Axion: Mass Dark Matter Abundance Relation

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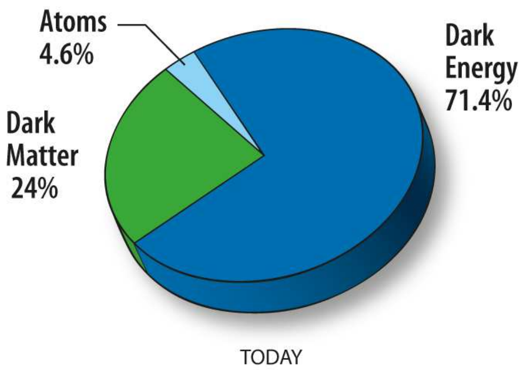
- Mystery 1: Dark Matter
- Mystery 2: T-symmetry of QED and QCD
- The Axion: a solution to both mysteries?
- Early Universe cosmology of the axion
- How to predict the axion mass if it's the dark matter

“Why move from Montréal to Darmstadt?”

Dark Matter: a Cosmic Mystery



Atoms: Standard Model.

Dark Energy: Cosmological Constant.
Strange value, but possible

Dark Matter: MYSTERY! NOT SM!

We only know 3 things about dark matter:

- It's **Matter**: gravitationally clumps.
- It's **Dark**: negligible electric charge, interactions too feeble to be detected except by gravity
- It's **Cold**: negligible pressure by redshift $z = 3000$

Another mystery: $\mathbf{T}$-symmetry in QED and QCD

$\mathbf{T}$ symmetry: "when you run a movie backwards, the *microphysics* is correct."

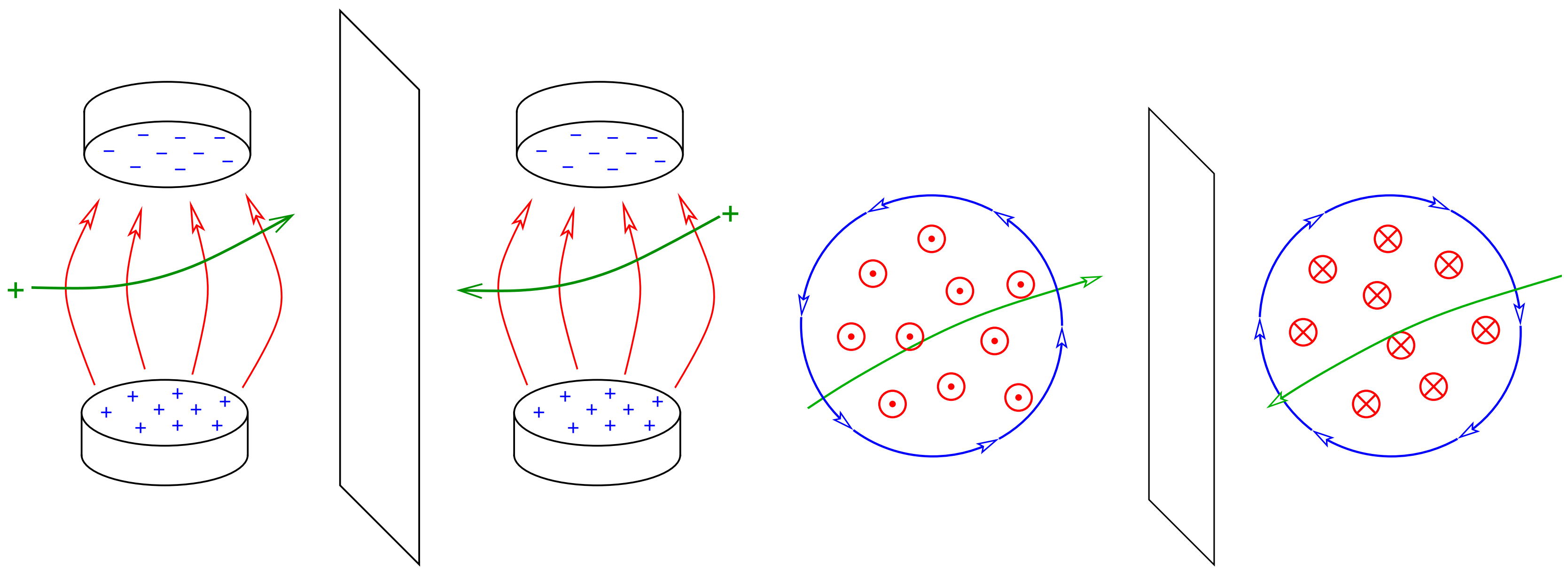
Statistical mechanics breaks $\mathbf{T}$.
But microphysics very nearly obeys it!

Weak physics breaks $\mathbf{T}$, but only through very small CKM effects. Observed in handful of experiments, all involving neutral meson oscillation.

No evidence for $\mathbf{T}$ viol in E&M or Strong interactions.

