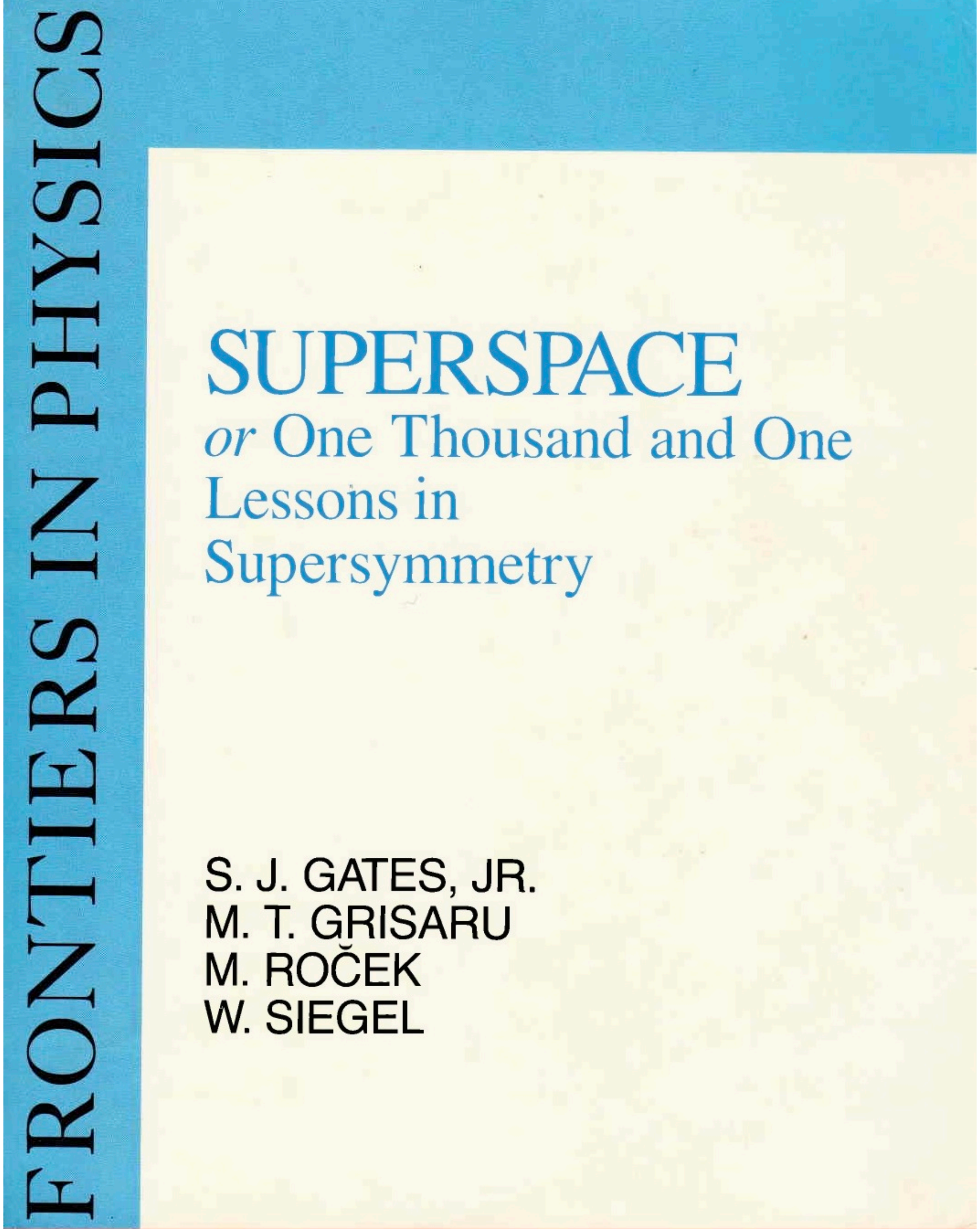
UV properties of N=8 and N=4 supergravities

Planck satellite and superconformal
supergravity from the sky

Renata Kallosh, Stanford

The workshop in honor of Professor Marc Grisaru

Workshop: Adventures in Superspace
April 19 - 20 2013
McGill University, Montreal, Canada



Great Book!

