Gravitational waves from low-energy inflation by particle production

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Fujita, RN, Obata & Tada, *in preparation*

Fujita, RN & Tada, PLB 778, 17 (2018), arXiv:1705.01533.

Past collaboration with
Barnaby, Hazumi, Hikage, Mukohyama, Namikawa, Peloso, Shiraishi, Shiu, Sorbo, Unal

Outline

1. Introduction

2. Model with $SU(2)$ Gauge Field

3. Summary and Conclusion

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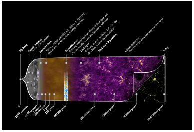
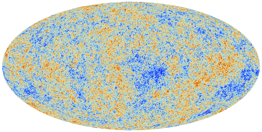
Introduction





◇ **Inflation** $\sim$ a good candidate paradigm to describe the primordial Univ.

- ▷ Solves the problems in hot BB cosmology
- ▷ Provides seeds for structure formation

horizon

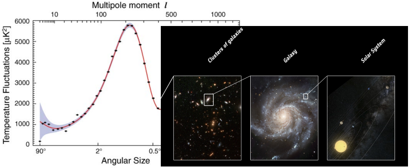
monopole flatness

⇕

Exponential expansion

⇕

$\phi(t)$



⇕

Inhomogeneities

⇕

$\delta\varphi$

Gravitational Waves from Inflation

Gravitational waves (GWs) h_{ij} – generic prediction of inflation

h_{ij} = (traceless & transverse part of δg_{ij}) = **tensor** mode

Tensor-to-scalar ratio

$$r \equiv \frac{\langle h\, h \rangle}{\langle \zeta\, \zeta \rangle}$$

ζ = (trace part of δg_{ij} (comoving gauge)) = **scalar** mode

A large number of experimental/observational efforts

- ◇ Planck, POLARBEAR, BICEP/Keck Array, SPIDER, . . .
- ◇ LiteBIRD, Simons Array, EBEX, PIXIE, COrE+ . . .
- ◇ **Future experiments aim for $\sigma(r) = \mathcal{O}(10^{-3})$**

Standard Prediction

Standard prediction for GWs from inflation

$$P_{\rm GW}(k) = \left.\frac{2H^2}{\pi^2 M_{\rm Pl}^2}\right|_{k=aH} , \qquad E_{\rm inflation} \cong 5 \cdot 10^{15}\,{\rm GeV} \left(\frac{P_{\rm GW}}{10^{-12}}\right)^{1/4}$$

Standard lore

Detectable GW $P_{\rm GW} \gtrsim 10^{-12}$ $\Longleftrightarrow$ Large $E_{\rm inflation} \gtrsim 10^{16}$ GeV

- ◇ Considered as direct probe of inflationary energy scale
- ◇ Slightly red-tilted $\sim$ decreasing H

?

Crucial assumptions

Source of GWs $=$ **vacuum fluctuations** of graviton

Evolution driven only by **expansion** of the universe

Evolution from Initial Quantum Vacuum

Initial Vacuum State

Initial

$\Downarrow$

deep inside the horizon

$\Downarrow$

$k \gg aH$

Vacuum

$\Downarrow$

no particle state

$\Downarrow$

$n_\lambda = 0$

$\Downarrow$

Classical evolution — governed by **expansion** $\sim H$

$\Downarrow$

$$P_{\rm GW}(k) = \left.\frac{2H^2}{\pi^2 M_{\rm Pl}^2}\right|_{k=aH}$$

- Quasi de Sitter $\Longrightarrow$ Quasi scale-invariance of $P_{\rm GW}$ (slight red tilt)

Question

How robust would this be upon detection?

- $E_{\text{inflation}}$?
- Quantum fluctuations of vacuum ?
- No interesting dynamics ?

