Bulk Reconstruction and Entropic Area Laws

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AdS/CFT and Quantum Gravity

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- But this definition is very indirect - need dictionary to reformulate boundary theory into gravitational language

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A slightly less vague question

In AdS/CFT, when and how does (semi)classical gravity emerge from boundary field theory?

AdS/CFT and Quantum Gravity

A few related questions:

- What does it mean for a field theory to be holographic? When is a field theory holographic? [Heemskerk, Penedones, Polchinski, Sully]
- Given a holographic field theory, what are the dynamics of the dual gravitational theory? [Lashkari, McDermott, Van Raamsdonk, ...]

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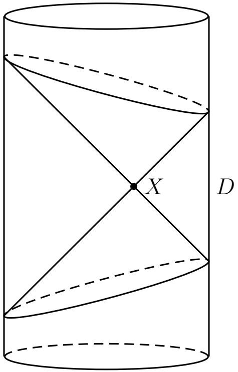
Recovering Bulk Operators

- In pure AdS, local field operators can be expressed in terms of local boundary operators by integrating against a kernel [Hamilton, Kabat, Lifschytz, Lowe]:

$$\phi(X) = \int_{D\subset\partial M} d^d x \, K(X|x)\mathcal{O}(x)$$



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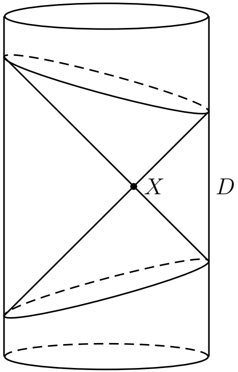

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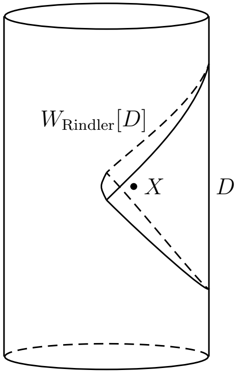

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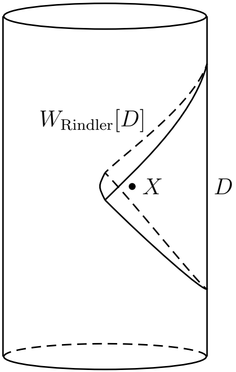
- Kernel may be taken to have support on different boundary regions D
- Subregion/subregion duality: a given boundary diamond D can reconstruct local operators in some subregion of the bulk



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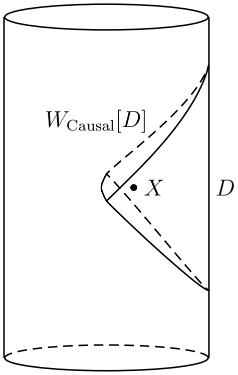

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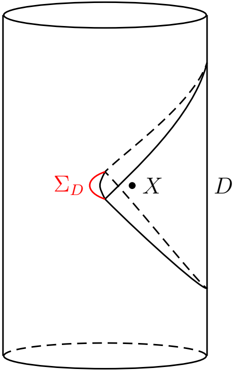

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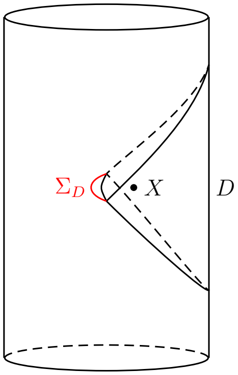

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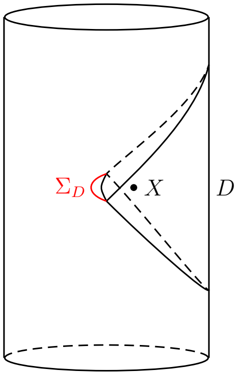
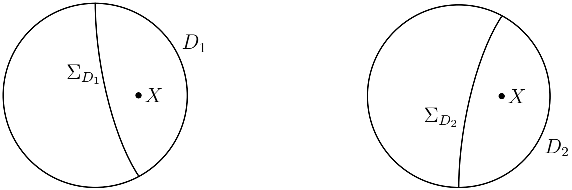
- Region that can be reconstructed is the entanglement wedge $W_E[D]$



