

# Pole Inflation in Jordan Frame SUGRA

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Based on arXiv:1709.03440 with

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# Outline

1. Basics of Pole Inflation

2. J-sugra and FKLMP Inflation Model

3. Pole Inflation in J-sugra Beyond FKLMP model

# 1. Basics of Pole Inflation

## 2. J-sugra and FKLMP Inflation Model

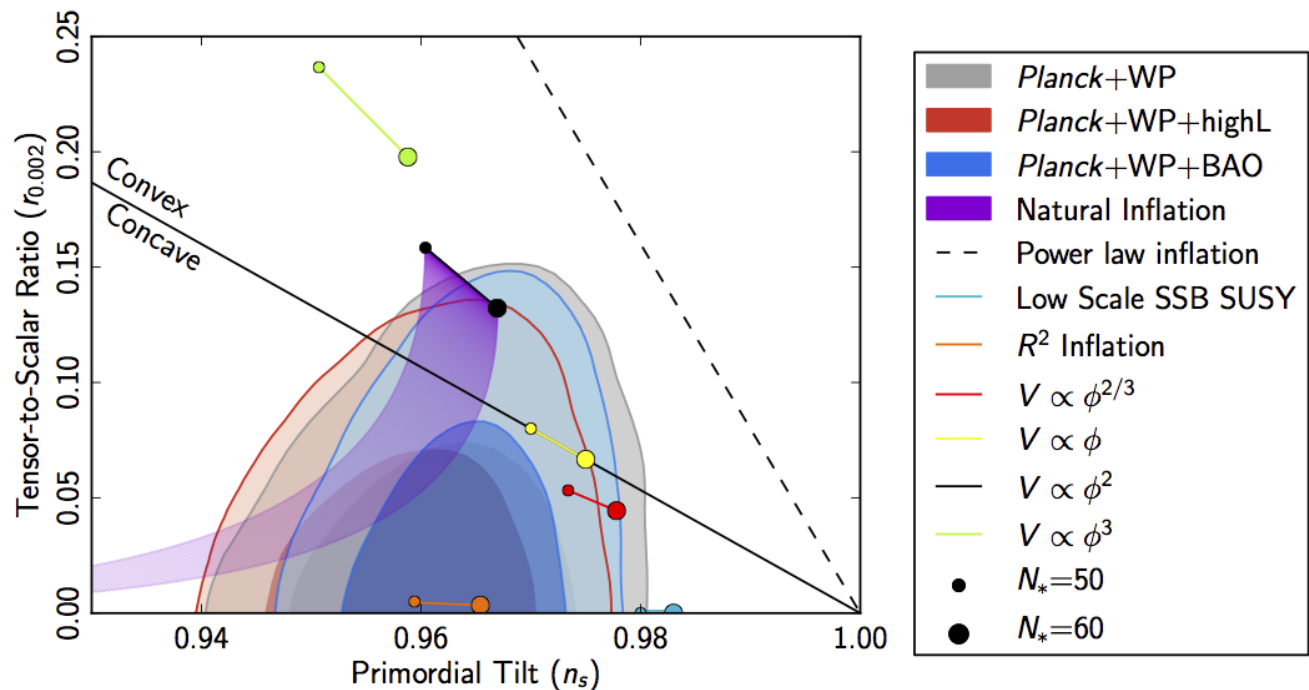
## 3. Pole Inflation in J-sugra Beyond FKLMP model

# Observation about Inflation

□ Observation of CMB gives us much information about inflation!

- Curvature power spectrum:  $P_\zeta \sim O(10^{-10})$
- Tensor to scalar ratio:  $r < 0.15$
- Spectral index:  $n_s \sim 0.96$

arXiv:1502.02114, Planck collaboration



# Basics of Slow Roll Inflation 1

- Simplest inflation model with canonical scalar field:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right]$$

- Inflation occurs when slow roll parameters are sufficiently small;

$$\epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2 \ll 1, \quad \eta = \frac{V''}{V} \ll 1$$

- Slow-roll inflation gives predictions in terms of a potential

$$P_\zeta = \frac{V}{24\pi^2\epsilon}, \quad n_s = 1 + 2\eta - 6\epsilon, \quad r = 16\epsilon$$

# Basics of Slow Roll Inflation 2

- ▣ Predictions strongly depend on potential:

$$P_\zeta = \frac{V}{24\pi^2\epsilon}, \quad n_s = 1 + 2\eta - 6\epsilon, \quad r = 16\epsilon \quad \epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2 \ll 1, \quad \eta = \frac{V''}{V} \ll 1$$