$\mathbf{T}$ in E&M

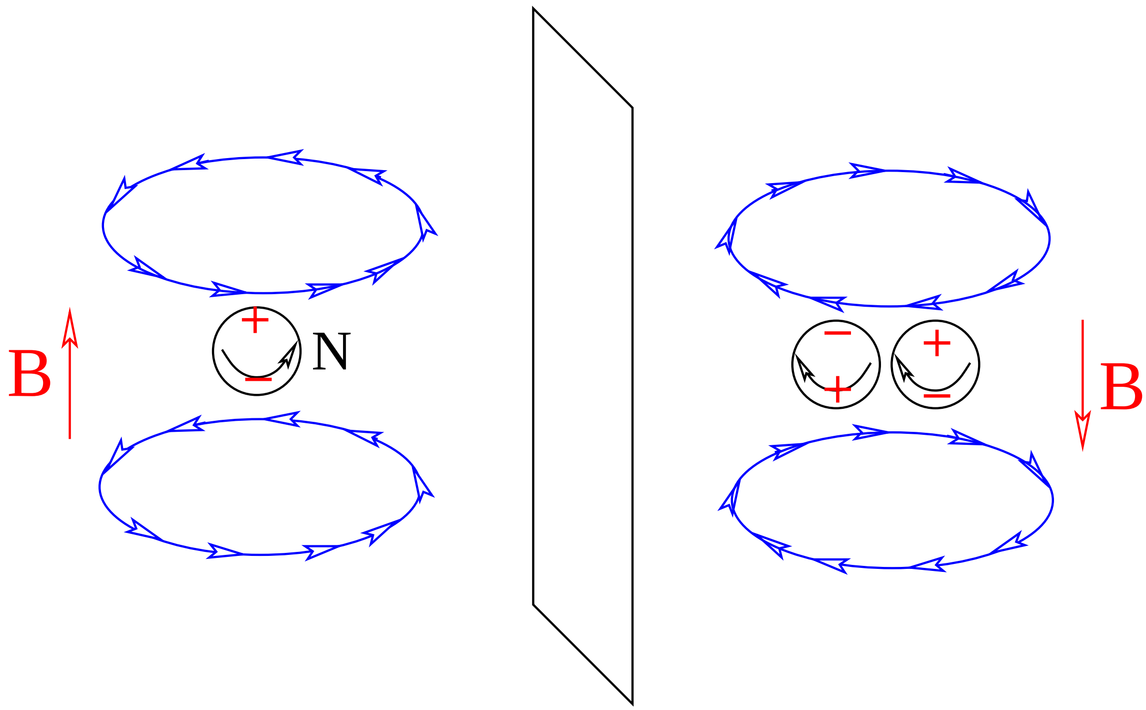
How do E, B fields change when you run movie backwards?



Q's unchanged, but J's flip. E same, but B flips.

Looking for $\mathbf{T}$: Neutron EDM

Put neutron in $\vec{B}$ field – spin lines up with $\vec{B}$.



Is there an Electric Dipole Moment (EDM) aligned with spin?

If so: looks different when movie runs backwards, $\mathbf{T}$ viol!

$\mathbf{T}$ and the E&M Action

Action $S \Rightarrow$ all physics. Local field thy: $S = \int \mathcal{L}\, d^4x$.

$\mathcal{L}$ a singlet (gauge symm) and spacetime scalar (Lorentz):

$$\mathcal{L} = \frac{\vec{B}^2 - \vec{E}^2}{2e^2} + \frac{\Theta}{4\pi^2}\vec{E}\cdot\vec{B} + \text{(electrons...)}$$

$\mathbf{T}$ flip: $\vec{E} \to \vec{E}$ and $\vec{B} \to -\vec{B}$:

$(B^2 - E^2) \to (B^2 - E^2)$ **BUT** $E \cdot B \to -E \cdot B$.

$$\mathcal{L} \xrightarrow{T} \frac{\vec{B}^2 - \vec{E}^2}{2e^2} - \frac{\Theta}{4\pi^2}\vec{E}\cdot\vec{B} + \text{(electrons...)}$$

Nonvanishing Θ is a $\mathbf{T}$ violation!

E&M $\mathbf{T}$ violation is Illusory!

The $\Theta \vec{E} \cdot \vec{B}$ term has no *consequences*!

$$\vec{E} \cdot \vec{B} = \frac{1}{4} \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta} = \partial^{\mu} K_{\mu} \,, \quad K^{\mu} \equiv \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} A^{\nu} F^{\alpha\beta}$$

I can integrate $\vec{E} \cdot \vec{B}$ to a boundary term.
Vanishes if $F^{\alpha\beta}$ vanishes on boundary. Alternately, EOM:

$$0 = \partial_{\mu} \left(\frac{1}{e^2} F^{\mu\nu} + \frac{\Theta}{8\pi^2} \epsilon_{\mu\nu\alpha\beta} \partial^{\alpha} A^{\beta} \right)$$

Second term zero by antisymmetry (**if** Θ constant)

QCD and its Lagrangian

QCD is like 8 copies of E&M, but with non-linearities:

$$\text{Field strength}: \quad F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu \,,$$

g: coupling. $a = 1 \ldots 8$. f_{abc} "structure constants"

$$S = \int dt \int d^3x \left(\frac{\vec{E}_a^2 - \vec{B}_a^2}{2g^2} + \frac{\Theta}{8\pi^2} \vec{E}_a \cdot \vec{B}_a \right)$$

where $\vec{E}_a \cdot \vec{B}_a$ still a total derivative:

$$\vec{E}_a \cdot \vec{B}_a = \partial^\mu K_\mu \,, \qquad 2K_\mu = \epsilon_{\mu\nu\alpha\beta} \left(A_a^\nu F_a^{\alpha\beta} + \frac{g f_{abc}}{3} A_a^\nu A_b^\alpha A_c^\beta \right)$$

Last term *need not* vanish on boundary even if $\vec{E}_a = 0 = \vec{B}_a$ there!
It's always $8\pi^2 N_I$ with N_I integer. So Θ mod 2π has *physical consequences*
G. 't Hooft, PRL 37, 8(1976); R. Jackiw and C. Rebbi, PRL 37, 172 (1976);

Callan Dashen and Gross, Phys Lett 63B, 334 (1976)

Neutron EDM and Θ

Theory: Neutron electric dipole moment should exist,

$$d_n = -3.8 \times 10^{-16}\, e\,\mathrm{cm} \times \Theta$$

so long as Θ is not zero! Guo *et al*, arXiv:1502.02295, assumes Θ, modulo 2π, is small

Experiment: Consistent with zero! Baker *et al* (Grenoble), arXiv:hep-ex/0602020

$$|d_n| < 2.9 \times 10^{-26}\, e\,\mathrm{cm}$$

Either $|\Theta| < 10^{-10}$ by (coincidence? accident?) or there is something deep going on here.

