step function response
- Scaling of storage
  - (consequence: power law relaxation to thermal equilibrium)

Based on low frequency FBE response

$$G_{low}(t) \propto t^{-H-1}$$

# 2050-2100: An uncertainty crisis

GCM's: for CO<sub>2</sub> doubling:

US National Academy of Science (1979): 1.5- 4.5°C

IPCC1	(1992):	1.5- 4.5°C
IPCC2	(1996):	1.5- 4.5°C
IPCC3	(2002):	1.5- 4.5°C
IPCC4	(2007):	2- 4.5°C
IPCC5	(2013):	1.5- 4.5°C

Diminishing returns....

# Two historical methods: simple and scaling

Projections: Hebert, Lovejoy, Tremblay, 2018

Forcing

$$T(t) = \lambda G_F * F$$

"\*" = convolution:

$$T(t) = \lambda \int_{-\infty}^t G_F(t-t') F(t') dt'$$

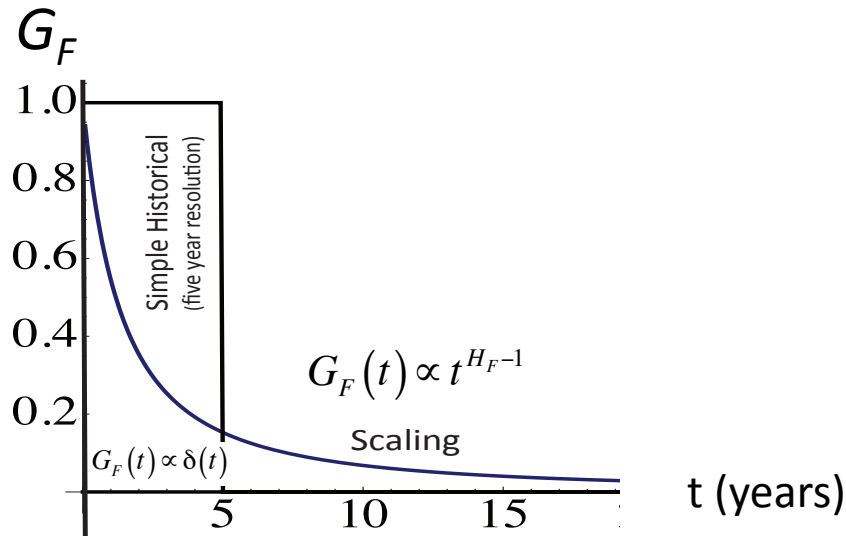
Simple  $G_F(t) \propto \delta(t)$

Scaling (long memory)  $G_F(t) \propto t^{H_F-1}; t \gg \tau$

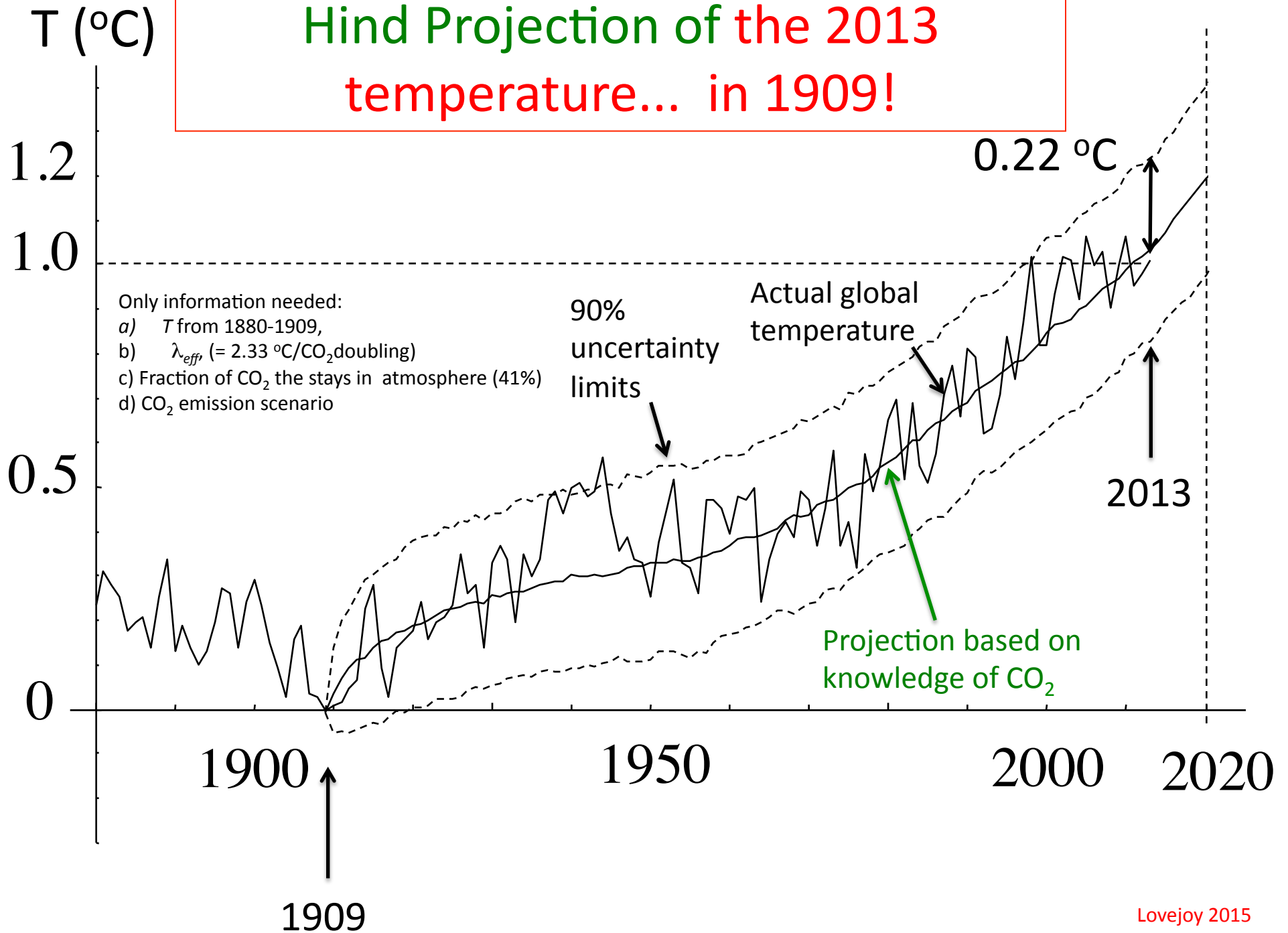
Scaling Climate Response  
Function = SCRF

Response exponent:  
 $H_F \approx -0.5 \pm 0.2$

High frequency truncation  $\tau$ :  
Land-ocean coupling time ( $\approx 2$  years)