- ▣ Example: power law potential  $V(\phi) = g\phi^n$ ,  
Number of e-folding is given by

$$N(\phi) := \log(a_e/a(\phi)) = \int_{\phi_e}^{\phi} d\phi \frac{V}{V'} \sim \frac{1}{2n} \phi^2$$

- $n_s$  and  $r$  can be represented by  $N$  as

$$n_s = 1 - \frac{n+2}{2N}, \quad r = \frac{4n}{N}$$

- This model is excluded from the observation.

$$r = \frac{8n}{n+2} (1 - n_s) \sim \frac{8n}{n+2} \times 0.04 = \begin{cases} 0.16 & \text{for } n = 2 \\ 0.19 & \text{for } n = 3 \end{cases}$$

# Pole Inflation

- pole inflation: inflation driven by the pole in kinetic term

Galante, Kallosh, Linde, Rosset (2015)

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} K(\rho) (\nabla \rho)^2 - V(\rho) \right]$$

- Kinetic term has a pole:

$$K(\rho) = \frac{a_p}{\rho^p} (1 + \mathcal{O}(\rho))$$

- Potential is finite at pole:

$$V(\rho) = V_0 (1 - c\rho + \mathcal{O}(\rho^2))$$

- Ideas

In terms of canonically normalized field, potential is stretched near pole.



- Flat potential is realized!
- Predictions do not depend on the detail of potential!

# Prediction of pole inflation with $p = 2$

□ Let us derive the prediction of pole inflation with  $p=2$

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} K(\rho) (\nabla \rho)^2 - V(\rho) \right]$$

$$K(\rho) = \frac{a_2}{\rho^2} (1 + \mathcal{O}(\rho))$$

$$V(\rho) = V_0 (1 - c\rho + \mathcal{O}(\rho^2))$$



$$\phi = e^{-\frac{\rho}{\sqrt{a_2}}}$$

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} (\nabla \phi)^2 - V(e^{-\frac{\phi}{\sqrt{a_2}}}) \right]$$

□ By using the result of slow-roll inflation, we obtain

$$N(\phi) = \int_{\phi_e}^{\phi} d\phi \frac{V}{V'} \sim \frac{a_2}{c} e^{\frac{\phi}{\sqrt{a_2}}} \quad \epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2 = \frac{a_2}{2} \frac{1}{N} \quad \eta = \frac{V''}{V} = -\frac{1}{N}$$

at  $\rho \rightarrow 0, \phi \rightarrow \infty, N \rightarrow \infty$

and

$$P_\zeta = \frac{V_0}{24\pi^2} \frac{2N^2}{a_2}, \quad n_s = 1 - \frac{1}{N}, \quad r = \frac{8a_2}{N^2}$$

Predictions do not depend on the detail of potential and coincide with observation with  $a_2 \sim \mathcal{O}(1)$ .  $(n_s \sim 0.96 \rightarrow r \sim a_2 * 0.013)$



# Pole inflation from Jordan frame

- If fundamental physics prefer to Jordan frame, it would be natural inflaton has the canonical kinetic term there.

This setting naturally leads to non-trivial kinetic term in Einstein frame!

$$\mathcal{L} = \sqrt{-g_J} \left( f(\rho) R^J - \frac{1}{2} g_J^{\mu\nu} \partial_\mu \rho \partial_\nu \rho - V_J(\rho) \right)$$



$$g_J^{\mu\nu} = f g_E^{\mu\nu}$$

$$\mathcal{L} = \sqrt{-g_E} \left( \frac{1}{2} R^E - K(\rho) g_E^{\mu\nu} \partial_\mu \rho \partial_\nu \rho + \dots \right)$$

So it would be possible to realize pole inflation in this frame work.

- We investigate this mechanism based on Jordan frame supergravity.