Θ from UV physics

Consider heavy Dirac quark $[Q^\alpha \; q_{\dot\alpha}]$ Two Weyl spinors
Q^α is 3, q^α is $\bar{3}$. Lagrangian:

$$\mathcal{L}(Q,q) = \frac{1}{2}\bar{Q}\not{D}Q + \frac{1}{2}\bar{q}\not{D}q + m q_\alpha Q^\alpha + m^* q^{\dot\alpha} Q_{\dot\alpha}$$

Mass m is in general complex.
Rotate $m = |m|e^{i\theta} \to |m|$ by rotating Q but not q.
Such a chiral rotation generates shift, $\Theta \to \Theta + \theta$.

Phase in mass of heavy quark becomes part of Θ_{QCD}.

Axion

Give Q^α, q^α different (global) U(1) charges (so $m = 0$)
Introduce complex φ with U(1) charge: can now write

$$\mathcal{L}_{\varphi qQ} = y\varphi q_\alpha Q^\alpha + y\varphi^* q^{\dot\alpha} Q_{\dot\alpha}$$

Symmetry-breaking potential for φ:

$$\mathcal{L}_\varphi = \mathcal{L}_{\varphi qQ} + \partial_\mu\varphi^*\partial^\mu\varphi + \frac{m^2}{8f_a^2}\left(2\varphi^*\varphi - f^2\right)^2$$

Phase $\varphi = e^{i\theta_A} f_a$ becomes part of Θ: $\Theta_{\text{eff}} = \Theta + \theta_A$ or

$$\mathcal{L}_\varphi = \partial_\mu\varphi^*\partial^\mu\varphi + V(\varphi^*\varphi) + \theta_A \frac{F^a_{\mu\nu}\tilde{F}^{\mu\nu a}}{32\pi^2} \text{[dim-5]}$$

How the axion works

φ, therefore θ_A, can evolve. What value is (free) energetically preferred? $W = \Omega V_{\mathrm{eff}}(\varphi) = -T\ln(Z_{\mathrm{Eucl}})$, so

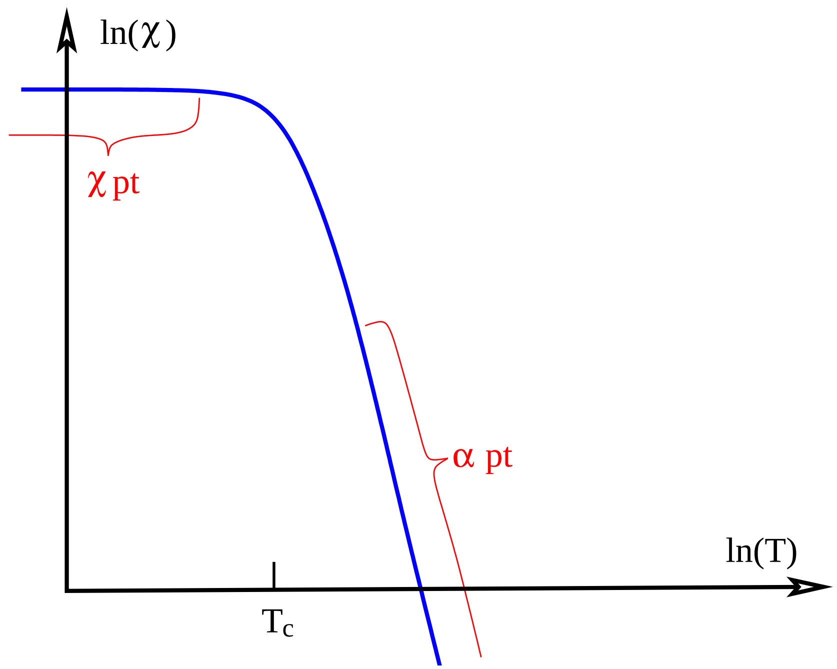
$$
\begin{aligned}
V_{\mathrm{eff}}(\theta_A) &= -\frac{T}{\Omega}\ln\int \mathcal{D}(A_\mu\bar{\psi}\psi)\,\mathrm{Det}\,(\not{D}+m)e^{-\int\frac{F^2}{4g^2}} \times e^{i(\Theta+\theta_A)\int\frac{F\tilde{F}}{32\pi^2}} \\
&\simeq \chi(T)(1-\cos[\Theta+\theta_A])\,, \\
\chi(T) &= \left\langle \int d^4x \frac{F\tilde{F}(x)}{32\pi^2}\,\frac{F\tilde{F}(0)}{32\pi^2}\right\rangle_\beta
\end{aligned}
$$

Nontrivial $\Theta+\theta_A$ (**T**-violation) $\rightarrow$ phase cancellation V_{eff} minimized when $\Theta_{\mathrm{eff}}=0 \rightarrow$ **T** valid.

Peccei Quinn PRL 38, 1440 (1977);

J. E. Kim, PRL 43, 103 (1979); Shifman Vainshtein and Zakharov, NPB 166, 493 (1980)

$\chi(T)$: what we expect



Low T: χ-pt works.
$\chi \simeq \frac{m_u m_d}{(m_u+m_d)^2} m_\pi^2 f_\pi^2$
Hi T: standard
pert-thy works(??)

Low T: $\chi(T \ll T_c) = (76 \pm 1\ \mathrm{MeV})^4$ Cortona *et al*, arXiv:1511.02867

High T: $\chi(T \gg T_c) \propto T^{-8}$ Gross Pisarski Yaffe Rev.Mod.Phys.53,43(1981)

but with much larger errors.

Axion in cosmology

Assume first: φ starts homogeneous [inflation]

Classical axion field!

Starts oscillating around

$t = \pi m_a^{-1}$. Damped:

- Hubble drag
- effect of dm_a/dt

Pressureless:

Acts Like Dark Matter!