General Arguments

$$P_{\mathrm{GW}} \equiv \frac{\mathrm{d}}{\mathrm{d}\ln k}\left\langle \left(\delta g_{ij}^{TT}\right)^2 \right\rangle \sim \frac{1}{\rho_{\mathrm{total}}} \frac{\mathrm{d}}{\mathrm{d}\ln k} \rho_{\mathrm{GW}} = \frac{\mathrm{d}}{\mathrm{d}\ln k} \Omega_{\mathrm{GW}}$$

GW power spectrum $\sim$ Spectrum of GW energy fraction Ω_{GW}

- **Standard lore:** GW generation determined only by expansion

$$\rho_{\mathrm{GW}} \sim H^4 \quad \Longrightarrow \quad \Omega_{\mathrm{GW}} \sim \frac{H^2}{M_{\mathrm{Pl}}^2}$$

- **In General:** There can be additional source for GW

$$\rho_{\mathrm{GW}} \not\sim H^4 \quad \Longrightarrow \quad \Omega_{\mathrm{GW}} \not\sim \frac{H^2}{M_{\mathrm{Pl}}^2}$$

In general...

Detectable GW $\neq$ **Large** $\boldsymbol{E}_{\textbf{inflation}}$

is possible.

This is such a simple argument...

Why hasn't this possibility been considered extensively?

Scalar-vector-tensor decomposition

Decomposition theorem (in cosmology)

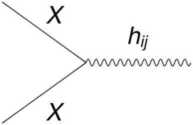
On **homogeneous** and **isotropic** background,
scalar, **vector** & **tensor** modes are decoupled
at the **1st-order** cosmological perturbations

$$\delta_1 S \ , \quad \delta_1 V_i \qquad \not\Rightarrow \qquad h_{ij}$$

Decomposition theorem

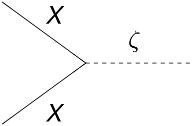
— Scalar/vector sources need to be 2nd order

$$\partial_i \delta S \, \partial_j \delta S \,, \quad \delta V_i \, \delta V_j \qquad \Longrightarrow \qquad h_{ij}$$



— They also source curvature (scalar) perturbations

$$(\delta S)^2 \,, \quad (\delta V_i)^2 \qquad \Longrightarrow \qquad \zeta$$



— But we know from observations

$$\frac{P_{\rm GW}}{P_\zeta} = r \ll 1$$

It is difficult for the source effects to become dominant

Standard Lore

Decomposition theorem

$\Downarrow$

No 1st-order sourcing for GWs

$\Downarrow$

2nd-order sourcing is necessary

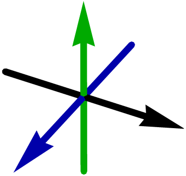
Cook & Sorbo '11; Senatore et al. '11; Cook & Sorbo '13; Ferreira & Sloth '14; Biagetti et al. '14; Mirbabayi et al. '14; Choi et al. '15; Ferreira et al. '15; Peloso et al. '16

ANY EXCEPTIONS ?

Exceptions to standard decomposition — requires additional “tensor”

- Introduce a new tensor field — e.g. bi-metric theory

- **Introduce an $SU(2)$ gauge field with a vev**



$$\langle A_\mu^a \rangle = A(t)\,\delta_\mu^a$$

— Isotropic ($SO(3)$ invariant) vev

— Perturbations δA_μ^a contain **“tensor” modes**

Maleknejad & Sheikh-Jabbari ’11

$$\delta A_i^a = (A + \delta A)\,\delta_{ia} + \partial_i \partial_a M + \partial_i M_a + \underset{\nearrow}{\boldsymbol{t_{ia}}}$$

“tensor” perturbation

— **“Tensor” modes** mix with GW h_{ij} at linear perturbations

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Model with $SU(2)$ Gauge Field

Model Criteria

- ***SU*(2) gauge field A_μ^a** $+$ **pseudo-scalar field χ**
- Unique interaction $\chi F\tilde{F}$
  - ▷ Necessary to prevent A_μ^a from decaying as $\rho_A \propto a^{-4}$
- **Decoupled from the inflaton sector**
  - ▷ Subdominant effects on inflationary dynamics
  - ▷ Interacts with the inflaton only gravitationally

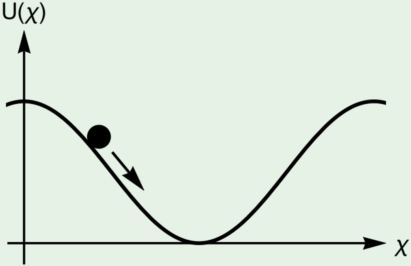
$$\mathcal{L} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{inflaton}} - \frac{1}{2}(\partial\chi)^2 - U(\chi) - \frac{1}{4}F_{\mu\nu}^a F^{a,\mu\nu} + \frac{\lambda}{4f}\chi F_{\mu\nu}^a \tilde{F}^{a,\mu\nu}$$

$$F_{\mu\nu}^a = 2\partial_{[\mu}A_{\nu]}^a - g\epsilon^{abc}A_\mu^b A_\nu^c \,, \qquad \tilde{F}^{a,\mu\nu} = \epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}/2$$