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# Bosonic part of J-SUGRA

- Action of bosonic part of Jordan frame super gravity (without gauge fields)

$$\begin{aligned} \frac{\mathcal{L}}{\sqrt{-g_J}} = & -\frac{1}{6}\Phi R_J + \left( \frac{1}{3}\Phi g_{\alpha\bar{\beta}} - \frac{\Phi_\alpha \Phi_{\bar{\beta}}}{\Phi} \right) g_J^{\mu\nu} \partial_\mu z^\alpha \partial_\nu \bar{z}^{\bar{\beta}} \\ & - \frac{1}{4\Phi} \left( \Phi_\alpha \partial_\mu z^\alpha - \Phi_{\bar{\beta}} \partial_\mu \bar{z}^{\bar{\beta}} \right) \left( \Phi_\gamma \partial_\nu z^\gamma - \Phi_{\bar{\delta}} \partial_\nu \bar{z}^{\bar{\delta}} \right) g_J^{\mu\nu} - V_J \end{aligned}$$

- Dynamical variables;

$z^\alpha$  : complex scalar fields

$g_{\mu\nu}^J$  : space time metric in Jordan frame

$$g_{\alpha\bar{\beta}} = \partial_\alpha \partial_{\bar{\beta}} \mathcal{K}$$

$$V_J = \frac{1}{9} \Phi^2 e^{\mathcal{K}} \left( -3W\bar{W} + g^{\alpha\bar{\beta}} \nabla_\alpha W \nabla_{\bar{\beta}} \bar{W} \right)$$

$$\nabla_\alpha W = \partial_\alpha W + (\partial_\alpha \mathcal{K}) W$$

$$\Phi_\alpha = \partial_\alpha \Phi$$

- Arbitrary functions in theory;

- Kähler potential  $\mathcal{K}(z^\alpha, \bar{z}^{\bar{\beta}})$
- Super potential  $W(z^\alpha, \bar{z}^{\bar{\beta}})$
- Frame function  $\Phi(z^\alpha, \bar{z}^{\bar{\beta}})$

# Einstein Frame SUGRA

- Einstein frame SUGRA is obtained by conformal transformation of metric

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = -\frac{1}{6}\Phi R_J + \left( \frac{1}{3}\Phi g_{\alpha\bar{\beta}} - \frac{\Phi_\alpha \Phi_{\bar{\beta}}}{\Phi} \right) g_J^{\mu\nu} \partial_\mu z^\alpha \partial_\nu \bar{z}^{\bar{\beta}}$$

$$- \frac{1}{4\Phi} \left( \Phi_\alpha \partial_\mu z^\alpha - \Phi_{\bar{\beta}} \partial_\mu \bar{z}^{\bar{\beta}} \right) \left( \Phi_\gamma \partial_\nu z^\gamma - \Phi_{\bar{\delta}} \partial_\nu \bar{z}^{\bar{\delta}} \right) g_J^{\mu\nu} - V_J$$

$$\Downarrow \quad g_{\mu\nu}^E = -\frac{1}{3}\Phi g_{\mu\nu}^J$$

$$\frac{\mathcal{L}}{\sqrt{-g_E}} = \frac{1}{2}R_E - g_{\alpha\bar{\beta}} g_E^{\mu\nu} \partial_\mu z^\alpha \partial_\nu \bar{z}^{\bar{\beta}} - V_E$$

$$g_{\alpha\bar{\beta}} = \partial_\alpha \partial_{\bar{\beta}} \mathcal{K}$$

$$V_E = e^{\mathcal{K}} \left( -3W\bar{W} + g^{\alpha\bar{\beta}} \nabla_\alpha W \nabla_{\bar{\beta}} \bar{W} \right)$$

- Frame function does not appear.

$\mathcal{K}(z^\alpha, \bar{z}^{\bar{\beta}})$  ,  $W(z^\alpha, \bar{z}^{\bar{\beta}})$  : arbitrary function of Einstein frame SUGRA

$\Phi(z^\alpha, \bar{z}^{\bar{\beta}})$  : Function to characterize Jordan frame

# FKLMP model

S.Ferrara, R.Kallosh, A.Linde, A.Marrani, A. Van Proeyen (2010)

□ Inflation model in Jordan frame super gravity.

□ FKLMP model

$$\mathcal{K}(z, \bar{z}) = -3 \log \left( -\frac{1}{3} \Phi \right)$$

$$\Phi(z, \bar{z}) = -3 + \delta_{\alpha\bar{\beta}} z^\alpha \bar{z}^{\bar{\beta}} + J(z) + \bar{J}(\bar{z})$$

Assuming fields configuration satisfies

$$z^\alpha = \bar{z}^{\bar{\alpha}} \quad \Phi_\alpha \partial_\mu z^\alpha - \Phi_{\bar{\beta}} \partial_\mu \bar{z}^{\bar{\beta}} = 0$$

$$\longrightarrow \frac{\mathcal{L}}{\sqrt{-g_J}} = -\frac{1}{6} \Phi R_J - \delta_{\alpha\bar{\beta}} g_J^{\mu\nu} \partial_\mu z^\alpha \partial_\nu \bar{z}^{\bar{\beta}} - V_J$$

Scalar fields have canonical kinetic terms !