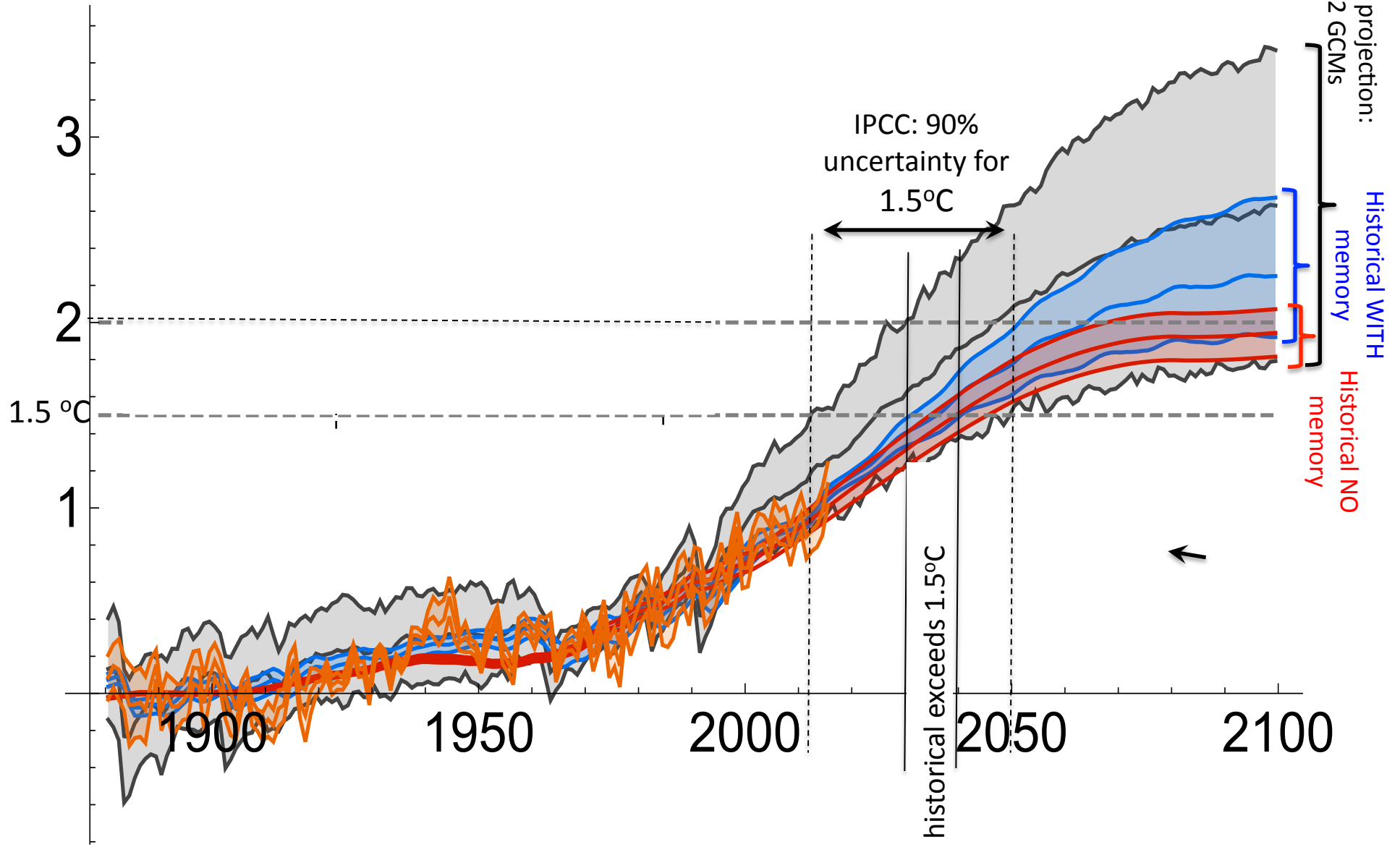
# Hind Projection of the 2013 temperature... in 1909!



