A SUPERSPACE ODYSSEY

Adventures in Superspace
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Bernard de Wit

Nikhef Amsterdam

Utrecht University

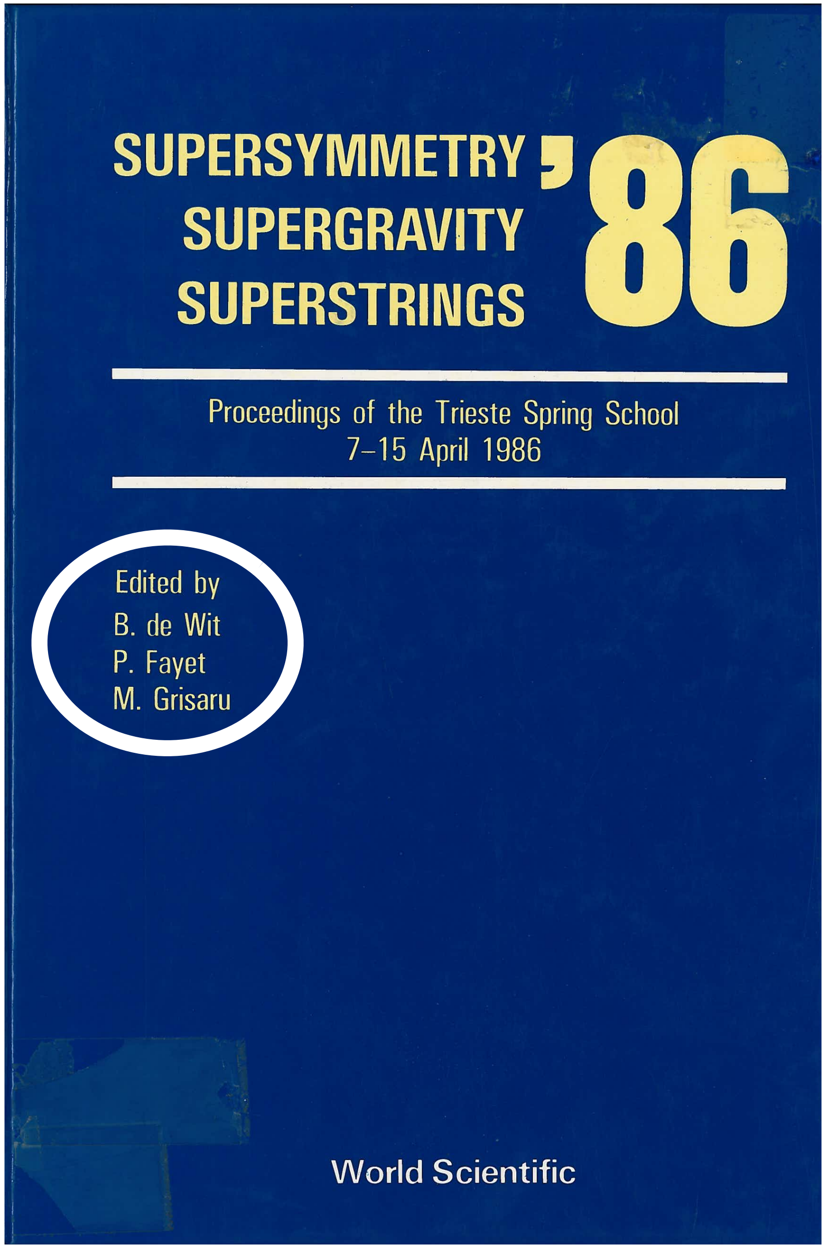
Marc and I met first at the Aspen Summer Workshop in 1977. In those days his Odyssey had already begun, although he was not yet fully in superspace.

During the subsequent years we met regularly, crossing many landscapes, although I must admit I was not always as daring as Marc.

Sometimes strange and unexpected things must have happened......

Marcia reminded
me that you were
laughing all the
time, while my
life was in
mortal danger

We also got involved in more down to earth activities as organizing conferences and editing proceedings!