□ In original paper, author investigate Higgs inflation in the context of NMSSM.  
Here we focus on simpler toy model than realistic Higgs inflation.

# FKLMP model with Single Scalar Field

□ Let us focus on FKLMP model with single field:  $z^\alpha = \phi$

$$\mathcal{K} = -3 \log \left( -\frac{1}{3} \Phi \right)$$

$$\Phi(\phi, \bar{\phi}) = -3 + \phi \bar{\phi} + J(\phi) + \bar{J}(\bar{\phi})$$

$$\phi = \bar{\phi} = \frac{\varphi}{\sqrt{2}}$$

□ For simplicity we focus on following choice of  $J(\phi)$ :

$$J(\phi) = -3 \left( \frac{1}{6} + \xi \right) \phi^2$$

□ Now action reduces to simple form:

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = \frac{1}{2} (1 + \xi \varphi^2) R_J - \frac{1}{2} g_J^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V_J$$

# FKLMP model as Pole Inflation

- $\xi$  model in Jordan frame:

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = \frac{1}{2} (1 + \xi\varphi^2) R_J - \frac{1}{2} g_J^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V_J$$

- $\xi$  model in Einstein frame  $g_{\mu\nu}^E = (1 + \xi\varphi^2) g_{\mu\nu}^J$

$$\frac{\mathcal{L}}{\sqrt{-g_E}} = \frac{1}{2} R_E - \frac{1}{2} K_E(\varphi) g_E^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{V_J}{(1 + \xi\varphi^2)^2}$$

Kinetic term has pole at  $\varphi \rightarrow \infty$   $\longrightarrow$  pole at  $\rho = 0$  with  $\rho = (1 + \xi\varphi^2)^{-1}$

$$K_E(\varphi)(\partial\varphi)^2 = \frac{2(1 + \xi\varphi^2) + 12\xi^2\varphi^2}{2(1 + \xi\varphi^2)^2} (\partial\varphi)^2 = \left( \frac{1}{4\xi} + \frac{3}{2} \right) \frac{(\partial\rho)^2}{\rho^2} + \frac{\rho^2}{\rho'^2}$$

- $V_E$  does not diverge or vanish at pole if  $V_J \propto \varphi^4$ .

Pole inflation works well:

$$n_s = 1 - \frac{2}{N}, \quad r = \frac{8}{N^2} \left( \frac{1}{4\xi} + \frac{3}{2} \right)$$

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# Beyond FKLMP

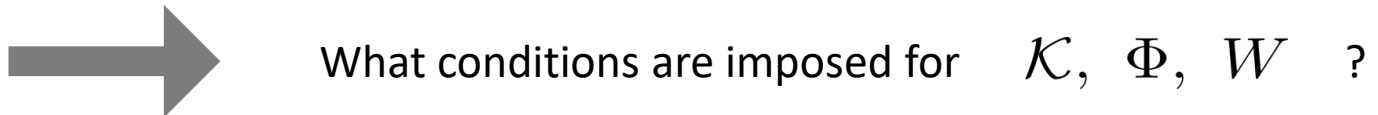
## □ FKLMP approach



Is FKLMP model unique one which satisfy these 2 conditions?

## □ Our approach

- Canonical Kinetic terms in Jordan frame
- Pole inflation



# Note: with FKLMP frame function

□ Action of J-frame sugra:

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = -\frac{1}{6}\Phi R_J + \left( \frac{1}{3}\Phi g_{\alpha\bar{\beta}} - \frac{\Phi_\alpha \Phi_{\bar{\beta}}}{\Phi} \right) g_J^{\mu\nu} \partial_\mu z^\alpha \partial_\nu \bar{z}^{\bar{\beta}} - \frac{1}{4\Phi} \left( \Phi_\alpha \partial_\mu z^\alpha - \Phi_{\bar{\beta}} \partial_\mu \bar{z}^{\bar{\beta}} \right) \left( \Phi_\gamma \partial_\nu z^\gamma - \Phi_{\bar{\delta}} \partial_\nu \bar{z}^{\bar{\delta}} \right) g_J^{\mu\nu} - V_J$$

□

• FKLMP frame function:  $\Phi(z, \bar{z}) = -3 + \delta_{\alpha\bar{\beta}} z^\alpha \bar{z}^{\bar{\beta}} + J(z) + \bar{J}(\bar{z})$

• canonical kinetic term:  $\frac{1}{3}\Phi g_{\alpha\bar{\beta}} - \frac{\Phi_\alpha \Phi_{\bar{\beta}}}{\Phi} = -\delta_{\alpha\bar{\beta}}$

→  $\delta\mathcal{K} := \mathcal{K} + 3 \log \left( -\frac{1}{3}\Phi \right)$

$$\delta\mathcal{K}_{\alpha\bar{\beta}} = 0 \quad \delta\mathcal{K} = h(z) + \bar{h}(\bar{z})$$

This is nothing but Kahler transformation from FKLMP Kahler potential!