Osc. frequency = axion mass: $\omega^2 = m_a^2 = \chi/f_a^2$

Dark matter density?

More dark matter if oscillations start larger or later:

$$\rho_{\mathrm{dm}} \propto f_a^{7/8}\, \theta_{A\,\mathrm{init}}^2 \qquad \text{(approximately)}$$

- Large f_a, (small m_a): later transition from cosmological constant to matter, more final energy density
- Initial $\theta_{A\,\mathrm{init}}$: larger value, larger starting amplitude.

Because $\theta_{A\,\mathrm{init}}$ unknown, scenario is **not predictive.**

Initial state of φ field?

most likely: **randomly different in different places!**

- Inflation stretches quantum fluctuations to classical ones: $\Delta\varphi \sim H_{\text{infl.}}$. If $N_{\text{efolds}} H^2 > f_a^2$, scambles field.

  If not: need $H < 10^{-5} f_a$ to avoid excess "isocurvature" fluctuations in axion field

- Gets scrambled *after* inflation if Universe was ever really hot $T > f_a \sim 10^{11}$ GeV.

Predictive: *if* axion=dark matter and *if* we can solve dynamics, *then* $\rightarrow$ predict f_a, m_a.

L. Visinelli and P. Gondolo, PRL 103, 011802 (2014)

Needed Ingredients

Predict relation between **Dark Matter Density** and **Axion Mass** assuming space-random starting angle.

Challenges:

1. $\chi(T)$: needs Lattice Gauge Thy.
2. Axion field dynamics: classical but with large scale hierarchy $f_a/H \sim 10^{30} \gg 1$

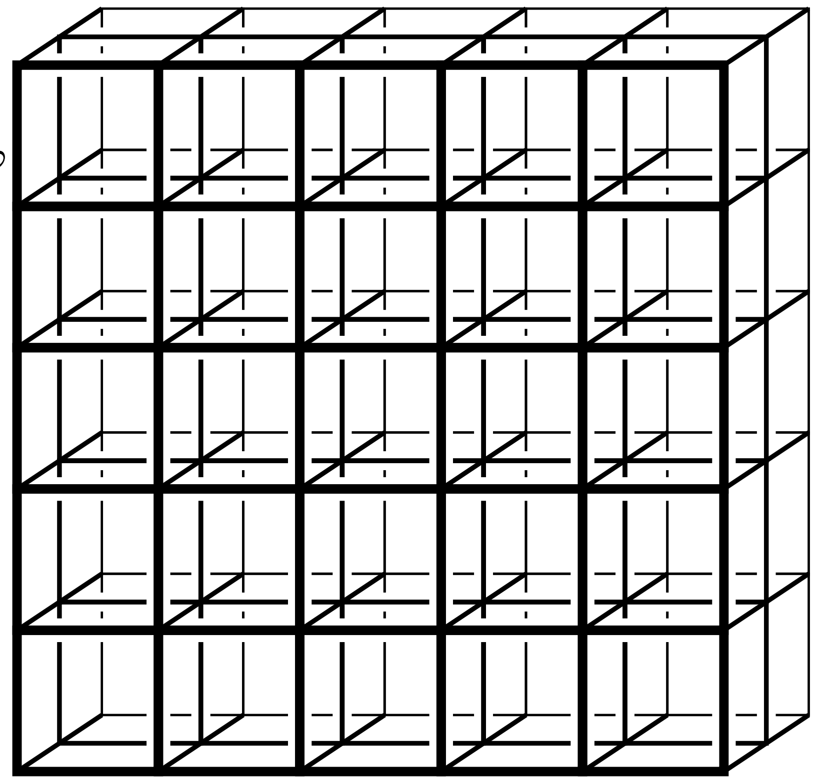
Would be valuable to resolve 1.

Claim: I can solve 2.

Solving space-inhomogeneous case

Put the Lagrangian

$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi + \frac{\lambda}{8}(2-\varphi^*\varphi)^2 - \chi(t)\,\mathrm{Re}\,\varphi$$



as classical field thy.

on **real-time lattice**,

$\varphi(t=0)$ random,

Hubble drag,

Count axions at end.

Nonperturbative approach.