And we wrote papers together, not bothered by the fact that Marc had become addicted to superspace while my own approach was usually more based on ‘components’.

One paper we wrote with Martin was:

Non-holomorphic corrections to the one-loop super Yang-Mills action

Phys. Lett. B374 (1996) 297

B. de Wit, M.T. Grisaru and M. Roček

The motivation: Marc and Martin had been calculating one-loop corrections to the Kähler potential of an N=2 non-abelian gauge theory in N=1 superspace. The result did not exhibit the holomorphic structure that one expects for N=2 supersymmetry.

Lesson drawn: the abelian holomorphic contributions and the non-abelian non-holomorphic contributions originate from two different N=2 invariants! The non-abelian contributions are originating from a higher-derivative invariant!

Hence we constructed the higher-order invariant as an N=1 superspace integral. In this exercise I obviously benefitted from the experience of my two collaborators who used superspace conventions where 1=2. Then we studied its consequences and resolved the puzzle.

Already a few years later I returned to the subject of higher-order derivative invariants in a different context, namely when attempting to calculate subleading corrections to BPS black hole entropy.

New techniques and ideas had become available, both on the microscopic and the field-theoretic side, enabling more and more detailed studies of black hole entropy. These precision studies require, however, more detailed knowledge of higher-derivative couplings in supergravity actions.

Here the comparison is between

Strominger, Vafa, 1996

microscopic/statistical entropy $\longleftrightarrow$

macroscopic/field-theoretic entropy

- microstate counting $\longrightarrow$ entropy $\quad S_{\text{micro}} = \ln d(q,p)$
- supergravity: Noether surface charge *Wald, 1993*

  first law of black hole mechanics *(BH thermodynamics)*

The limit of large charges coincides with the thermodynamic limit because the number of microstates increases. On the field theory side, this limit is described by general relativity, or, more generically, by field theories that are at most quadratic in space-time derivatives.

N=2 *supergravity:* *vector multiplet sector* (Wilsonian effective action)

vector multiplets contain scalars X^I

projectively defined: $X^I \longrightarrow Y^I$ (residual scale invariance)

Lagrangian encoded in a holomorphic homogeneous function: $\boxed{F(\lambda Y) = \lambda^2 F(Y)}$

Subleading corrections to Bekenstein-Hawking area law:

extend with one 'extra' complex field, corresponding to the Weyl supermultiplet $\boxed{\Upsilon}$

$$F(Y) \longrightarrow F(Y, \underset{\substack{\uparrow \\ \text{Weyl background}}}{\Upsilon})$$

homogeneity: $\boxed{F(\lambda Y, \lambda^2 \Upsilon) = \lambda^2 F(Y, \Upsilon)}$

Υ-dependence leads to terms $\propto (R_{\mu\nu\rho}{}^{\sigma})^2$ in effective action

BPS: supersymmetry at the horizon

near-horizon geometry: $\mathrm{AdS}_2 \times S^2$

$$\boxed{\begin{array}{ll} Y^I - \bar{Y}^I = \mathrm{i}p^I & \text{magnetic charges} \\ F_I - \bar{F}_I = \mathrm{i}q_I & \text{electric charges} \end{array}}$$

Ferrara, Kallosh, Strominger, 1996
Cardoso, dW, Käppeli, Mohaupt, 2000

covariant under dualities!

refers to the full function *F* !

Furthermore $\boxed{\Upsilon = -64}$ leads to subleading corrections

To ensure the validity of the first law of black hole mechanics, one must modify the definition of black hole entropy.

INSTEAD:
Use Wald's prescription based on a Noether surface charge.