Quantum Error Correction

- Proof of entanglement wedge reconstruction comes from combining [Jafferis, Lewkowykz, Maldacena, Suh] and quantum error correction [Almheiri, Dong, Harlow]
- There's redundancy in which boundary regions a local bulk operator can have support on:



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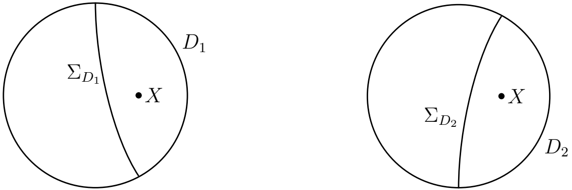

- The classical background identifies a subspace of states (the code subspace), and the different reconstructions are redundant only in this subspace
- Can then prove that any operator in $W_E[D]$ can be reconstructed (on code subspace) from D [Dong, Harlow, Wall; Faulkner, Lewkowycz]

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- Can try to reconstruct the full geometry, but this is hard. Partial progress:
    - Near boundary can just use Fefferman-Graham expansion
    - Hole-ography can do a little in 3D [Czech, Lamprou], though can't go too deep [Engelhardt, SF]
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- Instead, try recovering gravitationally interesting geometric features: area laws!

Why Area Laws?

- Properties of classical spacetimes, but connected to gravitational thermodynamics - presumably emerge from some coarse-graining mechanism
- Have some understanding of this for Bekenstein-Hawking entropy of BPS black holes [Strominger, Vafa]
- For dynamical black holes, less is known: interesting candidates are event horizon (globally defined) and holographic screens/apparent horizons (locally defined)

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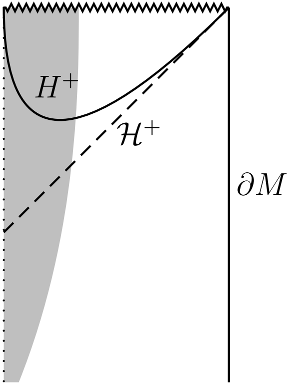

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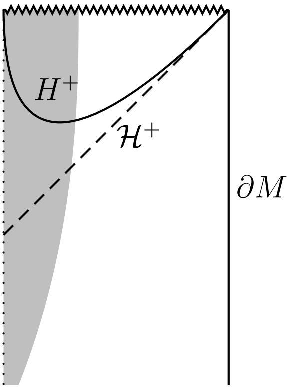

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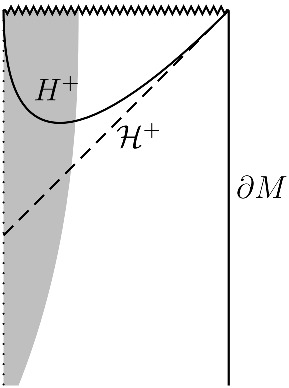

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- Try to come up with a more universal miscroscopic understanding

Coarse-Graining

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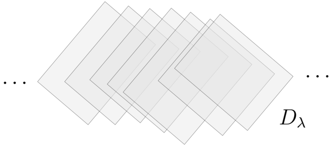

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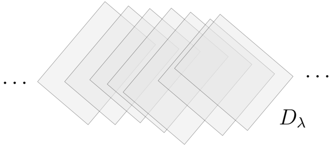

- Restricting a full state ρ to the set $\rho_F = \{\rho_{D_\lambda}\}$ of reduced states removes knowledge of correlations between points that aren't contained in any single diamond: $\rho \to \rho_F$ is coarse-graining