Dimastrogiovanni, Fujita & Fasiello '16

Model of Interest

$$\mathcal{L}_{\chi A} = -\frac{1}{2}(\partial\chi)^2 - U(\chi) - \frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \frac{\lambda}{4f}\chi F^a_{\mu\nu}\tilde{F}^{a,\mu\nu}$$



- Axionic field χ

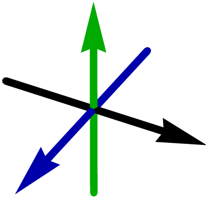
$$U(\chi) = \mu^4\left(1+\cos\frac{\chi}{f}\right)$$

- χ is in slow-roll

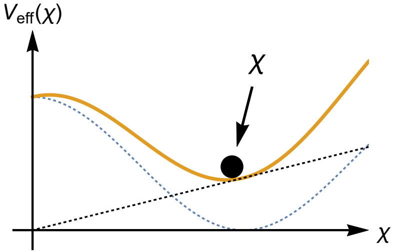
Background Attractor

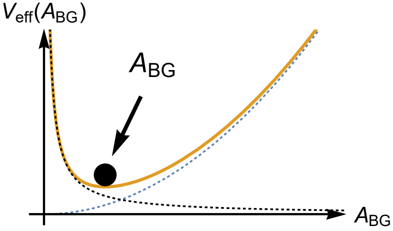
Isotropic vev

$$\langle A_0^a \rangle = 0 \, , \quad \langle A_i^a \rangle = a \, A_{\rm BG} \, \delta_i^a$$



Slow-Roll Attractor behavior





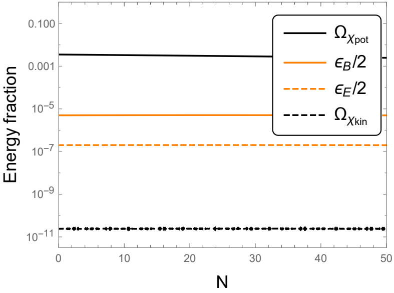
$$V_{\rm eff}(\chi) \simeq U(\chi) + \frac{3g\lambda H A_{\rm BG}^3}{f} \chi \qquad V_{\rm eff}(A_{\rm BG}) \simeq \frac{3H^2}{2} A_{\rm BG}^2 + \frac{H\mu^4 \sin(\chi/f)}{g\lambda A_{\rm BG}}$$

Key Parameters

Energy fraction: $\Omega_A \equiv \dfrac{g^2 A_{\rm BG}^4}{M_{\rm Pl}^2 H^2}$

“Mass”: $m_Q \equiv \dfrac{g A_{\rm BG}}{H}\ ,$

“Coupling”: $\xi \equiv \dfrac{\lambda \dot{\chi}}{2 f H} \underset{\substack{\uparrow \\ \text{attractor}}}{\simeq} m_Q + \dfrac{1}{m_Q}$



Control parameters

$\boldsymbol{m_Q\ ,\ \Omega_A}$

Tensor perturbations

- **A_μ^a perturbation decomposed into “scalar,” “vector,” “tensor”**

$$\delta A_0^a = \partial_a Y + Y_a \; , \quad \delta A_i^a = \delta A \, \delta_{ia} + \partial_i \partial_a M + \partial_i M_a + \underbrace{\boldsymbol{t_{ia}}}_{\textbf{“tensor”}}$$

- **Two tensor modes $h_{ij}^{\rm TT}$ & t_{ia} mix at the linear order**

- **Parity-violating operators**

$$\mathcal{L} \supset -\frac{1}{4} {\rm Tr}(\boldsymbol{F}^2) + \frac{\lambda\chi}{4f} {\rm Tr}(\boldsymbol{F}\tilde{\boldsymbol{F}}) \; \sim \; m_Q \, \epsilon^{abc} t_{al} \partial_b t_{cl} \; , \quad \xi \, \epsilon^{ijk} t_{il} \partial_j t_{kl}$$