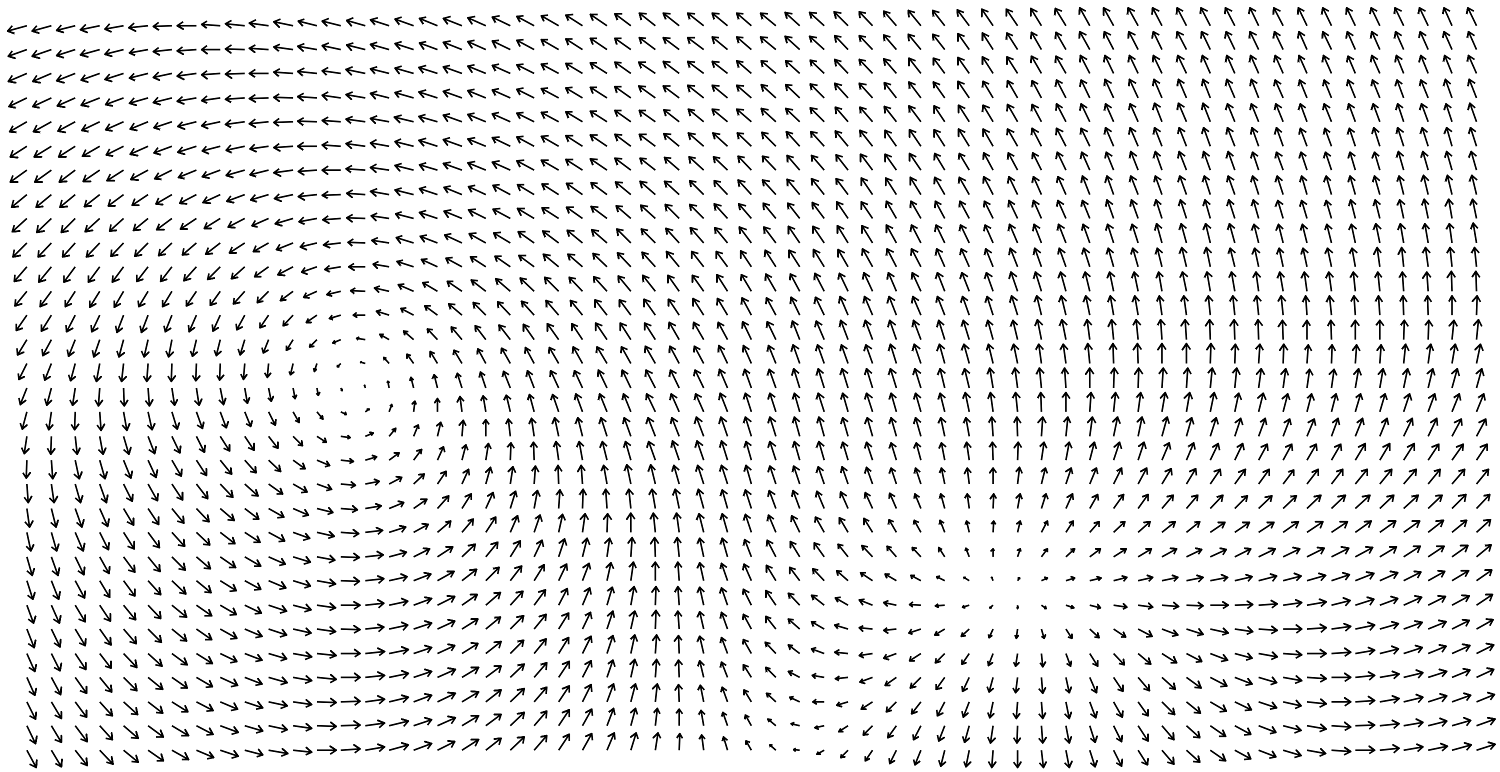
For now: Instanton Liquid for $\chi(T)$

Wantz Shellard arXiv:0908.0324; Borsanyi *et al* 1606.07494 comparable

Axions and Topology I

φ is a complex number – plot as a 2D arrow.

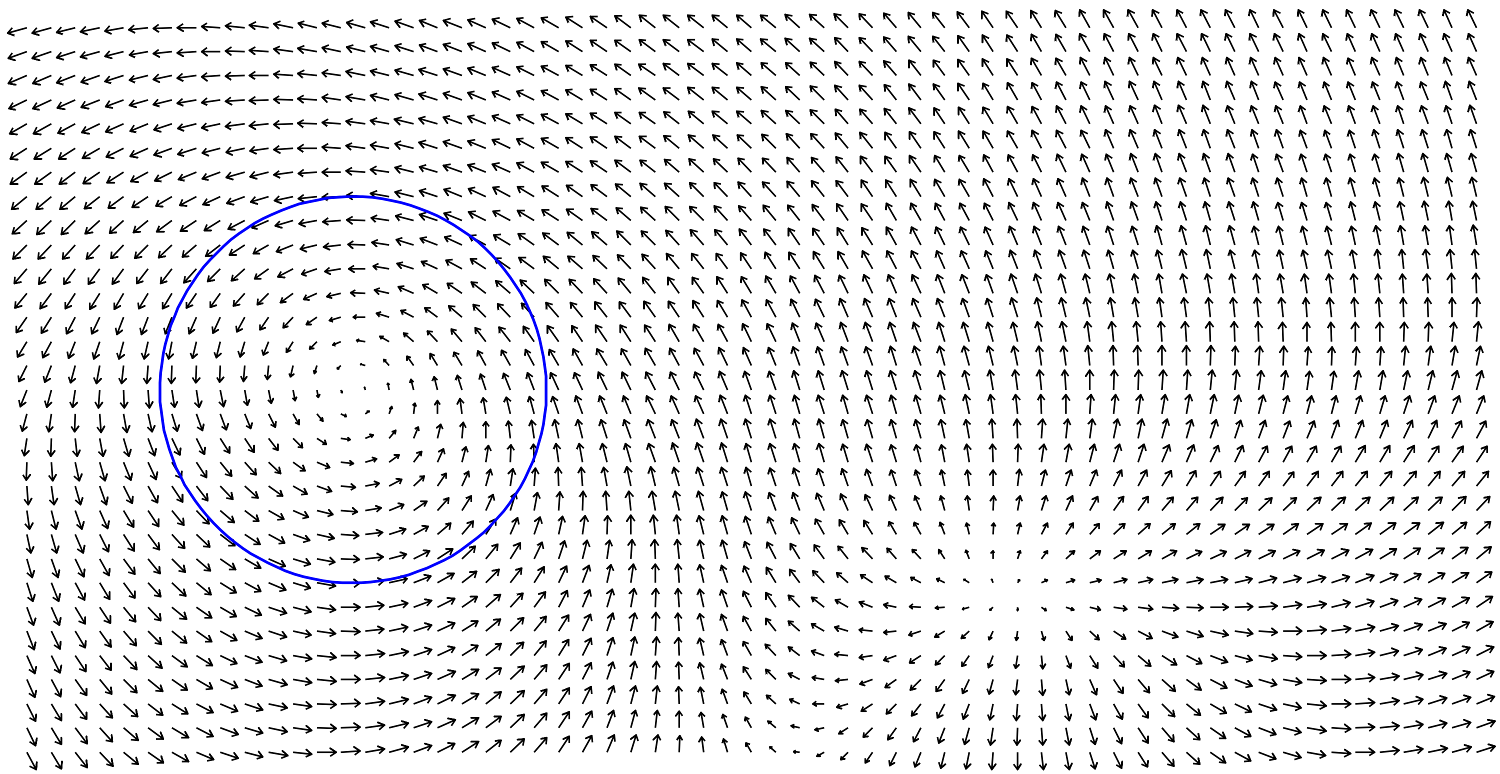
Axion field: a field of arrows. 2D slice for instance:



Field generically has vortices

Axions and Topology II

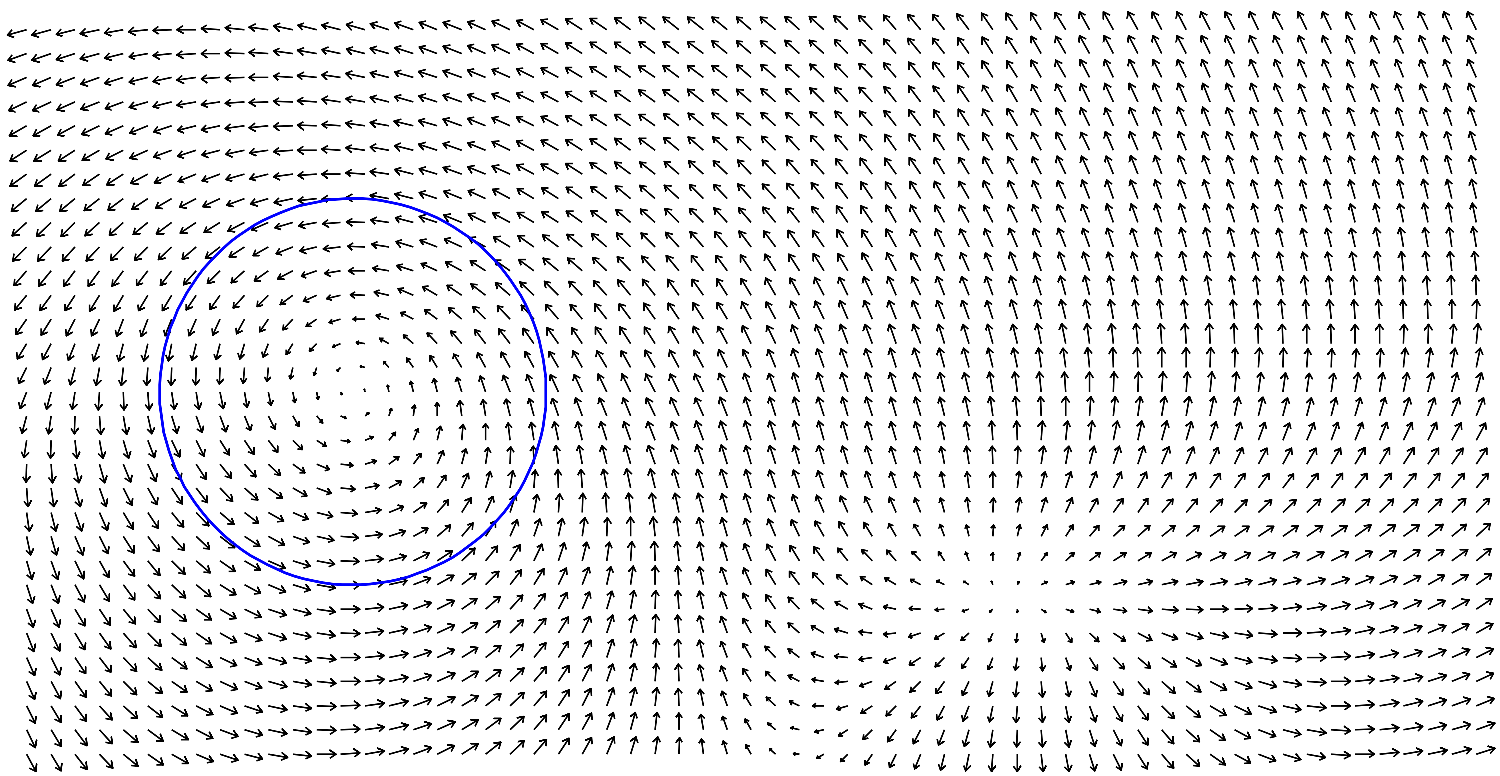
As you circle (anti)vortex, angle θ_A varies by $(-)2\pi$.



Continuity: angle θ_A *must* be undefined somewhere inside the circle. $\varphi = 0$ somewhere. Center of vortex.

Axions and Topology III

As you circle vortex, angle θ_A varies by 2π.



2D slice of a 3D picture: these "vortices" are 1D line structures.

Domain walls

2D slice of evolution, When the potential tilts:

Conventional Wisdom

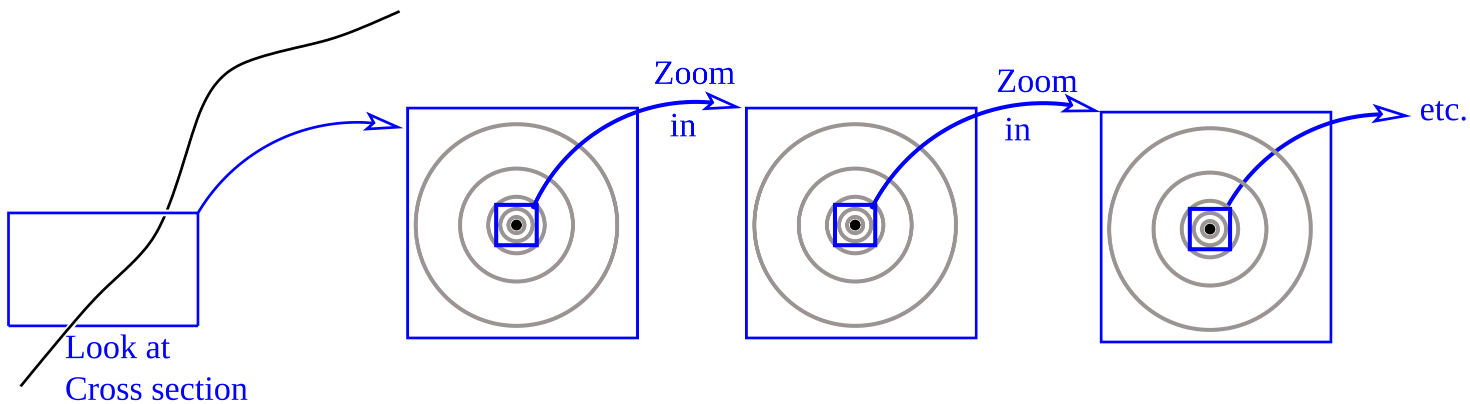
This dynamics produces axions! 3 sources:

- "misalignment" axions from $\theta_A \neq 0$ between strings
- "Wall" axions from breakup of axionic walls
- "String" axions radiated off of axionic strings

Axion production should be (much?) larger than angle-average of misalignment mechanism

Layers of String Energy

$$E_{\rm str} = \int dz \int d\phi \int r\,dr \left(\nabla\phi^* \nabla\phi \simeq f_a^2/2r^2 \right) \simeq \pi \ell f_a^2 \int_{\sim f_a^{-1}}^{\sim H^{-1}} \frac{r\,dr}{r^2}$$



Series of "sheaths" around string:
equal energy in each $\times 2$ scale, 10^{30} scale range! $\ln(10^{30}) \simeq 70$.

Log-large string tension $T_{\rm str} = \pi f_a^2 \ln(10^{30}) \equiv \pi f_a^2 \kappa$

Not reproduced by numerics (separation/core $\sim$ 400)

If $\kappa = 6$ not 70, does it matter?

Things which scale as κf_a^2:

- String tension, hence string inertia $T_{\rm str} \simeq \pi\kappa f_a^2$
- Energy in network $E \sim L_{\rm network}\pi\kappa f_a^2$

Things which do *not* scale with κ:

- string-string interaction: $dF/dl \sim f_a^2/r_{\rm sep}$
- power radiated from strings $dE/dldt \sim f_a^2 R_{\rm curv}^{-1}$

Small κ: strings "too floppy," physics wrong.

An effective description

The important axion-production physics is:

- Only long-range (light) degree of freedom is axion
- Axion strings: thin cores with high tension
  $T \simeq 70 \times \pi f_a^2$
- Correct string-field interactions.

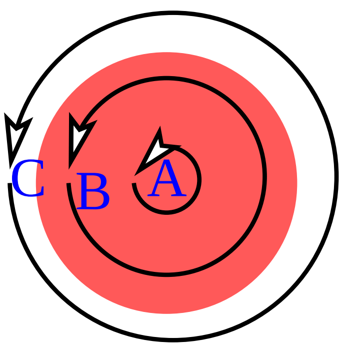
Any modified UV (high-mass) physics which does this is OK!

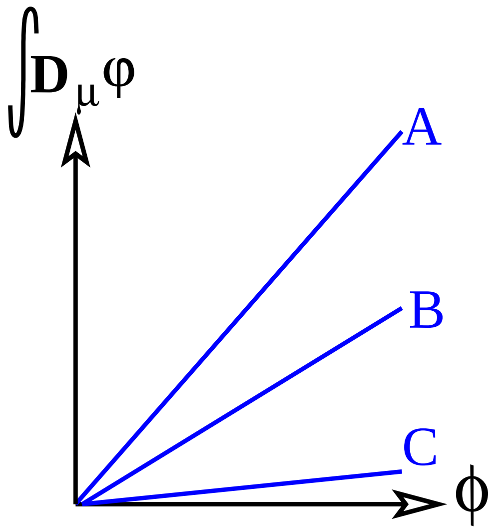
Find massive fields which somehow increase string tension

Abelian Higgs Model: Tension-Only Strings

$$\mathcal{L}(\varphi, A_\mu) = \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + (D_\mu \varphi)^*(D^\mu \varphi) + \frac{\lambda}{8}\left(2\varphi^*\varphi - f_a^2\right)^2$$

with $D_\mu = \partial_\mu - ieA_\mu$ covariant derivative





Magnetic flux centered on string. For large loops, compensates for $\nabla\varphi$ so $D\varphi \to 0$.

Finite tension $T \simeq \pi f_a^2$. No long-range interactions.

Abelian Higgs model

- Network of strings with tension $T \simeq \pi f_a^2$
- Only massive fields (Higgs, massive vector) outside cores
- No long-range interactions between strings
- Leads to dense networks, $\sim 10\times$ denser as...
- Look just like what we want "string cores" to look like

Trick: global strings, local cores

Hybrid theory with A_μ and two scalars

$$\begin{aligned}\mathcal{L}(\varphi_1, \varphi_2, A_\mu) \;=\; & \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\ & + \frac{\lambda}{8}\left[(2\varphi_1^*\varphi_1 - f^2)^2 + (2\varphi_2^*\varphi_2 - f^2)^2\right] \\ & + |(\partial_\mu - iq_1 e A_\mu)\varphi_1|^2 + |(\partial_\mu - iq_2 e A_\mu)\varphi_2|^2\end{aligned}$$