Wald, 1993

general N=2 formula:

$$S_{\mathrm{macro}} = \pi\,|Z|^2 - 256\,\mathrm{Im}\,F_\Upsilon(Y,\Upsilon)\Big|_{\Upsilon=-64}$$

$\dfrac{\mathrm{AREA}}{4\,G_{\mathrm{N}}}$ modification

EXAMPLE:

$$F(Y,\Upsilon) = -\frac{1}{6}\,\frac{C_{ABC}\,Y^A Y^B Y^C}{Y^0} - \frac{c_{2A}\,Y^A}{24\cdot 64\,Y^0}\,\Upsilon$$

confirms to the microscopic result

Maldacena, Strominger, Witten, 1997
Vafa, 1997

$$S_{\mathrm{macro}} = 2\pi\sqrt{\frac{1}{6}|\hat{q}_0|\left(C_{ABC}\,p^A p^B p^C + c_{2A}\,p^A\right)}$$

full agreement!

Cardoso, dW, Mohaupt, 1998

This establishes the semiclassical corrections to the black hole entropy.

Similar agreements are known, for instance, for N=4 BPS black holes (in an N=2 supergravity description).

Why is the agreement so good? There must be other supersymmetric couplings that will in principle contribute, also in the BPS limit!

This is why the study of supersymmetric higher-order derivative couplings is important!

First a simple N=0 classification.

Butter, dW, Kuzenko, Lodato (*in preparation*)

N=0 CONFORMAL 'SUPERSPACE' CLASSIFICATION

Consider the following fields:

Kaku, van Nieuwenhuizen, Townsend, 1977

a scalar field: $\phi \to \mathrm{e}^{w\Lambda_{\mathrm{D}}}\phi$ *(w Weyl weight)*

vierbein field: $e_\mu{}^a$

conformal connection fields:

spin connection: $\omega(e,b)_\mu{}^{ab}$

dilatations: b_μ

conformal boosts: $f_\mu{}^a = \frac{1}{2}\mathcal{R}(e,b)_\mu{}^a - \frac{1}{12}e_\mu{}^a\,\mathcal{R}(e,b)$

$$f \equiv f_\mu{}^a e_a{}^\mu = \frac{1}{6}\,\mathcal{R}(e,b)$$

Distinguish four superconformal 'classes' of conformal invariants.

The first three crucially depend on scalar fields:

① $$\mathcal{L} \propto \sqrt{g}\,\phi'\,\Box_c\phi = \sqrt{g}\big[-\mathcal{D}^\mu\phi'\,\mathcal{D}_\mu\phi + f\,\phi'\phi\big] \qquad w = 1$$

② $$\mathcal{L} \propto \sqrt{g}\,\phi'\,\Box_c\Box_c\phi$$ **linear**

$$= \sqrt{g}\,\mathcal{D}^2\phi'\,\mathcal{D}^2\phi + 2\sqrt{g}\,\mathcal{D}^\mu\phi'\big[2\,f_{(\mu}{}^a e_{\nu)a} - f\,g_{\mu\nu}\big]\mathcal{D}^\mu\phi$$

$$w = 0$$

③ $$\Box_c\Box_c\ln\phi = \left(\mathcal{D}^2\right)^2\ln\phi + 2\,\mathcal{D}^\mu\Big(\big[2\,f_{(\mu}{}^a e_{\nu)a} - f\,g_{\mu\nu}\big]\mathcal{D}^\nu\ln\phi\Big)$$

$$+\,w\big[\mathcal{D}^2 f + 2\,f^2 - 2\,(f_\mu{}^a)^2\big]$$ **non-linear**

arbitrary w

$$\tfrac{1}{6}\mathcal{D}^2\mathcal{R} - \tfrac{1}{2}\mathcal{R}^{ab}\,\mathcal{R}_{ab} + \tfrac{1}{6}\mathcal{R}^2$$

④ The fourth one contains the Weyl tensor:

$$C_{\mu\nu}{}^{\rho\sigma} = \mathcal{R}_{\mu\nu}{}^{\rho\sigma} - 2\,\delta_{[\mu}{}^{[\rho}\,\mathcal{R}_{\nu]}{}^{\sigma]} + \tfrac{1}{3}\delta_{\mu\nu}{}^{\rho\sigma}\,\mathcal{R}$$

SUPERSYMMETRIZATION:

The Weyl tensor is already contained in a reduced chiral (tensor) superfield

All the scalar fields are extended to full (reduced or generic) chiral superfields

First discuss chiral and full superspace integrals.

