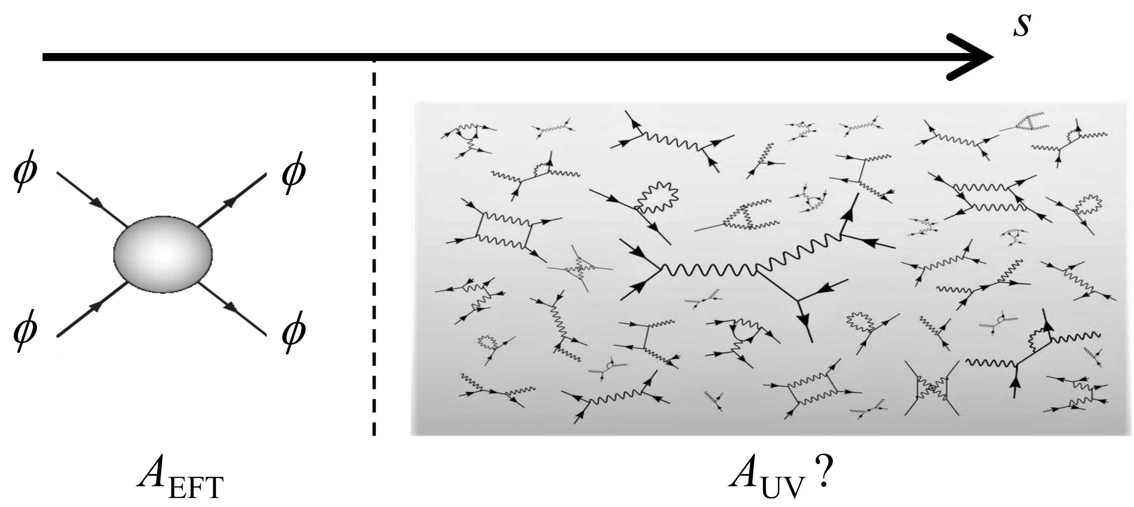
McGill CHEP Seminar 20 Oct 2017

Positivity Constraints on EFTs for Cosmology & Gravity

Scott Melville



Imperial College London

- **Effective Field Theory** — How it works and why it's useful
- **Positivity Bounds** — Imprints of unitarity/causality/locality/crossing
- **Applications** — Galileons, Massive gravity, Dark Matter

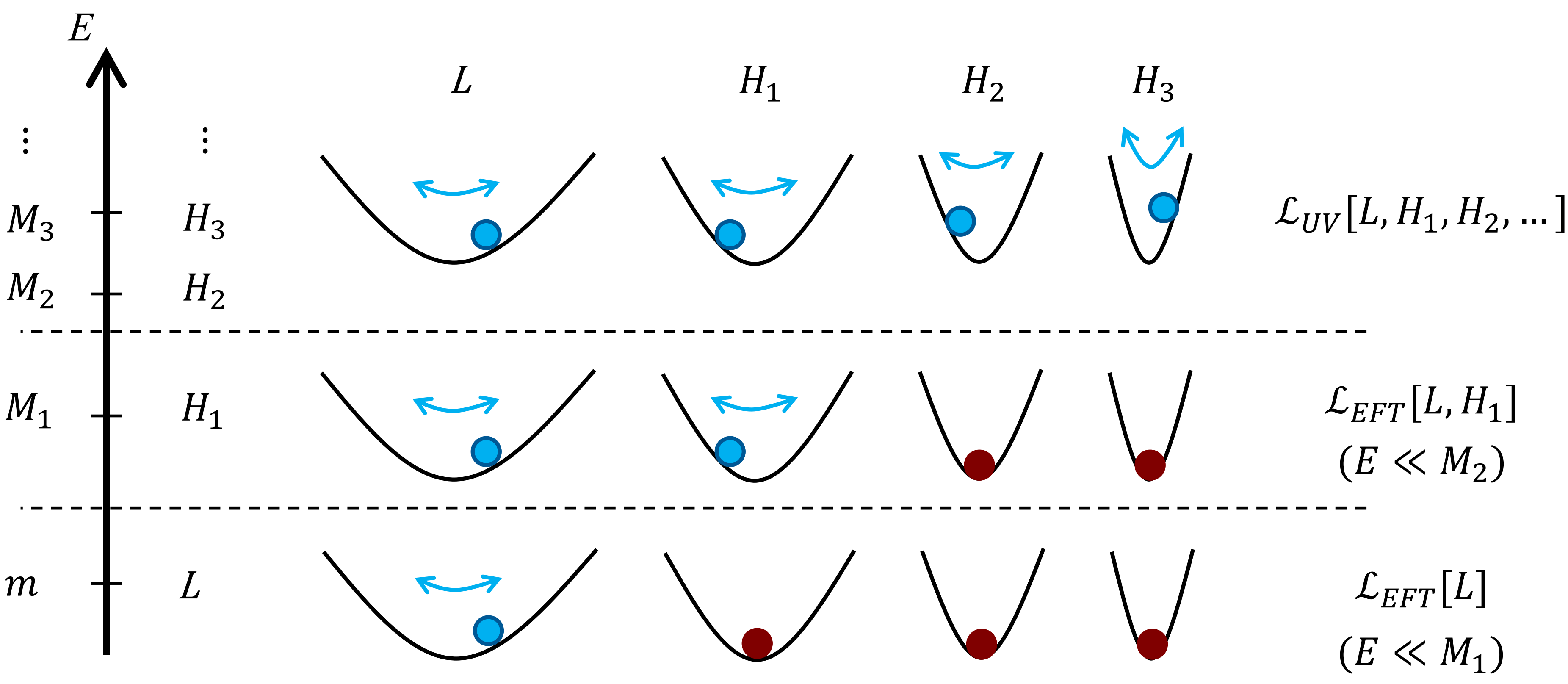
S Matrix Program, 1960's
Martin, Mandelstam, …

Adams et al., JHEP 10 (2006) 012
Causality, Analyticity and an IR Obstruction to UV Completion