Marc's papers on my desk
during the last few years
of the new wave of interest to
perturbative extended
supergravity

Scale Dependence and the Renormalization Problem of Quantum Gravity
Stanley Deser, Marcus T. Grisaru, P. van Nieuwenhuizen, C.C. Wu, 1975

Although Einstein theory is obtainable from a Weyl (scale) invariant model by a particular gauge choice, this imposes no conditions on its counterterms. The absence of certain non scale-invariant counterterms in Einstein–Yang–Mills (or Maxwell) theory is traced instead to invariance under vector field duality rotations.

In **2013** we are trying to understand the related issues, what is the reason for the 3-loop UV finiteness?

N=8 $E_{7(7)}$ duality

N=4 SU(1,1)x SU(4) duality and/or superconformal symmetry?

And/or the absence of the genuine higher derivative superinvarinats in N=4,…,8?

Reggeization and the Question of Higher Loop Renormalizability of Gravitation
Marcus T. Grisaru, P. van Nieuwenhuizen, C.C. Wu
1975

Improved Methods for Supergraphs
Marcus T. Grisaru, W. Siegel , M.Rocek ,
1979

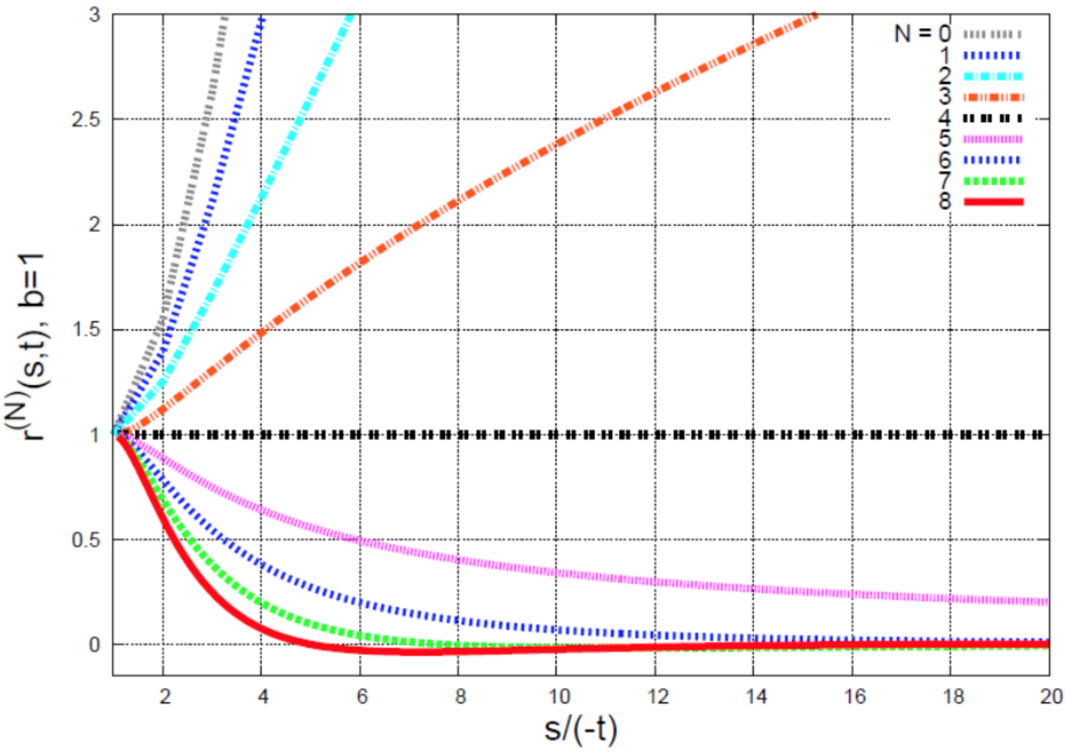
Some Properties of Scattering Amplitudes in Supersymmetric Theories
Marcus T. Grisaru, H.N. Pendleton 1977