Non holomorphic extensions are necessary to obtain beyond FKLMP model.

$$\Phi(z, \bar{z}) = -3 + \delta_{\alpha\bar{\beta}} z^\alpha \bar{z}^{\bar{\beta}} + J(z, \bar{z})$$

# Our Frameworks

- We consider following class of arbitrary functions with 2 scalar fields:

$$\mathcal{K} = \mathcal{K}(\phi + \bar{\phi}, S\bar{S}, S^2, \bar{S}^2),$$

$$\Phi = \Phi(\phi + \bar{\phi}, S\bar{S}, S^2, \bar{S}^2),$$

$$W = Sf(\phi)$$

- Inflaton direction:

$$\phi = \bar{\phi} = \frac{\varphi}{\sqrt{2}}, \quad S = \bar{S} = 0$$

- Note: stabilizer field  $S$  is needed to ensure positivity of the potential;

$$V_E = e^{\mathcal{K}} \left( -3W\bar{W} + g^{\alpha\bar{\beta}} \nabla_{\alpha} W \nabla_{\bar{\beta}} \bar{W} \right)$$

Kawasaki, Yamaguchi, Yanagida (2000)

← Negative term vanishes at  $S = 0$

$$= e^{\mathcal{K}} g^{S\bar{S}} |f|^2 \quad \text{on inflaton trajectory}$$

# Conditions for Pole Inflation

## □ Our 3 conditions:

- Inflation and stabilizer fields have canonical kinetic term in Jordan frame:

$$\frac{\Phi}{3} g_{\phi\bar{\phi}} - \frac{\Phi_{\phi}\Phi_{\bar{\phi}}}{\Phi} = -1 \quad \longrightarrow \quad g_{\phi\bar{\phi}} = \frac{3}{\Phi} \left( \frac{\Phi'(\varphi)^2}{2\Phi} - 1 \right)$$

$$\frac{\Phi}{3} g_{S\bar{S}} = -1 \quad \longrightarrow \quad g_{S\bar{S}} = -\frac{3}{\Phi}$$

- Kinetic term of inflaton in Einstein frame has pole structure:

$$-\frac{1}{2} g_{\phi\bar{\phi}} (\partial\varphi)^2 \rightarrow -\frac{1}{2} \frac{a_p}{\rho^p} (\partial\rho)^2 \quad \longrightarrow \quad g_{\phi\bar{\phi}} \rightarrow \frac{a_p}{\rho(\varphi)^p} \rho'(\varphi)^2$$

at  $\rho \rightarrow 0$  with some function  $\rho(\varphi)$

- Inflaton potential in Einstein frame is smooth at the pole:

$$V_E = -\frac{\Phi}{3} e^{\mathcal{K}} |f|^2 \rightarrow V_0 (1 - c\rho + \dots)$$

# Strategy

□ Differential equation,

$$g_{\phi\bar{\phi}} = \frac{3}{\Phi} \left( \frac{\Phi'(\varphi)^2}{2\Phi} - 1 \right)$$

$$g_{\phi\bar{\phi}} = \frac{a_p}{\rho(\varphi)^p} \rho'(\varphi)^2$$

➔  $(\rho'(\varphi))^2 = \frac{-2\Phi}{\frac{2a_p}{3\rho^p} \Phi^2 - (\partial_\rho \Phi)^2}$

1. Assuming functional form of  $\Phi(\rho)$ , solve above differential equation:

$$\rho = \rho(\varphi) \quad \longrightarrow \quad \Phi = \Phi(\varphi)$$

2. Determine a Kahler potential through

$$g_{\phi\bar{\phi}} = \frac{3}{\Phi} \left( \frac{\Phi'(\varphi)^2}{2\Phi} - 1 \right)$$

$$g_{S\bar{S}} = -\frac{3}{\Phi}$$

3. Determine super potential  $f$  through

$$-\frac{\Phi}{3} e^{\mathcal{K}} |f|^2 \rightarrow V_0(1 - c\rho + \dots)$$

# 1. Solve differential equation

1. Assuming functional form of  $\Phi(\rho)$ , solve differential equation:

- Assuming  $p = 2$  and  $\Phi(\rho) \rightarrow -A\rho^{-2}$ , our differential equation reduces to

$$(\rho'(\varphi))^2 = \frac{-2\Phi}{\frac{2a_p}{3\rho^p}\Phi^2 - (\partial_\rho\Phi)^2} \quad \longrightarrow \quad \rho'^2 = \tilde{\xi}\frac{\rho^4}{A} \quad \text{with} \quad \tilde{\xi} = \frac{3}{a_2 - 6}$$