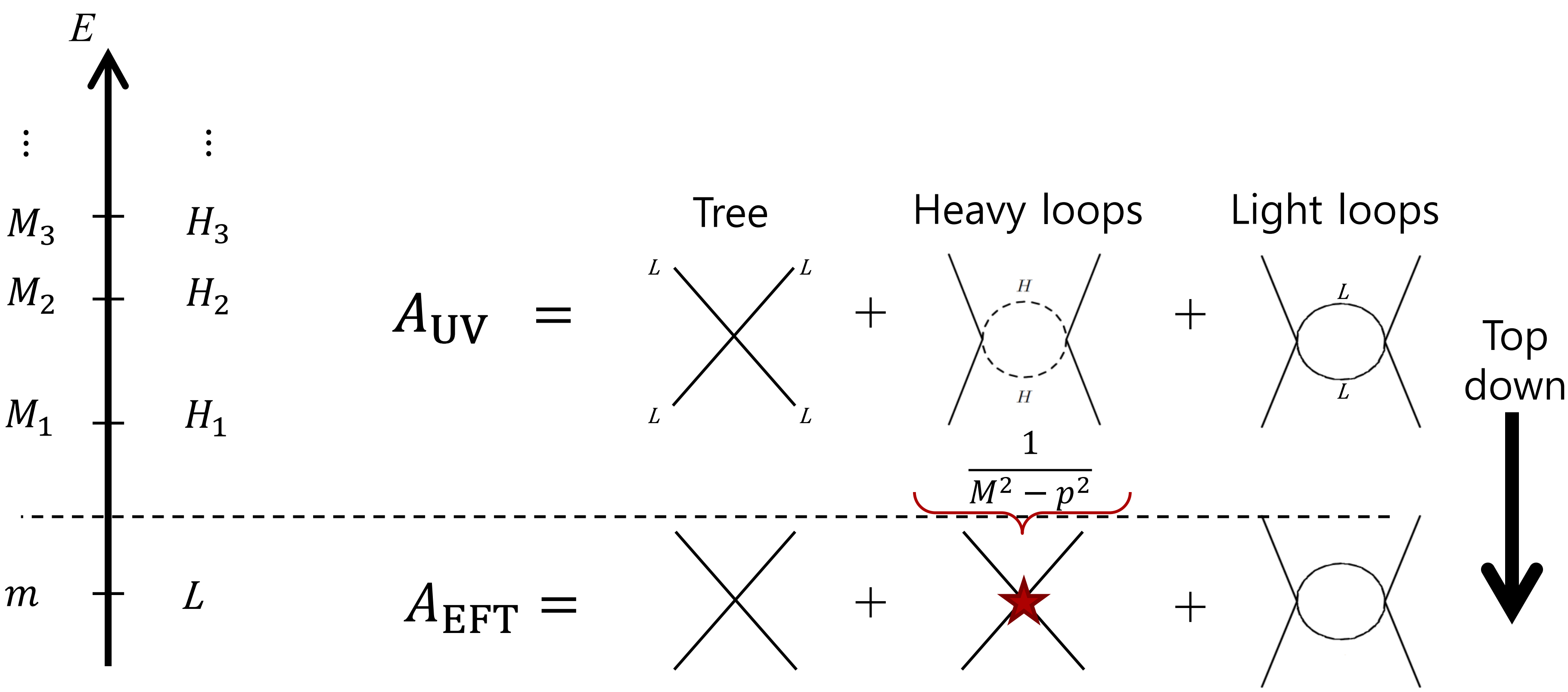
Bellazzini, JHEP 02 (2017) 034
Softness and Amplitudes' Positivity for Spinning Particles

SM, de Rham, Tolley, Zhou 2017
Positivity Bounds for Scalar Theories [1702.06134]
Massive Galileon Positivity Bounds [1702.08577]
Positivity Bounds for Spinning Particles [1706.02712]

Low Energy EFT

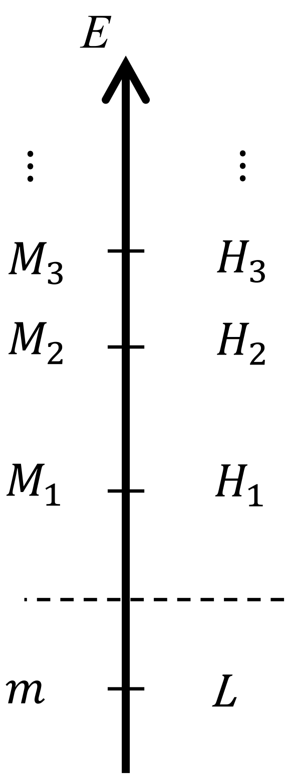


Low Energy EFT



$$\mathcal{O}(1) + \mathcal{O}\left(\frac{m^2}{M^2}\right) + \mathcal{O}\left(\frac{m^4}{M^4}\right) + \cdots + \mathcal{O}(g^2) + \cdots$$