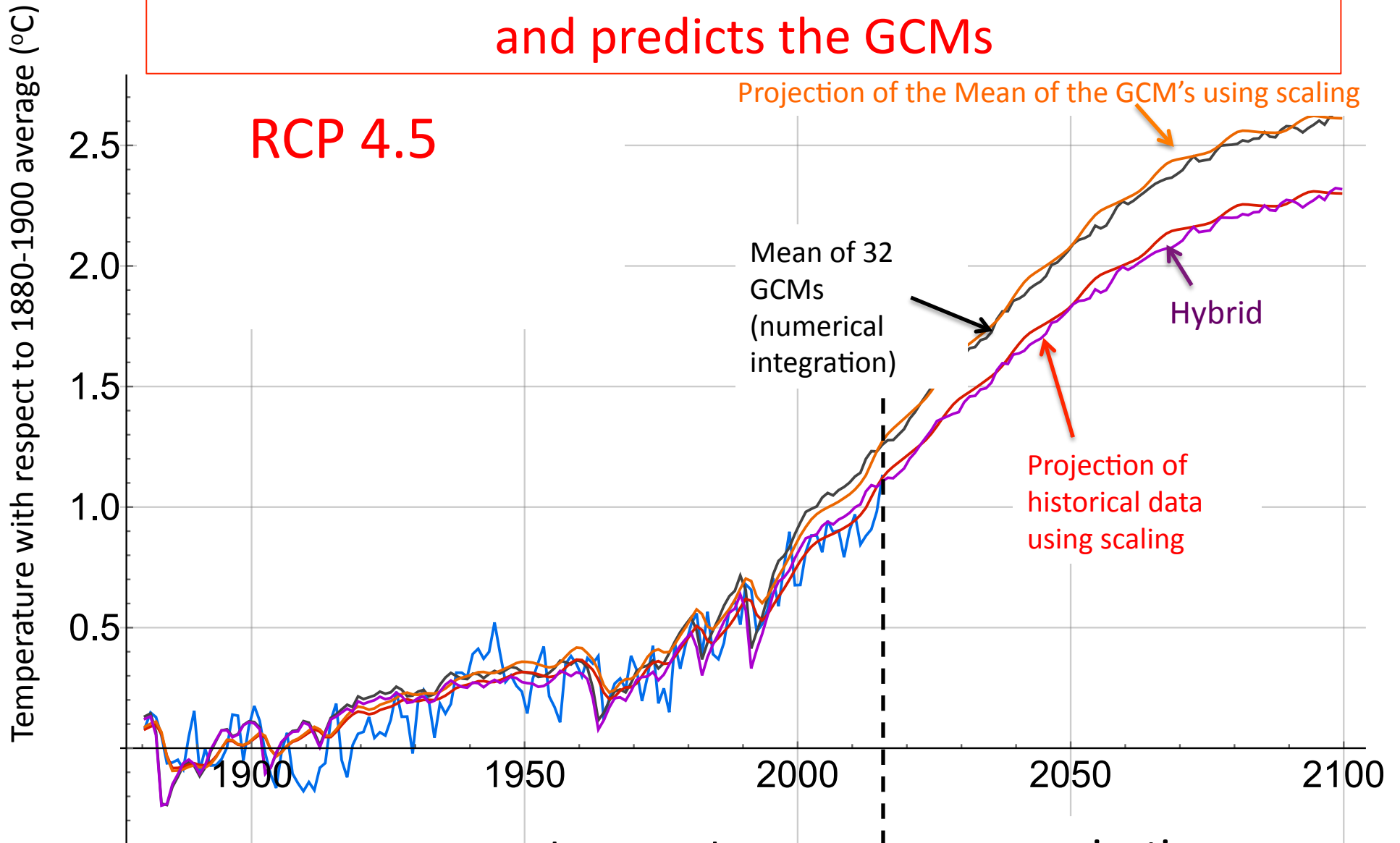


# Moderate mitigation scenario (RCP 4.5)

$\Delta T$  °C

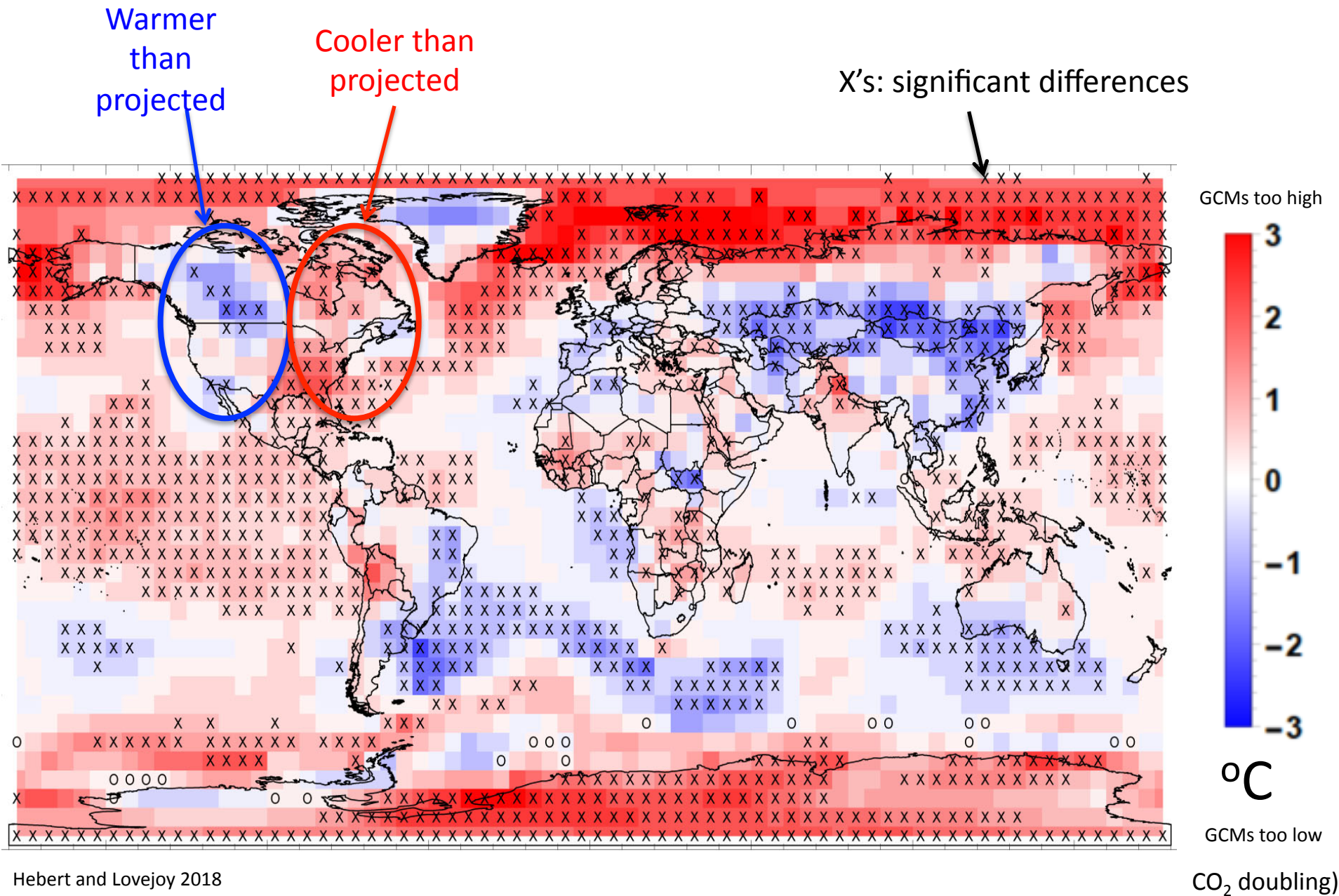


Validation: the historical method reproduces the past and predicts the GCMs



Conclusion: GCMs (nearly) linearly project the past... but their climates are unrealistic. Better to project the observations

# Differences: (GCM's)- (Historical Method)



# 2010's: Unity refound?

## Developments $\approx$ 1980's-2010:

**Empirical:** space-time statistical scaling analyses extended from small to global scales, (aircraft, satellites). [Also to numerical model outputs...](#)

**Theory:** Multifractals (intermittency), Generalized Scale Invariance, anisotropic scaling of governing equations (scaling stratification).

**Numerical:** Many numerical problems solved, NWP extended to climate: GCMs. Verification of scaling of GCM outputs.

## Post 2010:

GCM diminishing returns: climate sensitivity 1979 - present: 1.5-4.5°C /CO<sub>2</sub> doubling.  
Scaling for macroweather forecasting, including of GCM control runs.  
Scaling improves climate projections, reduces uncertainty.

# Climate Concepts: high versus low level laws (1)

	GCMs	Statistical Laws
What is Climate?	Control runs, strange attractors	Regime with fluctuations increasing with scale, beyond macroweather
Climate change?	Pullback-attractors	Change of climate states
Time scales	1 month (convenience), 30 years (fiat)	Objective transition scales $\tau_w, \tau_c$
Climate states	Average over 30 years	Average over $\tau_c$
Macroweather states	Monthly anomalies	Average of anomalies over weather scales ( $\tau_w$ ) w.r.t. the current climate state (scale $\tau_c$ ).
Equilibrium Climate Sensitivity	Asymptotic response to a step-function increase in forcing	Linear relation between forcing and response (memory can be estimated from internal variability).