- Solutions can be written as

$$\rho(\varphi) = \frac{A}{\sqrt{\tilde{\xi}}(C + \varphi)} \quad \text{with integration constant } C$$

- Then frame function can be obtained as

$$\Phi(\varphi) = -\tilde{\xi}C^2 \left(1 + \frac{1}{C}\varphi\right)^2 = -3 \left(1 + \sqrt{\frac{\tilde{\xi}}{3}}\varphi\right)^2$$

Here integration constant is chosen as

$$C = \sqrt{3/\tilde{\xi}} \quad \text{so that} \quad \frac{\mathcal{L}}{\sqrt{-g_J}} \supset -\frac{\Phi}{6}R^J = \frac{1}{2}R^J + \text{nonminimal couplings}$$

# 2. Determine a Kahler potential

## 2. Determine a Kahler potential

$$g_{\phi\bar{\phi}} = \frac{3}{\Phi} \left( \frac{\Phi'(\varphi)^2}{2\Phi} - 1 \right)$$



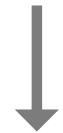
$$g_{\phi\bar{\phi}} = \partial_{\phi} \partial_{\bar{\phi}} \mathcal{K} = \frac{1}{2} \partial_{\varphi} \partial_{\varphi} \mathcal{K}(\varphi)$$

$$\Phi(\varphi) = -3 \left( 1 + \sqrt{\frac{\tilde{\xi}}{3}} \varphi \right)^2$$

$$\mathcal{K} = -12 \left( 1 + \frac{1}{2\tilde{\xi}} \right) \log \left( 1 + \sqrt{\frac{\xi}{3}} \varphi \right)$$

on inflaton trajectory

$$\phi = \bar{\phi} = \frac{\varphi}{\sqrt{2}}, \quad S = \bar{S} = 0$$



$$\mathcal{K} = -12 \left( 1 + \frac{1}{2\tilde{\xi}} \right) \log \left( 1 + \sqrt{\frac{\xi}{6}} (\phi + \bar{\phi}) \right)$$

Note: here  $S$  dependence are omitted.

# 3. Determine a super potential

3. Determine a super potential  $W = Sf(\phi)$

- Function  $f$  can be determined from the requirement

$$V_E = -\frac{\Phi}{3} e^{\mathcal{K}} |f|^2 \rightarrow V_0(1 - c\rho + \dots) \quad \text{at } \rho \rightarrow 0$$

- Left hand side can be evaluated as

$$V_E = - \left( 1 + \sqrt{\frac{\tilde{\xi}}{3}} \varphi \right)^{-10 - \frac{6}{\tilde{\xi}}} |f|^2$$

- In order for  $V_E$  to be constant at  $\varphi \rightarrow \infty$  ( $\rho \rightarrow 0$ ),

$$f(\phi) = \lambda \phi^m \quad \text{with } m = 5 + \frac{3}{\tilde{\xi}}$$

$$V_E \rightarrow |\lambda|^2 \left( \frac{3}{2\tilde{\xi}} \right)^m \left( 1 - 2m \sqrt{\frac{3(1 + 2\tilde{\xi})}{A}} \rho + \dots \right) \quad \text{at } \rho \rightarrow 0$$



# Note: Consistency check

□ We assumed following two conditions;

- Stabilizer field also has canonical kinetic term in Jordan frame:

$$g_{S\bar{S}} = -\frac{3}{\Phi}$$

- Inflaton direction is  $Re \phi$ .