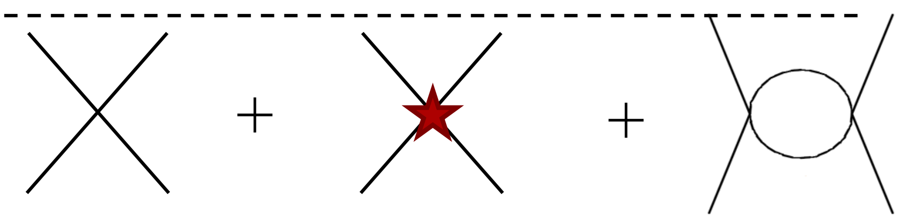
Low Energy EFT



$$A_{\text{UV}} = ???$$

Bottom up

$$A_{\text{EFT}} =$$

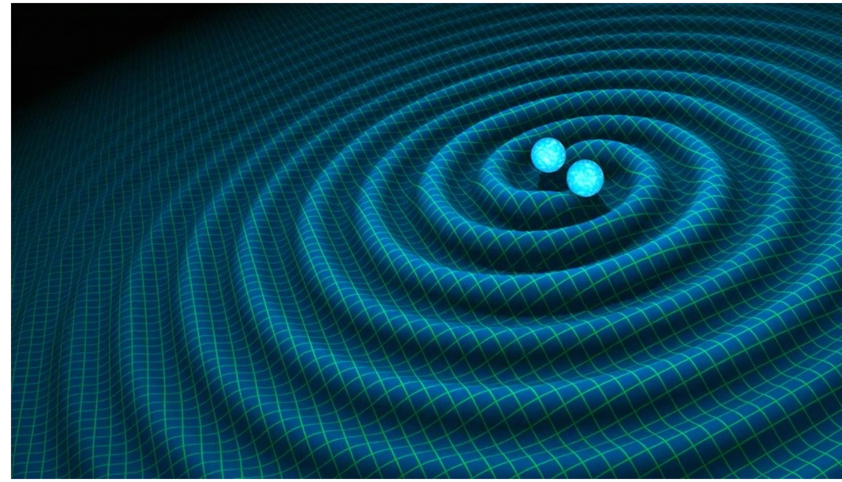


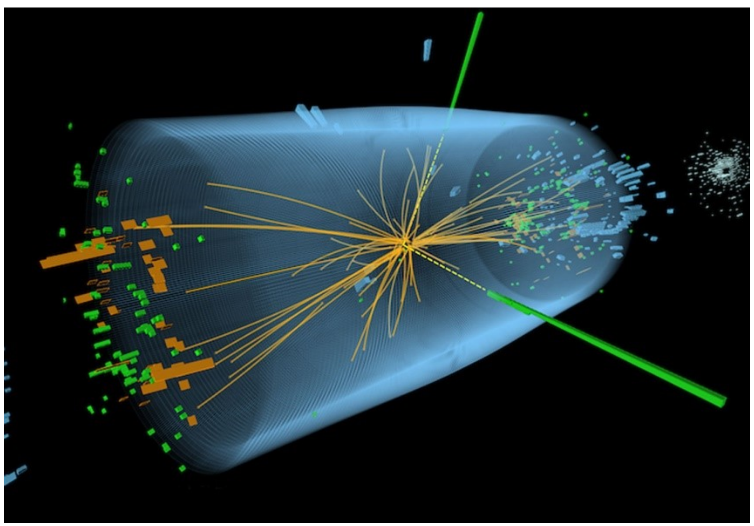
$$O(1) + O\left(\frac{m^2}{M^2}\right) + O\left(\frac{m^4}{M^4}\right) + \cdots + O(g^2) + \cdots$$

Wilson Coefficients (LECs) $\rightarrow$ c_0, c_2, c_4

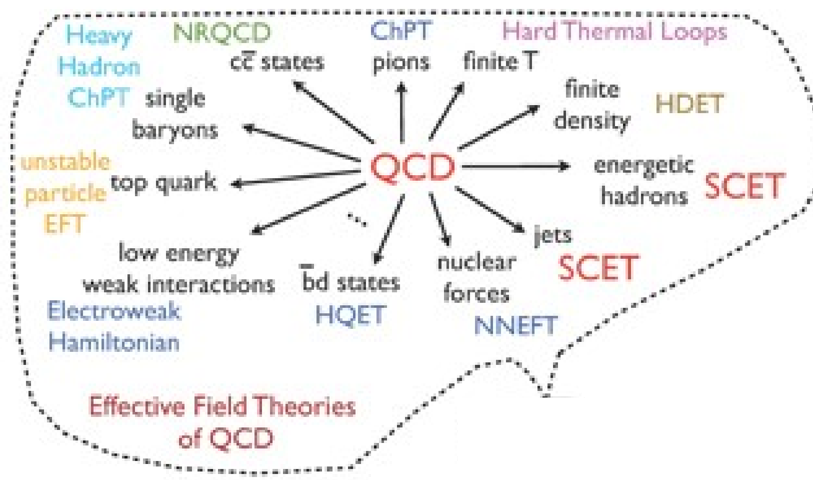
Cosmology

- EFT of Inflation
- EFTs for Reheating
- EFT of Large Scale Structure
- EFTs of Dark Matter
- EFT of Dark Energy
- …