The One Loop Four Particle S Matrix In Extended Supergravity
Marcus T. Grisaru, W. Siegel, 1982

Bound State Regge Trajectories In N=8 Supergravity
Marcus T. Grisaru, H.J. Schnitzer ,1981

Leading Regge Double Logs

- $[t/s \ln^2(-t/s)]^L$ recently resummed to all orders for any number of supersymmetries Bartels, Lipatov, Sabio Vera, 1208.3423
- Note that these terms are heavily power-suppressed, by $(t/s)^L$, with respect to the leading eikonal behavior.
- N=8 Regge terms most heavily damped in HE limit
- N=4 Regge terms are totally boring…



Supergraphity. Part 1. Background Field Formalism
Marcus T. Grisaru, W. Siegel, 1981

Supergraphity. 2. Manifestly Covariant Rules and Higher Loop Finiteness
Marcus T. Grisaru, W. Siegel , 1982

Compensating Fields And Anomalies
B. de Wit, Marcus T. Grisaru , 1985

→ Just used in ↓

Conjecture on Hidden Superconformal Symmetry of N=4 Supergravity
Sergio Ferrara ,Renata Kallosh, Antoine Van Proeyen 2012

Ask hep-th: a Grisaru and t anomaly get 15 papers

The topic is even more confusing (but even more relevant and important) today: what is the role of the U(1) 1-loop anomaly in N=4 supergravity? What is the reason for a 3-loop UV finiteness?

On the U(1) duality anomaly and the S-matrix of N=4 supergravity
J.J.M. Carrasco, R. Kallosh, R. Roiban, A.A. Tseytlin, 2013

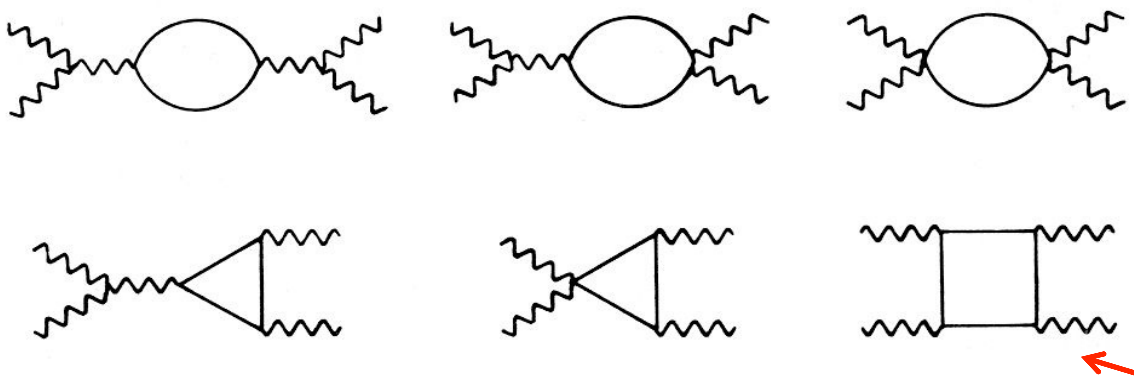
Calculating 1-loop N=4 supergravity scattering amplitudes using color/kinematics duality and the double-copy construction we find that a particular U(1) symmetry which was present in the tree-level amplitudes is broken at the 1-loop level.

Whether such anomalous amplitudes affect the UV behavior of the theory (which, at four loops, will be unambiguously determined by an explicit calculation currently in progress) remains an open question.

THE ONE-LOOP, FOUR-PARTICLE S-MATRIX IN EXTENDED SUPERGRAVITY

GRISARU, SIEGEL (1981)

Using N=1 superfield background, the calculations are hardest for N = 1 supergravity and become progressively easier for N>1 *culminating in an absolutely trivial determination of the one-loop, four-particle S-matrix in N = 8 supergravity*



In N=8 only one simple scalar box graph, all other vanish!