$$\phi - \bar{\phi} = S = \bar{S} = 0$$

□ These two conditions are satisfied if we includes  $S$  dependence in Kahler potential as

$$\mathcal{K} = -3 \log \left[ \left( 1 + \sqrt{\frac{\tilde{\xi}}{6}} (\phi + \bar{\phi}) \right)^2 - \frac{1}{3} |S|^2 \right] - 6 \left( 1 + \frac{1}{\tilde{\xi}} \right) \log \left( 1 + \sqrt{\frac{\tilde{\xi}}{6}} (\phi + \bar{\phi}) \right) - \frac{3}{4} \zeta |S|^4$$

$$\frac{m_{Im(\phi)}^2}{H_{inf}^2} = \frac{2(3 + 5\tilde{\xi})}{1 + 2\tilde{\xi}} > 1 \quad \frac{m_S^2}{H_{inf}^2} = \zeta \tilde{\xi}^2 \varphi^4 \gg 1$$

$$H_{inf}^2 = V_E/3 = \frac{|\lambda|^2}{3} \left( \frac{3}{2\tilde{\xi}} \right)^m$$



Masses of  $Im(\phi)$  and  $S$  are sufficiently large!

# Results

□ We have derived all arbitrary functions of Jordan frame supergravity;

$$\Phi(\varphi) = -3 \left( 1 + \sqrt{\frac{\tilde{\xi}}{6}} (\phi + \bar{\phi}) \right)^2$$

$$\mathcal{K} = -12 \left( 1 + \frac{1}{2\tilde{\xi}} \right) \log \left( 1 + \sqrt{\frac{\tilde{\xi}}{6}} (\phi + \bar{\phi}) \right)$$

Omitting S dependence

$$W = \lambda S \phi^m \quad \text{with} \quad m = 5 + \frac{3}{\tilde{\xi}} \quad m \text{ is integer if } \tilde{\xi} = 1 \text{ or } 3$$

- Inflaton has canonical kinetic term in Jordan frame
- Pole inflation works well

$$\frac{\mathcal{L}}{\sqrt{-g_E}} = \frac{1}{2} R - \frac{a_2}{\rho^2} (\partial\rho)^2 - V_0 (1 - c\rho) + \dots$$