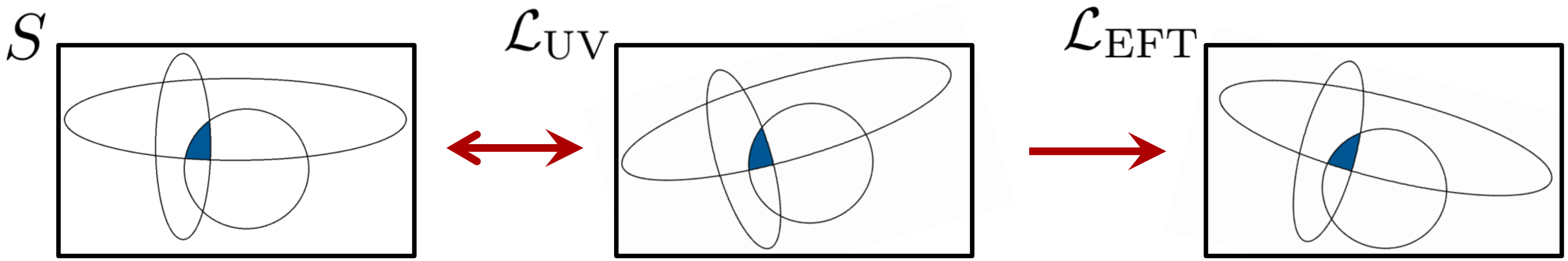


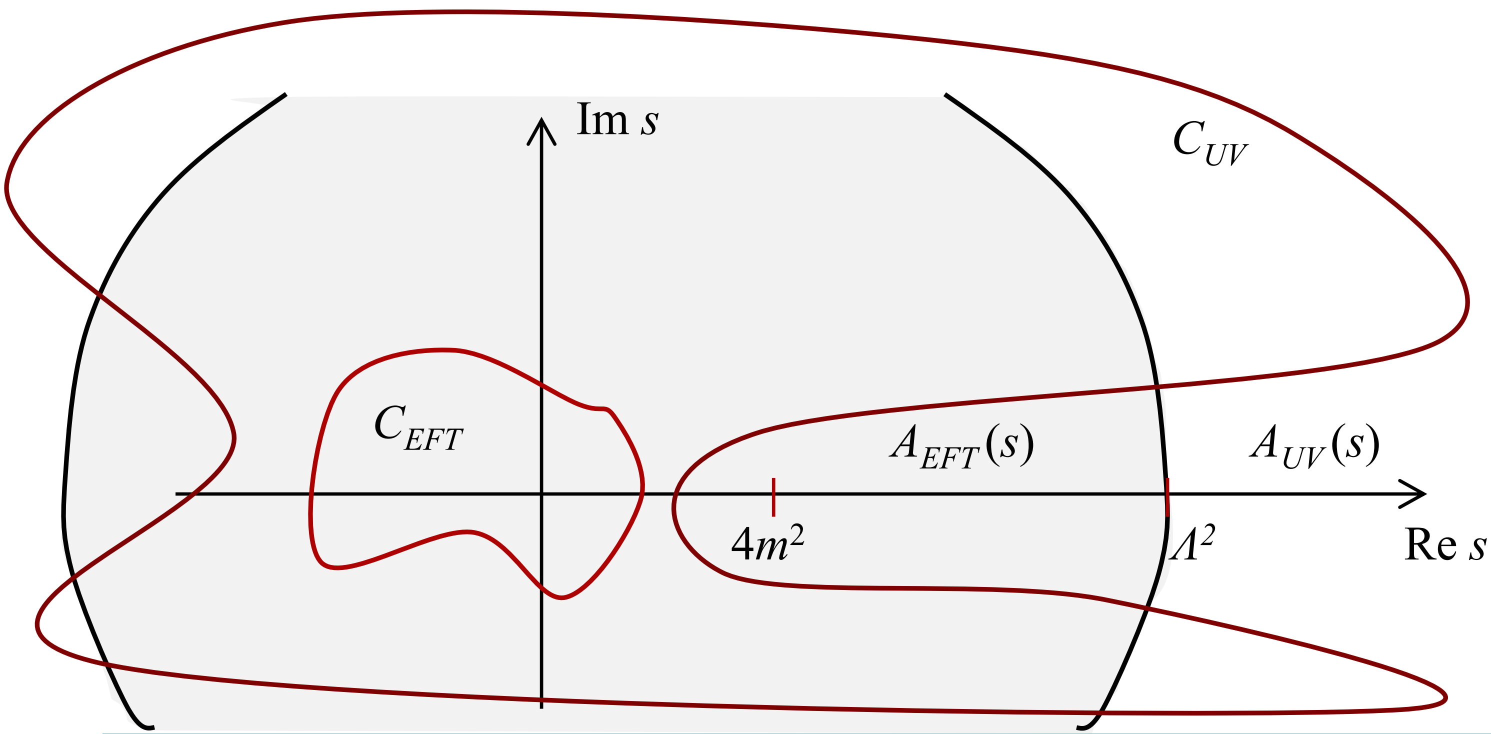


Particle Physics



1960 S Matrix Program (1102.0168) hep-ph/9607351 hep-th/0602178 1702.06134 1706.02712 2017





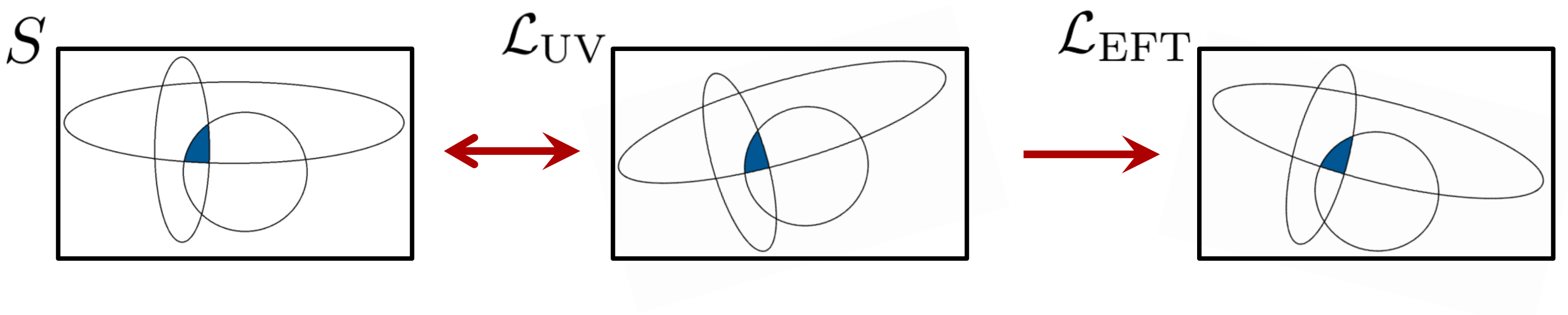
Analyticity

$$\oint_{C_{EFT}} = \oint_{C_{UV}}$$

Unitarity

$$\oint_{C_{UV}} > 0$$

			hep-ph/9607351	1702.06134	
1960	*S* Matrix Program	(1102.0168)	hep-th/0602178	1706.02712	2017

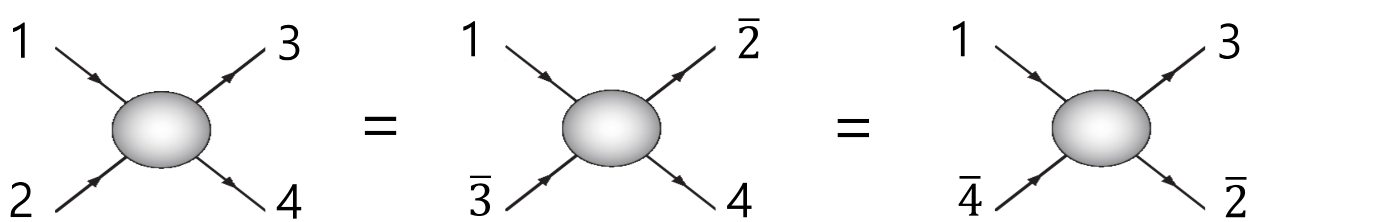


1. Unitarity $\quad S^\dagger S = 1 \quad \Rightarrow \quad \mathrm{Im}\, A(s,t)$ **positive** for $s > 4m^2$ $\quad (0 \leq t < 4m^2)$

2. Causality $\quad [\,\mathcal{O}(x), \mathcal{O}(y)] = 0$ if $(x-y)^2$ spacelike $\quad \Rightarrow \quad A(s,t)$ **analytic** in s at fixed t

3. Locality $\quad \int \mathrm{d}k \;\; e^{ikx} A(k)$ exists $\quad \Leftarrow \quad A(s,t)$ **polynomially bounded**

4. Crossing



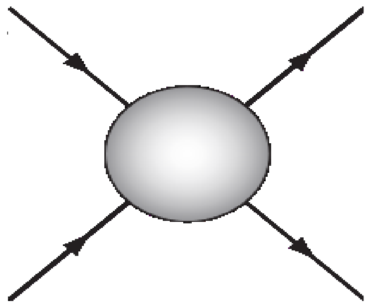
Kinematics

16 components – 10 (Poincaré) – 4 (on-shell) = 2 (ω, k, θ)