Bulk Picture

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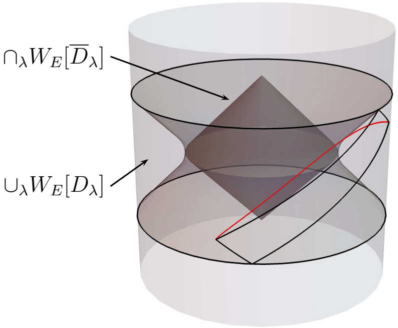
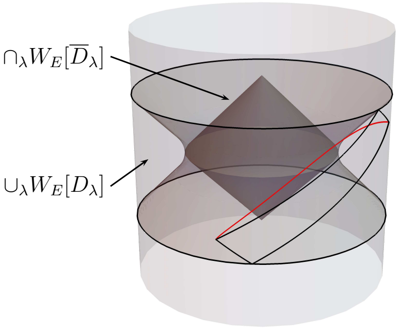


Bulk Picture

- If the QFT state has a geometric bulk dual, subregion/subregion duality tells us what this corresponds to in the bulk
- A "deep bulk" region is completely unrecoverable, but can recover local operators near the asymptotic region (related to [Nomura, Rath, Salzetta])
- Consistent with rough interpretation of e.g. BH entropy as arising from ignorance of interior of black hole



Differential Entropy and Hole-ography

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- From family of regions F can define differential entropy:

$$S_{\text{diff}}[F] = \lim_{n \to \infty} \sum_{i=1}^{n} \left(S[D_i] - S[D_i \cap D_{i+1}] \right)$$

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- $S_{\text{diff}}[F]$ computes the length of some curve(s) σ_F in the bulk constructed from the entanglement wedges of $\{D_\lambda\}$ [Balasubramanian, Chowdhury, Czech, de Boer, Heller; Headrick, Myers, Wien]:

$$S_{\text{diff}}[F] = \frac{\text{Length}[\sigma_F]}{4G_N\hbar}$$

- No general physical interpretation of $S_{\text{diff}}[F]$, but partial one is as the cost of a constrained state swapping protocol [Czech, Hayden, Lashkari, Swingle]

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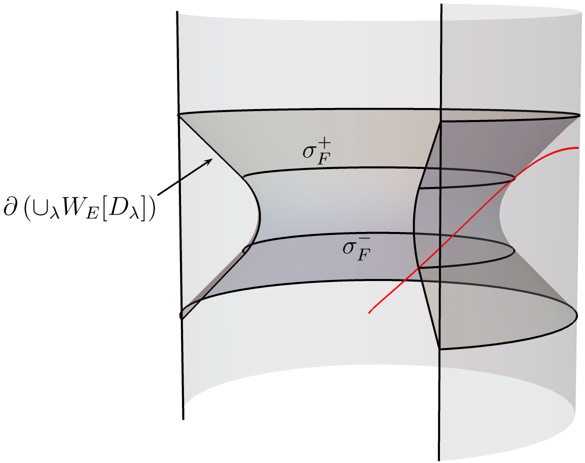
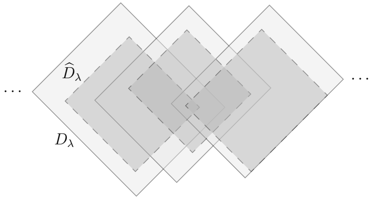


Monotonicity from SSA

- What happens as we further coarse-grain $F = \{D_\lambda\}$ to $\widehat{F} = \{\widehat{D}_\lambda\}$ with $\widehat{D}_\lambda \subset D_\lambda$? ("Weakening the QECC")



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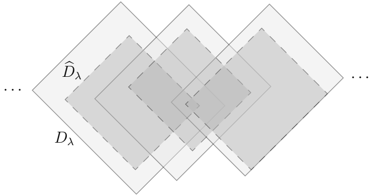

- Recall strong subadditivity of entanglement entropy:

$$S[AB] + S[BC] - S[ABC] - S[B] \geq 0$$

Implies irreversibility under removal of subsystems: in terms of mutual information, $I(A|B) \leq I(A|BC)$