Parity violation $\Longleftrightarrow$ right-handed mode $\neq$ left-handed mode

$$h_{ij}^{\rm TT} = \sum_{P=R,L} e_{ij}^P h_P \; , \qquad t_{ij} = \sum_{P=R,L} e_{ij}^P t_P$$

L and R sectors are decoupled

$$\mathcal{T}_{R/L} = \left(h_{R/L}\,,\ t_{R/L}\right)$$

$$\mathcal{L}_{R/L} \cong \frac{1}{2}\left(\mathcal{T}'^{\dagger}_{R/L}\mathcal{T}'_{R/L} - \mathcal{T}^{\dagger}_{R/L}\mathbf{\Omega}^{2}_{R/L}\mathcal{T}_{R/L}\right)$$

$$\left(\mathbf{\Omega}^{2}_{R/L}\right)_{11} \cong k^2 - \frac{a''}{a} \qquad \textbf{— metric tensor}$$

$$\left(\mathbf{\Omega}^{2}_{R/L}\right)_{22} \cong a^2H^2\left[\frac{k^2}{a^2H^2} \pm 4m_Q\frac{k}{aH} + 2m_Q^2\right] \qquad \textbf{— } \boldsymbol{SU(2)} \textbf{ tensor}$$

$$\left(\mathbf{\Omega}^{2}_{R/L}\right)_{12} \cong 2a^2H^2\sqrt{\Omega_A}\left(\mp\frac{k}{aH} - \xi\right) \qquad \textbf{— mixing}$$

t_L amplification: $\left(2-\sqrt{2}\right)m_Q \lesssim \frac{k}{aH} \lesssim \left(2+\sqrt{2}\right)m_Q$

Mixing with h_L: $\propto \sqrt{\Omega_A}$

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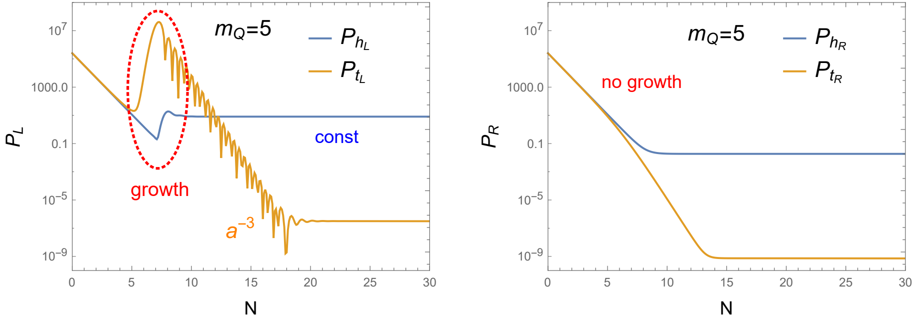
$SU(2)$ energy fraction < 1

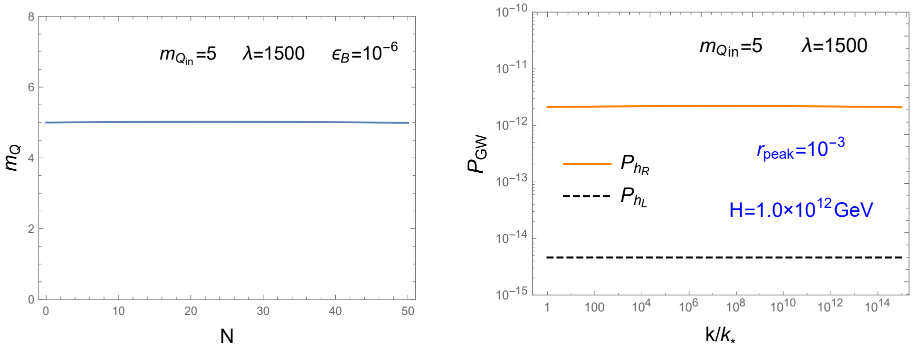
Evolution of *R*- and *L*-handed modes



- Tachyonic (exponential) growth in t_L for a finite duration
- Energy transfer to h_L is suppressed by $\sqrt{\Omega_A}$
- After enhancement, h_L becomes constant
- After enhancement, t_L damps due to mass $> H$
- Energy transfer $h_L \to t_L$ sustains t_L constant after $T_L/H_L \sim \sqrt{\Omega_A}/m_Q$