$$a_2 = 6 \left( 1 + \frac{1}{2\tilde{\xi}} \right)$$

$$V_0 = |\lambda|^2 \left( \frac{3}{2\tilde{\xi}} \right)^m$$

$$c = 2m \sqrt{\frac{3(1+2\tilde{\xi})}{A}}$$

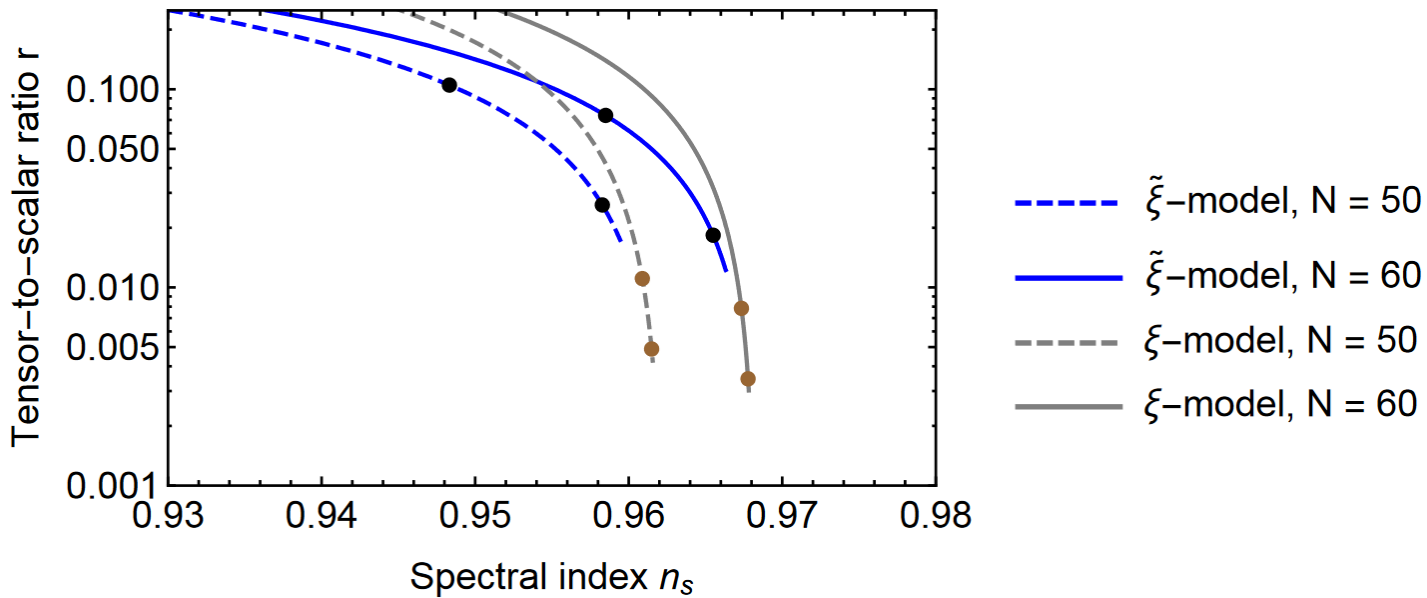
# Prediction of our model

From the general argument of pole inflation,

$$n_s = 1 - \frac{2}{N} \quad r = \frac{48}{N^2} \left( 1 + \frac{1}{2\tilde{\xi}} \right) \quad \mathcal{P}_\zeta = \frac{|\lambda|^2 \tilde{\xi} N^2}{36\pi^2 (1 + 2\tilde{\xi})} \left( \frac{3}{2\tilde{\xi}} \right)^m$$

Numerical calculations

at leading order of N



Our model has lower bound of  $r$ ;  $r > 48/N^2$

# Relation with alpha attractor model

## □ Sumper symmetric $\alpha$ attractor model:

Cecotti, Kallosh (2014)

Inflation model based on Einstein frame sugra with free parameter  $\alpha > 0$  with

$$\mathcal{K} = -3\alpha \log(T + \bar{T})$$

omitting stabilizer field

→  $r = \frac{12}{N^2} \alpha$  without lower bound

## □ Our (and FKLMP) model reduces to alpha attractor model in Einstein frame:

$$\mathcal{K} = -12 \left(1 + \frac{1}{2\tilde{\xi}}\right) \log \left(1 + \sqrt{\frac{\tilde{\xi}}{6}} (\phi + \bar{\phi})\right)$$

Comparing the Kahler potential of  $\alpha$  attractor model, we find

$$\alpha = 4 \left(1 + \frac{1}{2\tilde{\xi}}\right) \quad T = \frac{1}{2} + \sqrt{\frac{\tilde{\xi}}{6}} \phi$$

Now parameter  $\alpha$  has lower bound:  $\alpha > 4$ , which corresponds to  $r > 48/N^2$ .

## □ Note: super potential is different in each theory and prediction is not equivalent at subleading order.

# Origin of the lower bound of $r$

- If we start from  $\alpha$  attractor model in Einstein frame, any positive value of  $\alpha$  should be allowed.

Then where does our constraint  $\alpha > 4$  come from?

- It is clear from the frame function (= the conformal factor ) in our Jordan frame.

$$\Phi = -3 \left( 1 + \sqrt{\frac{1}{3(\alpha - 4)}} (\phi + \bar{\phi}) \right)^2$$

which is complex valuable when  $\alpha < 4$  and then Jordan frame metric is ill defined.

- Thus lower bound of  $\alpha$ , and hence that of tensor-to-scalar ratio  $r$ , are key observable quantity to distinguish the model based on Jordan frame from other models which related by conformal transformation.

$$\begin{array}{lll} r > 48/N^2, & r > 12/N^2, & r > 0 \\ \text{Our model,} & \text{FKLMP model,} & \text{alpha attractor model} \end{array}$$



# Summary

## □ Our findings:

- Non-holomorphic extension of frame function is necessary to construct Inflation models beyond FKLMP.  $\Phi(z, \bar{z}) = -3 + \delta_{\alpha\bar{\beta}} z^\alpha \bar{z}^{\bar{\beta}} + J(z, \bar{z})$

- We give one example of pole inflation model in J-sugra.

$$\Phi = -3 \left( 1 + \sqrt{\frac{\xi}{6}} (\phi + \bar{\phi}) \right)^2 \quad \mathcal{K} = -12 \left( 1 + \frac{1}{2\xi} \right) \log \left( 1 + \sqrt{\frac{\xi}{6}} (\phi + \bar{\phi}) \right) \quad W = \lambda S \phi^{5+3/\xi}$$

where inflaton has canonical kinetic term in Jordan frame.

In this model  $r$  has lower bound:  $r > 48/N^2$ .

- Kinetic structure of our model and FKLMP model are equivalent with that of super symmetric  $\alpha$ -attractor model with a lower bound of  $\alpha$ , which comes from positivity of a conformal factor.

## □ Discussions:

- Is the log type Kahler potential natural? Are there any preference from high energy theory?
- We use ad-hoc assumptions like  $\Phi(\rho) \rightarrow -A\rho^{-2}$ . Is there room to construct yet another pole inflation models based on J-sugra.