Obviously ① and ④ originate from chiral superspace integrals, while ② and ③ are based one full superspace integrals.

Hence the N=4 supersymmetric version of the Gauss-Bonnet term is therefore a linear combination of a chiral and a full superspace integral:

$$\mathcal{L}_{\mathrm{GB}} = \sqrt{g}\{C^{abcd}C_{abcd} + \frac{4}{w}\Box_{\mathrm{c}}\Box_{\mathrm{c}}\ln\phi\}$$

The kinetic multiplet

N=1 conformal superspace:

Ferrara, van Nieuwenhuizen, 1978

$$\int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\; \Phi\, \bar{\Phi}' \approx \int \mathrm{d}^2\theta\; \Phi\, \mathbb{T}(\bar{\Phi}')$$

$$\mathbb{T}(\bar{\Phi}') = \bar{D}^2 \bar{\Phi}' \qquad w = 2$$

$w = 1$

dW, van Holten, Van Proeyen, 1983

dW, Katmadas, van Zalk, 2011

N=2 conformal superspace:

$$\int \mathrm{d}^4\theta\, \mathrm{d}^4\bar{\theta}\; \Phi\, \bar{\Phi}' \approx \int \mathrm{d}^4\theta\, \Phi\, \mathbb{T}(\bar{\Phi}')$$

$$\mathbb{T}(\bar{\Phi}) \propto \bar{D}^4 \bar{\Phi} \qquad w = 2$$

$w = 0$

Weyl weights for N=0 coincide with those for N=2 !

For instance, the highest component of $\mathbb{T}(\bar\Phi)$:

$$
\begin{aligned}
C|_{\mathbb{T}(\bar\Phi)} = {} & 4(\Box_{\rm c} + 3\,D)\Box_{\rm c}\bar A - \tfrac{1}{2}D_a\big(T^{ab}{}_{ij}\,T_{cb}{}^{ij}\big)\,D^c\bar A + \tfrac{1}{16}(T_{abij}\varepsilon^{ij})^2\bar C \\
& + D_a\big(\varepsilon^{ij}D^aT_{bcij}\,F^{+bc} + 4\,\varepsilon^{ij}T^{ab}{}_{ij}\,D^cF^+_{cb} - T_{bc}{}^{ij}\,T^{ac}{}_{ij}\,D^b\bar A\big) \\
& + \big(6\,D_bD - 8\mathrm{i}D^aR(A)_{ab}\big)\,D^b\bar A + \cdots
\end{aligned}
$$

The chiral fields Φ and $\bar\Phi'$ can be represented by composite chiral fields. These are homogeneous functions of reduced chiral multiplets, i.e. vector multiplets and the Weyl multiplet.

In this way we, for instance, determined the simplest F^4-invariant with all the supergravity corrections included.

Bosonic terms of the F^4-Lagrangian with supergravity corrections:

$$
\begin{aligned}
e^{-1}\mathcal{L} = {}& \\
&\mathcal{H}_{IJ\bar K\bar L}\Big[\tfrac14\big(F_{ab}^{-\,I}\,F^{-ab\,J} - \tfrac12 Y_{ij}{}^{I}\,Y^{ij\,J}\big)\big(F_{ab}^{+\,K}\,F^{+ab\,L} - \tfrac12 Y^{K\,ij}\,Y^{L}{}_{ij}\big) \\
&\qquad +4\,\mathcal{D}_aX^I\,\mathcal{D}_b\bar X^K\big(\mathcal{D}^aX^J\,\mathcal{D}^b\bar X^L + 2\,F^{-\,ac\,J}\,F^{+\,b}{}_{c}{}^{L} - \tfrac14\eta^{ab}\,Y_{ij}^{J}\,Y^{L\,ij}\big)\Big] \\
&+\Big\{\mathcal{H}_{IJ\bar K}\Big[4\,\mathcal{D}_aX^I\,\mathcal{D}^aX^J\,\mathcal{D}^2\bar X^K - \big(F^{-ab\,I}\,F_{ab}^{-\,J} - \tfrac12 Y_{ij}^{I}\,Y^{Jij}\big)\big(\Box_{\rm c}X^K + \tfrac18 F_{ab}^{-\,K}\,T^{abij}\varepsilon_{ij}\big) \\
&\qquad +8\,\mathcal{D}^aX^I F_{ab}^{-\,J}\big(D_cF^{+\,cb\,K} - \tfrac12\mathcal{D}_c\bar X^K T^{ij\,cb}\varepsilon_{ij}\big) - \mathcal{D}_aX^I\,Y_{ij}^{J}\,\mathcal{D}^aY^{K\,ij}\Big] + \text{h.c.}\Big\} \\
&+\mathcal{H}_{I\bar K}\Big[4\big(\Box_{\rm c}\bar X^I + \tfrac18 F_{ab}^{+\,I}\,T^{ab}{}_{ij}\varepsilon^{ij}\big)\big(\Box_{\rm c}X^K + \tfrac18 F_{ab}^{-\,K}\,T^{abij}\varepsilon_{ij}\big) + 4\,\mathcal{D}^2X^I\,\mathcal{D}^2\bar X^K \\
&\quad +8\,\mathcal{D}_aF^{-\,abI}\,\mathcal{D}_cF^{+c}{}_{b}{}^{K} - \mathcal{D}_aY_{ij}{}^{I}\,\mathcal{D}^aY^{K\,ij} + \tfrac14 T_{ab}{}^{ij}\,T_{cdij}\,F^{-ab\,I}\,F^{+cd\,K} \\
&\quad +\big(\tfrac16 R(\omega,e) + 2\,D\big)Y_{ij}{}^{I}\,Y^{ij\,K} + 4\,F^{-ac\,I}\,F^{+}{}_{bc}{}^{K}\,R(\omega,e)_a{}^{b} \\
&\quad -\big[\varepsilon^{ik}\,Y_{ij}{}^{I}\,F^{+ab\,K}\,R(\mathcal{V})_{ab}{}^{j}{}_{k} + \text{h.c.}\big] \\
&\quad -\big[\mathcal{D}_c\bar X^K\big(D^cT_{ab}{}^{ij}\,F^{-\,I\,ab} + 4\,T^{ij\,cb}\,D^aF_{ab}^{-\,I}\big)\varepsilon_{ij} + \text{h.c.}\big] \\
&\quad +8\big(R^{\mu\nu}(\omega,e) - \tfrac13 g^{\mu\nu}R(\omega,e) + \tfrac14 T^{\mu}{}_{b}{}^{ij}\,T^{\nu b}{}_{ij} + \mathrm{i}R(A)^{\mu\nu} - g^{\mu\nu}D\big)\mathcal{D}_\mu X^I\,\mathcal{D}_\nu\bar X^K\Big]
\end{aligned}
$$

dW, Katmadas, van Zalk, 2011

Extension:

The kinetic multiplet of the non-linear chiral multiplet,

$$\mathbb{T}(\ln\bar{\Phi}) \propto \bar{D}^4 \ln\bar{\Phi}$$

is itself a linear superconformal chiral field!
Hence the supersymmetric version of the Gaus-Bonnet invariant and related Lagrangians can be determined by the same manipulations as before.

Butter, dW, Kuzenko, Lodato (*in preparation*)

This analysis has eventually important consequences for N=2 BPS black hole entropy. For instance, the orginal determination of the sub-leading corrections was based on a Lagrangian of type 4 . There have been claims that instead one should base the calculation on an invariant proportional to the Gauss-Bonnet term. Actually, both type of couplings play a role in the evaluation of the logarithmic corrections to BPS black hole entropy.