Large coupling — Scale-invariant $P_{\rm GW}$

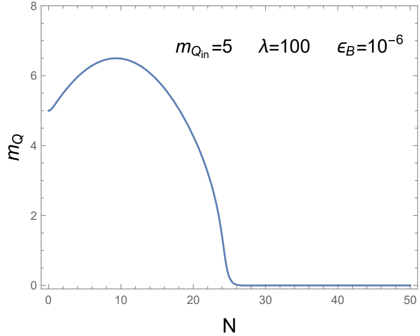
Large coupling λ $\Longleftrightarrow$ large friction $\Longleftrightarrow$ $m_Q = $ const. $\Longleftrightarrow$ scale invariant

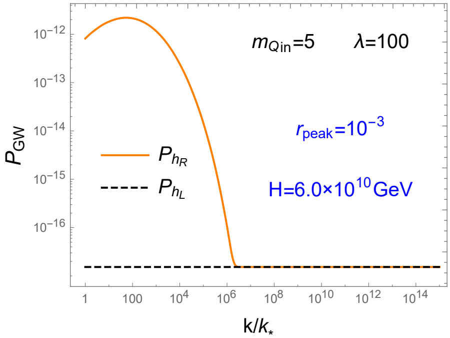


$$f = 10^{-3} M_{\rm Pl} \, , \quad g = 1.1 \cdot 10^{-2} \, , \quad \frac{\mu^2}{f} = 3.1 \cdot 10^{13}\,\text{GeV}$$

Small coupling — Scale-dependent $P_{\rm GW}$

Small coupling $\lambda \iff$ small friction $\iff m_Q \neq$ const. $\iff$ scale variant





$$f = 10^{-3}\,M_{\rm Pl}\ , \quad g = 6.1\cdot 10^{-4}\ , \quad \frac{\mu^2}{f} = 6.9\cdot 10^{11}\,\text{GeV}\ ,$$

Detectable *r* for low-scale inflation

GW spectrum: $P_{\rm GW} \simeq P_{h_L} = \underbrace{\Omega_A \, F(m_Q)}_{\text{growth}} \times \frac{2H^2}{\pi^2 M_{\rm Pl}^2}\,, \quad F(m_Q) \sim \exp\left(1.2\pi m_Q\right)$

Tensor-to-scalar: $r = \frac{P_{\rm GW}}{P_\zeta} = \underbrace{\Omega_A \, F(m_Q)}_{\text{growth}} \times r_{\rm standard}$

- Suppressed by fractional energy density, $\Omega_A \ll 1$
- **Exponentially enhanced by m_Q**

Message

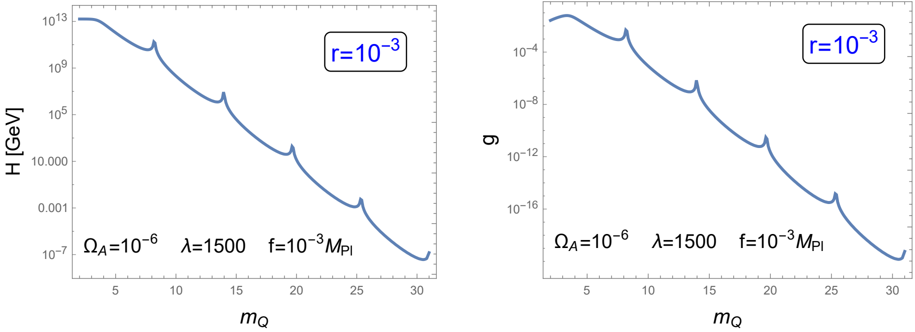
- Relation between $P_{\rm GW}$ & H is no longer one-to-one
- Exponentially enhanced compared to the standard prediction
- For a given r, required value of H is exponentially smaller

Detectable *r* for low-scale inflation

- For given values of $\{r, m_Q, \Omega_A\}$, H and g scale as

$$\boxed{H \sim 10^{13}\,\mathrm{GeV} \times \mathrm{e}^{-0.6\pi m_Q} \sqrt{\frac{r}{\Omega_A}} \,, \quad g \sim 10^{-5}\, \mathrm{e}^{-0.6\pi m_Q} \frac{\sqrt{r}}{\Omega_A}}$$