$(\omega^2 = k^2 + m^2)$

p_1 $(\omega, 0{,}0, k)$

$-p_3$ $(\omega, k\sin\theta\,, 0, k\cos\theta)$



$(\omega, 0{,}0, -k)$ p_2

$(\omega, -k\sin\theta\,, 0, -k\cos\theta)$ $-p_4$

$$
\begin{aligned}
s &= 4\omega^2 &&= -(p_1+p_2)^2 \\
t &= 2k^2(1+\cos\vartheta) &&= -(p_1+p_3)^2 \\
u &= 2k^2(1-\cos\theta) &&= -(p_1+p_4)^2
\end{aligned}
$$

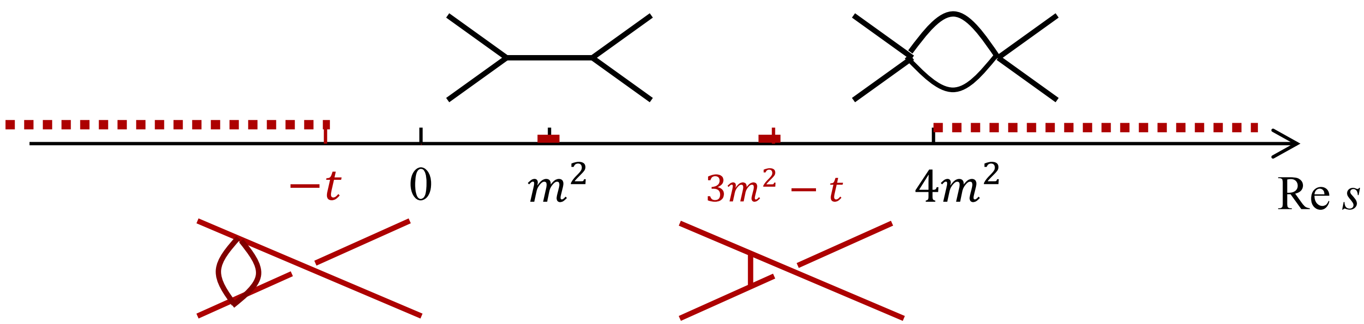
$(s+t+u = 4m^2)$

$$S = \mathbb{1} + iT$$

$$A_{12\to 34} = \langle p_3 p_4 | T | p_1 p_2 \rangle = A(s,t)$$

Analyticity

$$A(s,t) = \sum_{p,q} s^p\, t^q + \text{poles} + \text{branch cuts}$$



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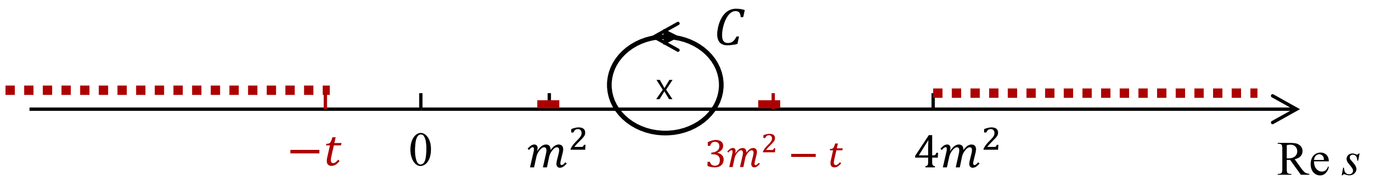
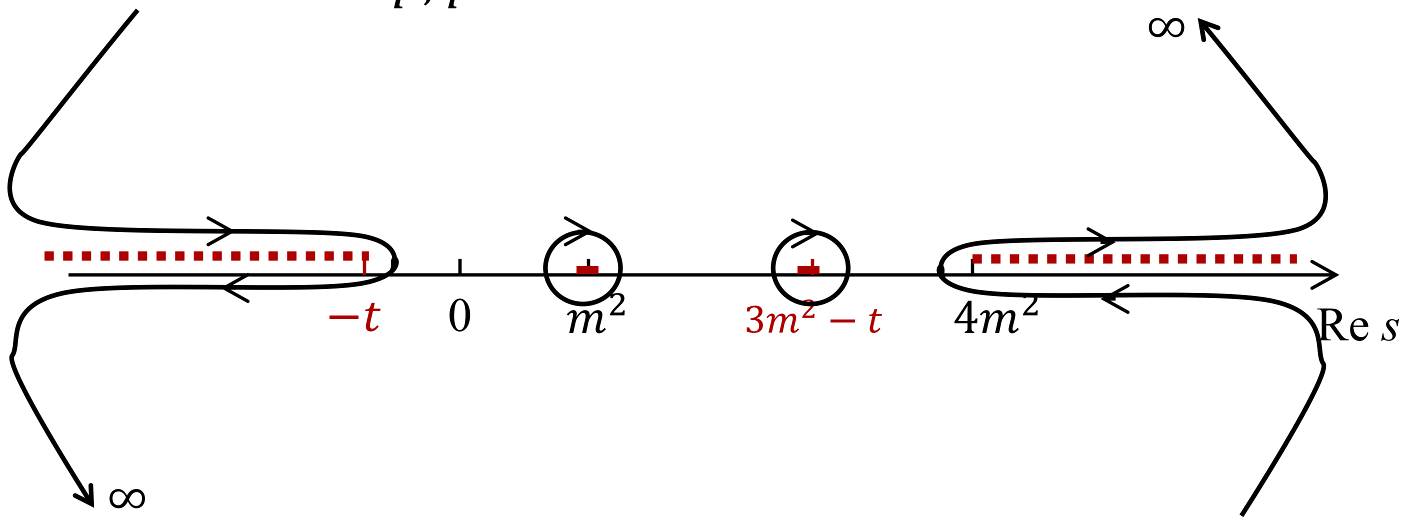