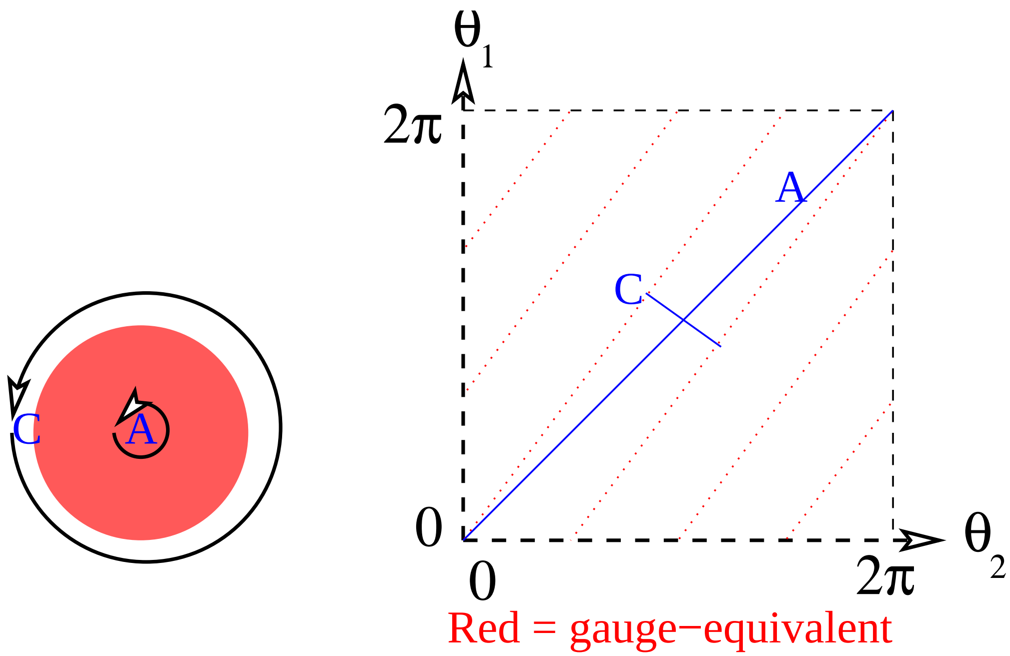
Pick $q_1 \neq q_2$, say, $q_1 = 4$, $q_2 = 3$.

Two rotation symmetries, $\varphi_1 \to e^{i\theta_1}\varphi_1$, $\varphi_2 \to e^{i\theta_2}\varphi_2$

$q_1\theta_1 + q_2\theta_2$ gauged, $q_2\theta_1 - q_1\theta_2$ global (Axion)

Two scalars, one gauge field

String where *each* scalar winds by 2π:



B-field *almost* compensates gradients outside string.

$f_a^2 = f^2/(q_1^2+q_2^2)$.

$$T \simeq 2\pi f^2 \,, \qquad \frac{dF}{dl} = \frac{f^2}{(q_1^2+q_2^2)r} \,, \qquad \kappa_{\rm eff} = 2(q_1^2+q_2^2) \,.$$

Higher tension $=$ higher initial density, longer lasting, hardier loops

Results

- $10\times$ string tension leads so $3\times$ network density but
- only 40% more axions than with axion-only simulation,
- **Fewer** (78%) axions than $\theta_{A\,\mathrm{init}}$-averaged misalignment
- Usual argument **wrong**, involves **double-counting**
- energy in "walls" is *part* of misalignment energy
- Walls get "eaten" by strings, energy lost for axions
- Strings bad at making axions.

Conclusions

- Dark matter and $\mathbf{T}$ in QCD both mysteries
- Axion could explain both!
- Axion dynamics in early Universe: string defects
- Need auxiliary DOF to get high string tension.

Put in known dark matter density, vacuum $\chi(0)$
Recent lattice result for $\chi(T)$, these results:
result: $m_{\mathrm{ax}} \simeq 26 \pm 3\,\mu\mathrm{eV}$ (6 Ghz)

Lattice: Why is $\chi(T \sim 1\text{GeV})$ Hard?

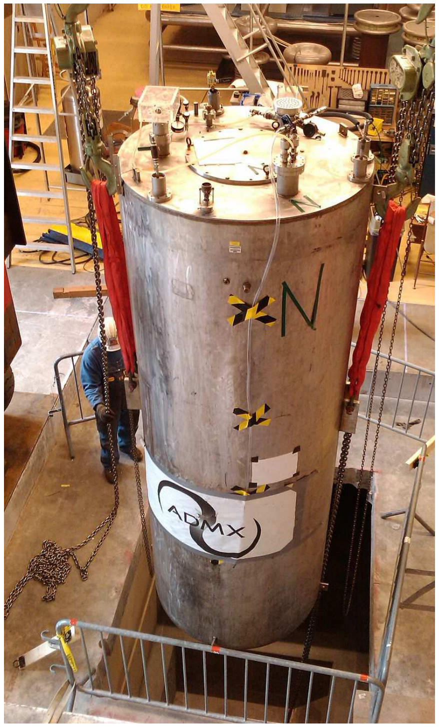
Lattice Monte-Carlo, find fraction of configs with instanton. But

- Instantons get rare fast, $\chi \sim T^{-7\ldots-8}$. Statistics??
- Slowing-down of algorithm to change N_I
- Instanton counting must have very low false-positive rate!

Chiral limit *not* a problem: $\chi \propto (m_u m_d m_s / T^3)$, if $m \ll T$.
Use $T \gg m \gg m_u$, multiply by $m_u m_d m_s / m^3$.

How do experimentalists look for Axions?

Most sensitive method: resonant cavity in magnetic field



Microwave cavity inside superconducting solenoid. $\vec{E}$ field of cavity mode aligned with $\vec{B}$ of solenoid. Cavity oscillation: oscillating $\vec{E} \cdot \vec{B}$. If cavity resonance matches $m_A/\hbar$: cavity resonance driven. Squid readout – tuneable cavity ...

What about Anthropic Principle?

Trendy Explanation for "coincidences" or "tunings"

Why is Cosmological Constant so small?
If it were 100 times bigger, matter would fly apart or collapse before life could evolve. Nature plays dice, universes with all values occur, but only universes with life get observed.

Why does **QCD** respect $\mathbf{T}$ symmetry?
If **QCD***violated* $\mathbf{T}$*, something would go wrong with nuclear physics, which would make life impossible. Nature plays dice, only universes where life is possible get observed.*
Except that life is fine in a world where $\Theta = 10^{-2}$!