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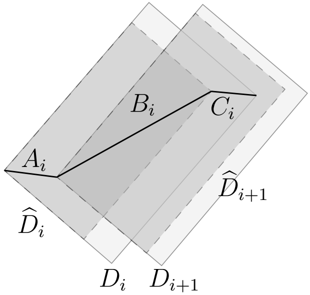

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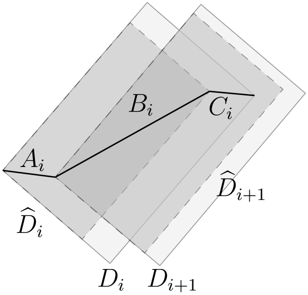

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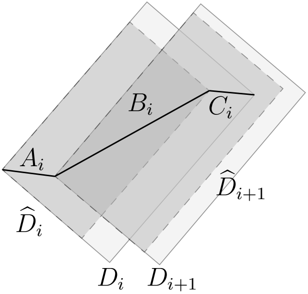

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- $S_{\text{diff}}[\widehat{F}] \geq S_{\text{diff}}[F] \Rightarrow$ area law

Area Laws

Take-home Message

In (2+1)-bulk dimensions, we obtain a family of area laws which are a precise manifestation of strong subadditivity! Coarse-graining comes from removing long-distance correlators on the boundary*.

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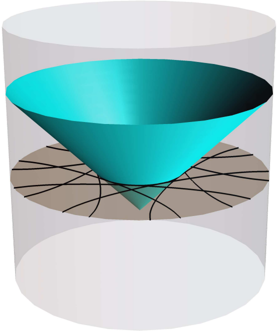
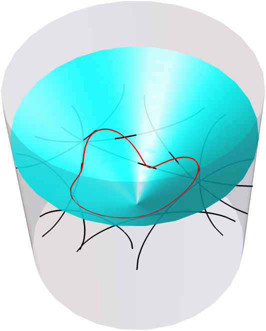
*Caveat: interpretation in terms of coarse-graining isn't quite correct due to vacuum rigidity; if ρ is vacuum, ρ_F is sufficient to tell you're in vacuum

Some Examples

Null; include Hawking area law for a simple causal horizon





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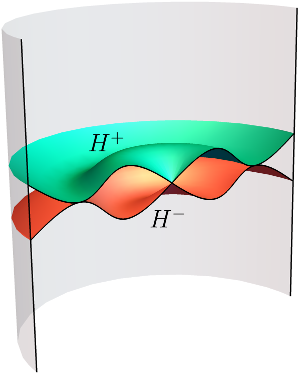
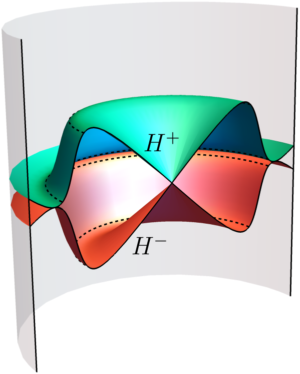
Spacelike



Some Examples

Mixed-signature; signature change similar to holographic screens



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- Future work!

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- But a quantum generalization of the precise connection using SSA is still lacking, and would presumably include something like differential entropy of bulk

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- Work in progress with N. Bao, C. Cao, C. Keeler: using techniques from [Alexakis, Balehowsky, Nachman], for a (3+1) bulk, seems that knowledge of areas of arbitrary perturbations of a foliation of boundary-anchored extremal surface is sufficient to guarantee uniqueness of metric (still dotting "i"s and crossing "t"s, though!)

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- In certain contexts, these laws match the Hawking area law for event horizons, and they show the same mixed-signature behavior of holographic screens - suggests a universal microscopic mechanism

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- How far into the bulk can they reach? Can they always reproduce the familiar area laws, or only sometimes?