$$A(s,t) = \oint_C \frac{ds'}{2\pi i} \frac{A(s',t)}{s'-s}$$

Analyticity

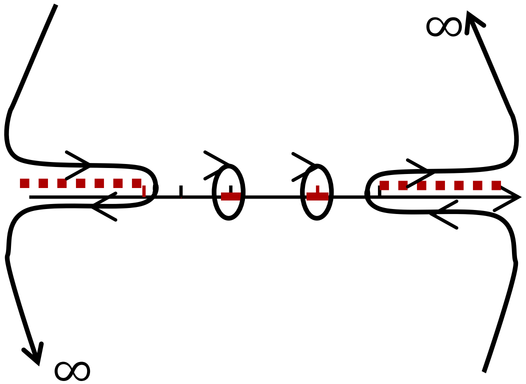
$$A(s,t) = \sum_{p,q} \; s^p \, t^q + \text{poles} + \text{branch cuts}$$



$$A(s,t) = \frac{\text{Res}}{s-m^2} + \frac{\text{Res}}{u-m^2} + \left(\int_{4m^2}^{\infty} + \int_{-\infty}^{-t} \right) \frac{A(s'+i\varepsilon,t) - A(s'-i\varepsilon,t)}{s'-s} + \int_{C^\pm} \frac{A(s',t)}{s'-s}$$

Dispersion Relation

$$A(s,t) = \sum_{p,q} s^p \, t^q + \text{poles} + \text{branch cuts}$$



1. Subtract poles: $B(s,t) = A(s,t) - \text{poles}$
2.
3.
4.

$$A(s,t) = \frac{\text{Res}}{s - m^2} + \frac{\text{Res}}{u - m^2} + \left(\int_{4m^2}^{\infty} + \int_{-\infty}^{-t} \right) \frac{A(s' + i\varepsilon, t) - A(s' - i\varepsilon, t)}{s' - s} + \int_{C^{\pm}} \frac{A(s', t)}{s' - s}$$

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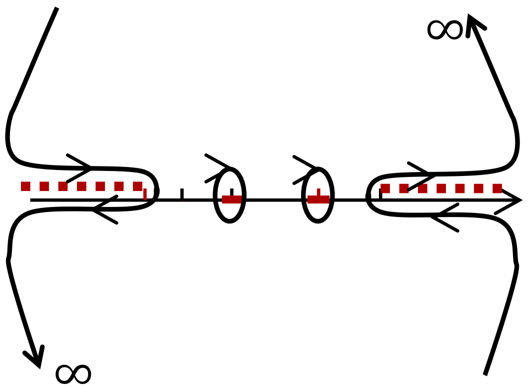
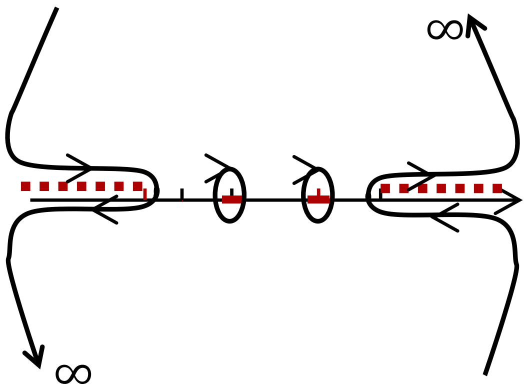


1. Subtract poles: $B(s,t) = A(s,t) - \text{poles}$
2. Schwarz Reflection: $A(s^*) = A^*(s)$
3.
4.

$$B(s,t) = \left(\int_{4m^2}^{\infty} + \int_{-\infty}^{-t} \right) \overbrace{\frac{A(s'+i\varepsilon,t) - A(s'-i\varepsilon,t)}{s'-s}}^{2i\,\mathrm{Im}\,A(s',t)} + \int_{C^\pm} \frac{A(s',t)}{s'-s}$$

Dispersion Relation

$$A(s,t) = \sum_{p,q} s^p t^q + \text{poles} + \text{branch cuts}$$



1. Subtract poles: $B(s,t) = A(s,t) - \text{poles}$
2. Schwarz Reflection: $A(s^*) = A^*(s)$
3. Crossing: $A(s,t) = A(u,t)$
4.

$$B(s,t) = \int_{4m^2}^{\infty} ds' \, \text{Im}\, A(s',t) \left(\frac{1}{s'-s} + \frac{1}{s'-u} \right) + \int_{C^\pm} \frac{A(s',t)}{s'-s}$$

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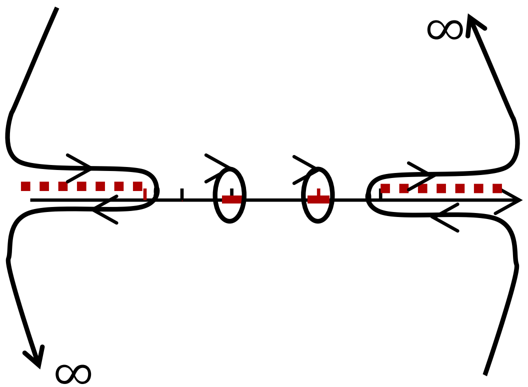
$$A(s,t) = \sum_{p,q} s^p t^q + \text{poles} + \text{branch cuts}$$



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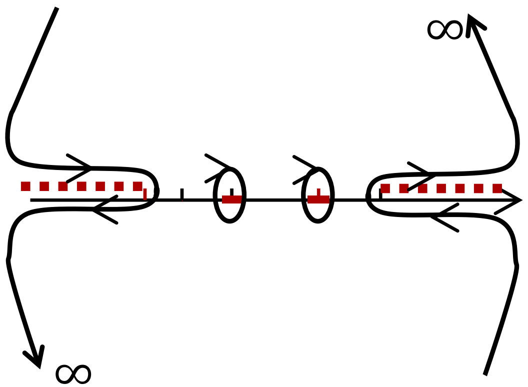

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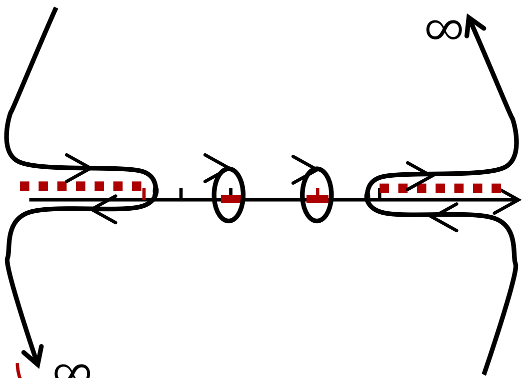
$$\frac{1}{2}\partial_s^2 B(s,t) = \int_{4m^2}^{\infty} ds' \, \text{Im}\, A(s',t) \left(\frac{1}{(s'-s)^3} + \frac{1}{(s'-u)^3} \right) + \int_{C^\pm} \frac{A(s',t)}{(s'-s)^3}$$