Sen, 2006, 2011

In principle the functions in these Lagrangins are not known and require physics input. This is different when performing dimensional reduction on an invariant that does not depend on an arbitrary function.

For instace, take the 5D higher-derivative action:

$$\begin{aligned}
8\pi^2\,\mathcal{L}_{\rm vww} \;=\;& \tfrac{1}{4}\,E\,c_A Y_{ij}{}^{A}\,T^{CD}\,R_{CDk}{}^{j}(V)\,\varepsilon^{ki} \\
&+E\,c_A\sigma^A\left[\tfrac{1}{64}R_{CD}{}^{EF}(M)\,R_{EF}{}^{CD}(M)+\tfrac{1}{96}R_{MNj}{}^{i}(V)\,R^{MN}{}_{i}{}^{j}(V)\right] \\
&-\tfrac{1}{128}\mathrm{i}\varepsilon^{MNPQR}\,c_A W_M{}^{A}\,\left[R_{NP}{}^{CD}(M)\,R_{QRCD}(M)+\tfrac{1}{3}\,R_{NPj}{}^{i}(V)\,R_{QRi}{}^{j}(V)\right] \\
&+\tfrac{3}{16}E\,c_A\big(10\,\sigma^A T_{BC}-F_{BC}{}^{A}\big)R(M)_{DE}{}^{BC}\,T^{DE}+\cdots
\end{aligned}$$

Hanaki, Ohashi, Tachikawa, 2007

yields the following 4D actions under dimensional reduction:

Banerjee, dW, Katmadas, 2011

One action corresponds to an addition to the chiral superspace density invariant

$$F(X,(T_{ab}{}^{ij}\varepsilon_{ij})^2) = -\frac{1}{2}\frac{C_{ABC}X^AX^BX^C}{X^0} - \frac{1}{2048}\frac{c_A X^A}{X^0}(T_{ab}{}^{ij}\varepsilon_{ij})^2$$

Bergshoeff, de Roo, dW, 1981

Another one is the action that we considered in 1996, but now coupled to supergravity. Note the logarithmic terms which we also considered at the time based on the renormalizability of the gauge theory.

$$\mathcal{H}(X,\bar{X}) = \tfrac{1}{384}\mathrm{i}c_A\left(\frac{X^A}{X^0}\ln\bar{X}^0 - \frac{\bar{X}^A}{\bar{X}^0}\ln X^0\right)$$

corresponding to a full superspace density invariant, described in terms of a Kähler potential. This class of actions does not contribute to the electric charges and the entropy of BPS black holes.

dW, Katmadas, van Zalk, 2010

Here the logarithmic factor is associated with the presence of the non-linear invariant.

Some characteristic terms:

$$
\begin{aligned}
8\pi^2\,\mathcal{L}_{\rm vww} \;\to\;& -\tfrac{1}{384}\mathrm{i}c_A t^A\left[\tfrac{2}{3}\mathcal{R}_{ab}\mathcal{R}^{ab} + R(\mathcal{V})^{+i}_{ab\,j}\,R(\mathcal{V})^{+abj}{}_i\right] \\
& -\tfrac{1}{768}\mathrm{i}c_A(t^A - \bar{t}^A)\,(X^0)^{-1}\varepsilon_{ij}T^{cdij}\,R(M)^{ab}{}_{cd}\,F^{-0}_{ab} \\
& +\mathrm{h.c.}
\end{aligned}
$$

where $\mathcal{R}_{\mu\nu}$ is the Ricci tensor

Its structure was not fully known at the time. Neither is it known whether this coupling is subject to some non-renormalization theorem. The *N*=2 supersymmetric Gauss-Bonnet invariant is related to this class.

Butter, dW, Kuzenko, Lodato (*in preparation*)

It is great seeing you here in Montreal! And best wishes for the future!