# Climate Concepts: high versus low level laws (2)

	GCMs	Statistical Laws
Externally Forced variability	The response to processes outside the climate system that increase or decrease energy fluxes into it.	The response to deterministic forcing: the ensemble average of the response.
Internal variability	Variability due to dynamics internal to the climate system.	The response to stochastic innovations: the difference between the actual state and the ensemble averaged state.
Uncertainty	-“Structural uncertainty”(each model has different climate), -Initial condition uncertainty	Stochastic forcings, part of theory/model.
Uncertainty (climate projections)	The dispersions of GCMs about Multi-Model Ensemble.	The dispersion in the reconstructions of historic forcings and historic responses.
Predictability limits	Deterministic limits	Stochastic limits

A consequence of relying of GCMs: theory is not empirically informed.  
Ex.: The missing quadrillion.

# The future of climate science

- GCMs are research tools, each with its own climate. Not always the best tools for forecasting or projecting.
- Relying on a unique tool (e.g. for projecting to 2050) is weak: grounds for skepticism

- Beyond deterministic predictability limit, GCM's are stochastic.

*All they require are realistic grid scale statistics. This could be done at much lower resolutions, and with today's computers.*

Deterministic, mechanistic small scale details are not needed: *irrelevant!*

Modelling structures at 1km that live for 15 minutes and then averaging everything over a factor of a million to make a decadal projection is an unnecessary waste of resources.

-Stochastic scaling models are already the most realistic for macroweather and climate temperature forecasts and projections ... they could be possibly merged with GCM approaches for even greater accuracy.

- Better empirical grounding of theory.
- Better understanding of reality... and models!