Dispersion Relation

$$A(s,t) = \sum_{p,q} s^p\, t^q + \text{poles} + \text{branch cuts}$$



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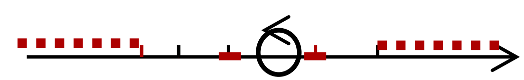
$$\int_{4m^2}^{\infty} \frac{ds'}{\pi} \operatorname{Im} A(s',t) \left(\frac{1}{(s'-s)^3} + \frac{1}{(s'-u)^3} \right) = \tfrac{1}{2} \partial_s^2 B(s,t)$$



Scalar Positivity Bounds

$$\partial_v^{2N} B(v,t)\Big|_{v=0} \propto \int_{4m^2}^{\infty} ds' \frac{\mathrm{Im}\, A(s',t)}{\left(s'+\frac{t}{2}\right)^{2N+1}} \qquad \left(v = s - 2m^2 + \frac{t}{2}\right)$$

$$\boxed{\partial_s^{2N} B(s,t) > 0}$$

$$\left(\frac{2N+1}{\mathcal{M}^2} + \partial_t\right)\partial_v^{2N} B(v,t)\Big|_{v=0} \propto \int_{4m^2}^{\infty} ds' \left[\partial_t - \frac{1}{s'+t} + \frac{1}{\mathcal{M}^2}\right] \frac{\mathrm{Im}\, A(s',t)}{\left(s'+\frac{t}{2}\right)^{2N+1}}$$

$$\boxed{\left(\frac{2N+1}{\mathcal{M}^2} + \partial_t\right)\partial_v^{2N} B(v,t)\Big|_{v=0} > 0} \qquad \mathcal{M}^2 = \left(s' + \frac{t}{2}\right)\Big|_{\text{branch point}}$$

$$\boxed{\partial_t^M \partial_v^{2N} B + \sum_{k=0}^{M/2} \sum_{r=0}^{M/2} \frac{c_{r,k}}{\mathcal{M}^{2k}} \partial_t^{M-2r} \partial_v^{2N+2r-2k} B > 0}$$

Spinning Positivity Bounds

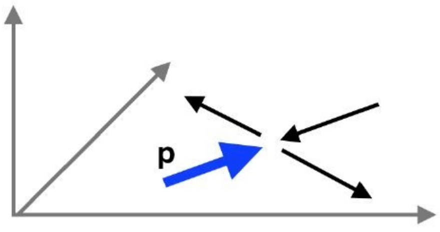
Kinematic Singularities

$$\omega \sin\theta = \frac{\sqrt{stu}}{s-4m^2} \quad \Rightarrow \quad (s-4m^2)^{\#}\left(\; A(\theta) + A(-\theta) \; \right)$$

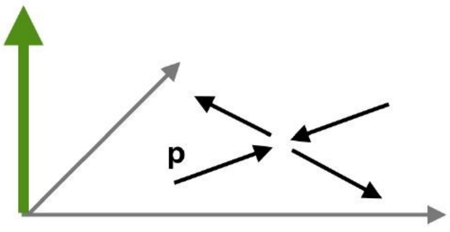
Crossing

$$A_{\tau_1\tau_2\tau_3\tau_4}(s,t) = e^{i(\dots)} A_{-\tau_1-\tau_2-\tau_3-\tau_4}(u,t)$$

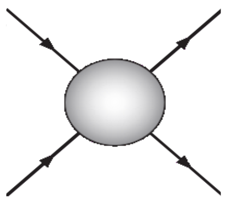
Helicity



Transversity



Shift symmetric scalar



$$\mathcal{L} = -\tfrac{1}{2}(\partial\varphi)^2 + c(\partial\varphi)^4 + \ldots$$