Example parameters



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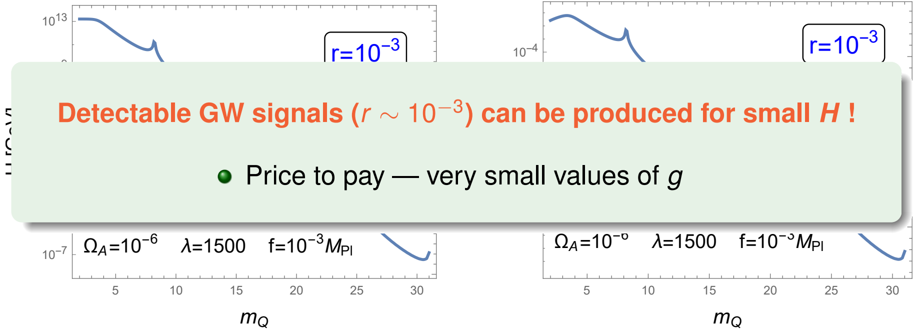


Detectable GW signals ($r \sim 10^{-3}$) can be produced for small H !

- Price to pay — very small values of g

Consistencies and Constraints

Production of gauge field δA_μ^a is very efficient. For large values of m_Q,

- ▷ The validity of our calculation may break down
- ▷ Too large production may be constrained by other observables

We need to ensure a parameter space to avoid such pathologies

CHECK LIST

1. Backreaction of δA_μ^a to the background dynamics is negligible
2. Our perturbative calculation is justified
3. Constraints on curvature (scalar) perturbations are respected

Backreaction to background dynamics

- Tensor backreaction to **Friedmann equation**

$$3M_{\rm Pl}^2H^2 = \rho_\phi + \rho_\chi + \rho_A + \langle \delta\rho_{\rm tensor} \rangle$$

Require: $\langle \delta\rho_{\rm tensor} \rangle \ll \rho_A$

- Tensor backreaction to **background equations of motion**
  - EOM of χ
  - EOM of $A_{\rm BG}$

— **Require:** Produced δA_μ^a has negligible contribution to them

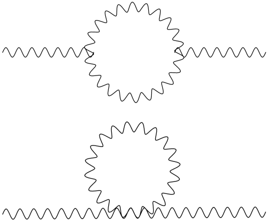
Perturbativity

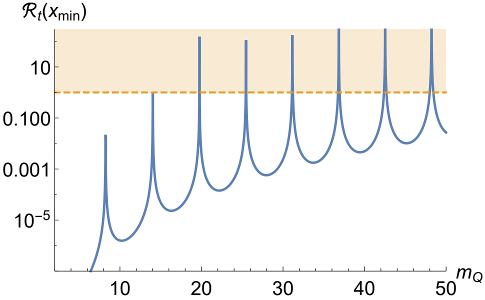
Our perturbative calculation is justified if: $R_t \equiv \dfrac{\langle (t_{ij})^2 \rangle_{\text{1-loop}}}{\langle (t_{ij})^2 \rangle_{\text{tree}}} \ll 1$

3-pt. & 4-pt. interactions

$$H_I^{(3)} = g \times \mathcal{O}(t_{ij}^3)$$

$$H_I^{(4)} = g^2 \times \mathcal{O}(t_{ij}^4)$$





Curvature perturbation (*in progress*)

Curvature perturbation ζ is produced by $\delta\phi$, because

— **“Tensor” modes t_{ia} do not source ζ at linear level**

— “Scalar” modes of δA^a_μ are negligible (mass suppression)

— $\delta\chi$ is negligible as long as it does not become a curvaton

Two possible contributions

1. Spectral index can be modified by the presence of A^a_μ background vev

$$n_s - 1 = 2(\eta_\phi - 2\epsilon_\phi - \epsilon_H) \simeq 2(\eta_\phi - \Omega_A)$$