$$A(s,t) \sim c\,(s^2 + t^2 + u^2)$$

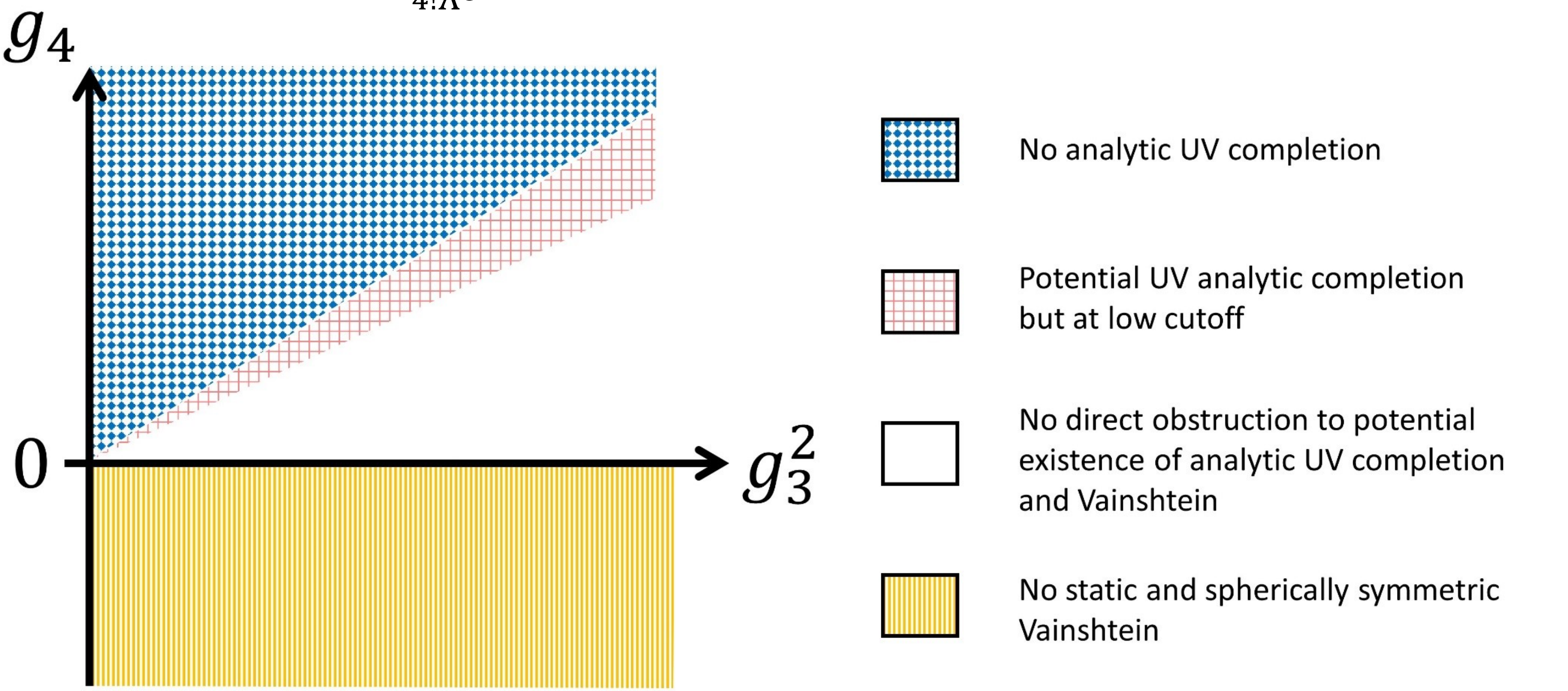
$$c > 0$$

DBI $\mathcal{L} = -\sqrt{1+(\partial\varphi)^2} = -\tfrac{1}{2}(\partial\varphi)^2 + \tfrac{1}{4}(\partial\varphi)^4 + \cdots$ ✓

Anti-DBI $\mathcal{L} = \sqrt{1-(\partial\varphi)^2} = -\tfrac{1}{2}(\partial\varphi)^2 - \tfrac{1}{4}(\partial\varphi)^4 + \cdots$ ✗

Massive Galileon

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \frac{g_3}{3!\Lambda^3}\varphi(\mathrm{tr}[(\partial\partial\varphi)^2] - \mathrm{tr}[\partial\partial\varphi]^2) + \frac{g_4}{4!\Lambda^6}\varphi(2\mathrm{tr}[(\partial\partial\varphi)^3] - 3\mathrm{tr}[\partial\partial\varphi](\mathrm{tr}[\partial\partial\varphi])^2 + \mathrm{tr}[\partial\partial\varphi]^3)$$



Massive Gravity

$$\mathcal{L} = M_P^2 \, R\left[\eta + \frac{h}{M_P}\right] - V(h)$$

$$V(h) = m^2[h^2] + \frac{c_3 m^4}{\Lambda_3^3}[h^3] + \frac{d_5 m^6}{\Lambda_3^6}[h^4]$$

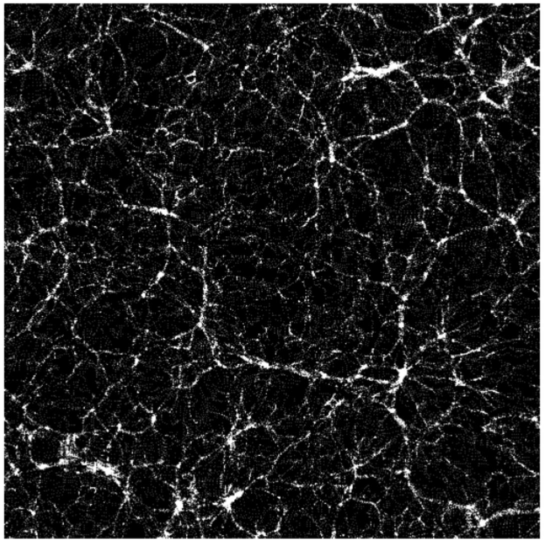
Work in Progress

Self-Interacting Dark Matter

to resolve small scale problems in simulations

$$0.1\text{cm}^2/\text{g} < \frac{\sigma}{m} < 1\text{cm}^2/\text{g}$$

to match observations



- cusp-core
- missing satellite
- too-big-to-fail
- diversity problem

- bullet cluster
- halo ellipticity
- substructure mergers
- merging clusters

Self-Interacting Dark Matter

to resolve small scale problems in simulations

$$0.1\text{cm}^2/\text{g} < \frac{\sigma}{m} < 1\text{cm}^2/\text{g}$$

to match observations

$$\frac{\sigma_{NR}}{m} = \underbrace{f(c_n)}_{\text{Positivity}} \underbrace{\frac{m^3}{\Lambda^6}}_{\Lambda > m} < \# \; m^{-3}$$

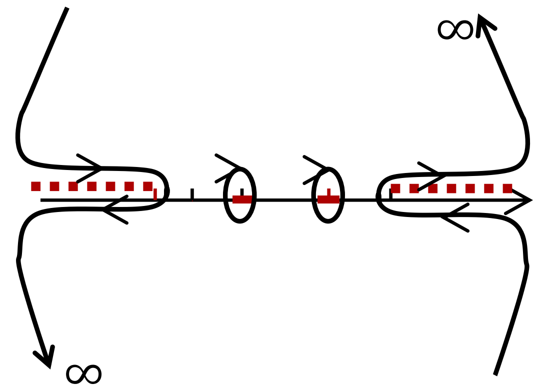
Work in Progress

Positivity Bounds

1. Unitarity
2. Causality
3. Locality
4. Crossing

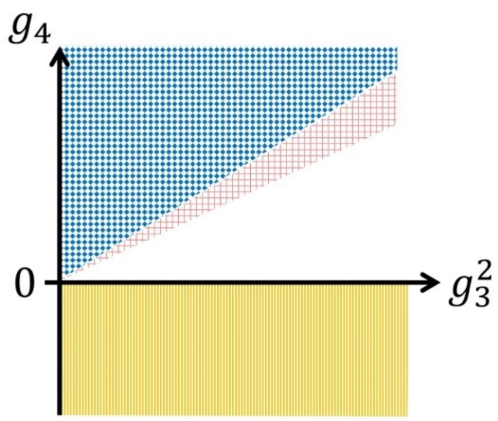
$$\sum_{i,j} \partial_t^i \partial_s^j \left(A(s,t) - \text{poles} \right) > 0$$

$$(s-4m^2)^{\#}\left(A_{\tau_1\tau_2\tau_1\tau_2}(\theta) + A_{\tau_1\tau_2\tau_1\tau_2}(-\theta)\right)$$



Applications

Massive Galileon



Massive Gravity

Self-Interacting Dark Matter

Work in Progress