▷ **Requires:** $\Omega_A \lesssim 10^{-2}$

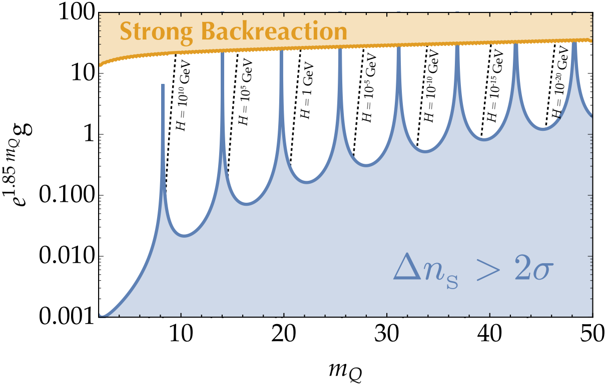
2. Second-order effects $\mathcal{O}(t_L^2)$ on scalar perturbations — *work in progress*

Viable Parameter Space for $r = 10^{-3}$

Backreaction/perturbativity constraints: $g \ll \mathcal{G}(m_Q) \propto \mathrm{e}^{-0.6\pi m_Q}$

Spectral index constraint: $g \gtrsim 10^{-3}\,\mathrm{e}^{-0.6\pi m_Q}\sqrt{r}$

Fujita, RN & Tada '17



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$m_Q \equiv g A_{BG}/H$

Signal Distinguishability

Lesson thus far: Detection of GW would **not** necessarily fix $E_{\text{inflation}}$

- Is this a bad news ?
- Would detected signals be indistinguishable from standard prediction ?

No.

- GW signals from this model are very **unique**
- **Look into other observables**

1 GW (tensor) non-Gaussianity

- ◇ Tensor three-point correlation

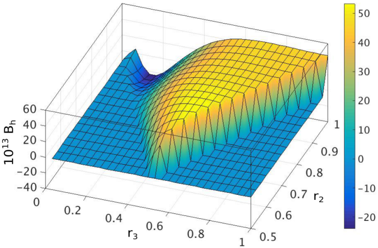
$$B_h = \langle hhh \rangle$$

- ◇ Undetectable in standard case
- ◇ Non-linearity parameter

$$f_{\rm NL}^{\rm tensor} \sim \frac{B_h}{P_\zeta^2} \approx \frac{r^2}{\Omega_A}$$

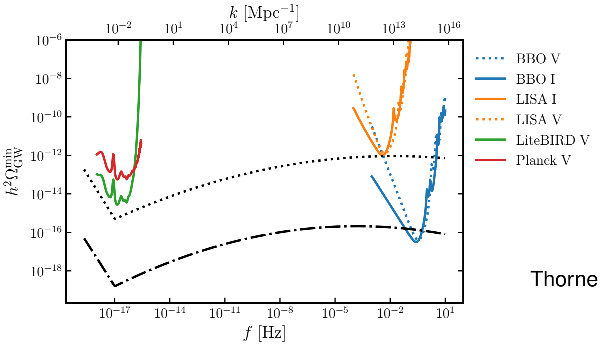
- ◇ Important probe of the origin of GW

Agrawal, Fujita & Komatsu '17



❷ **Parity-violating GW signals** — Left-handed $\neq$ Right-handed

- ◇ Induces correlations that would be otherwise null
- ◇ **CMB**
  - ▷ Temperature & B-mode $\langle TB \rangle$
  - ▷ E-mode & B-mode $\langle EB \rangle$
- ◇ **GW interferometers**
  - ▷ Aim for direct detection of parity violation



Thorne et al. '17

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Summary and Conclusion

- Future observations aim for $\sigma(r) = \mathcal{O}\left(10^{-3}\right)$
- **Standard notion for inflationary GW:**
  - ▷ Vacuum fluctuation of graviton & evolution only by expansion
  - ▷ $E_{\text{inflation}} \cong 10^{16}\,\text{GeV} \times (r/0.01)^{1/4}$ — detection of r implies high $E_{\text{inflation}}$
- **$SU(2)$ can induce dominant GW signals — $r \not\Longrightarrow$ high $E_{\text{inflation}}$**
  - ▷ $SU(2)$ perturbations can source GWs at linear order
  - ▷ Background motion induces parity violation in the perturbations
  - ▷ Scale-invariant/variant features in spectra
- **Non-Gaussianity & parity violation can distinguish the origin of GW**
  - ▷ Neither NG nor PV $\Longrightarrow$ Standard inflation
  - ▷ Both NG and PV $\Longrightarrow$ $SU(2)$ origin
  - ▷ Only one of NG & PV $\Longrightarrow$ something else ??

. . . Let's hope for detection of r