At the other extreme, the N = 0 theory (ordinary Einstein gravity) would seem to require a major computer calculation.

Today, 32 years later

Lance Dixon talk about his work with Z. Bern, J.J. Carrasco, H. Johansson & R. Roiban at Symmetries and Quantum Gravity Hermann Nicolai-Fest – MPI Potsdam September 7, 2012 (no significant news yet)

N=8 Supergravity: Scattering and Ultraviolet Behavior through Four Loops

What if it's finite?

• Of course N = 8 is the simplest ☺

• Then we should determine the finite values of N=8 scattering amplitudes near D=4.
• These all have IR divergences, but fortunately they exponentiate, much more simply than in Yang-Mills theory

$$\ln \frac{\mathcal{M}_4}{M_4^{\text{tree}}} = \left(\frac{\kappa}{8\pi}\right)^2 \frac{M_4^{1-\text{loop}}}{M_4^{\text{tree}}} + \mathcal{F}_4$$

divergent finite

Two-loop finite remainder

$$\mathcal{F}_4 = \left(\frac{\kappa}{8\pi}\right)^4 F_4^{(2)} + \cdots$$

$$F_4^{(2),\mathcal{N}=8}\Big|_{s-\text{channel}} = 8\left\{t\,u\left[f_1\left(\frac{-t}{s}\right) + f_1\left(\frac{-u}{s}\right)\right] + s\,u\left[f_2\left(\frac{-t}{s}\right) + f_3\left(\frac{-t}{s}\right)\right]\right.$$
$$\left. + s\,t\left[f_2\left(\frac{-u}{s}\right) + f_3\left(\frac{-u}{s}\right)\right]\right\},$$

• f_1 is a very simple, maximal (weight 4) transcendental function of $x = -t/s$:

$$f_1(x) = \zeta_4 + i\pi\zeta_3 - \int_x^1 \frac{dt}{t(1-t)}\left[\frac{1}{6}\ln^3 t + \frac{i\pi}{2}\ln^2 t\right]$$

• f_2 and f_3 are related to f_1 by crossing symmetry

Checks and gravity amplitude

- Unitarity cuts of new N=4 SYM integrand agree with those of an old form computed without BCJ [1008.3327].
- To get N=8 SUGRA, we use double copy formula:

$$\mathcal{A}_4^{(L)} = \sum_{i\in\Gamma} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i C_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

$$\mathcal{M}_4^{(L)} = \sum_{i\in\Gamma} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i^2}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

- Cuts of new N=8 supergravity amplitude also agree with a previous (KLT driven) construction [0905.2326]

Based on most recent talks by Dixon, Bern, Dennen and discussions with them

3 loop summary:
N=8 no worse than N=4 SYM in UV

Manifest quadratic representation at 3 loops – same as N=4 SYM – implies same critical dimension (as for $L = 2$):

$$I_3^{\text{quad.}} \sim \int \frac{(d^6 l_i)^3 \; l_i^2}{[(l_i)^2]^{10}} \sim \ln \Lambda$$

$$D_c = 4 + \frac{6}{L} = 6$$

$$M_4^{(3),D=6-2\epsilon}\Big|_{\text{pole}} = \frac{1}{\epsilon} \frac{5\zeta_3}{(4\pi)^9} \left(\frac{\kappa}{2}\right)^8 (s_{12}s_{13}s_{14})^2 M_4^{\text{tree}}$$

$\mathcal{D}^6 R^4$ counterterm

Also recovered via string theory argument (up to factor of 9?) Green, Russo, Vanhove, 1002.3805; talk by Green?

4 loop N=4 SYM UV pole

$$\mathcal{A}_4^{(4)}(1,2,3,4)\Big|_{\text{pole}}^{SU(N_c)} = -6\, g^{10}\, \mathcal{K}\, N_c^2 \Big(N_c^2\, V_1 + 12\, (V_1 + 2\, V_2 + V_8) \Big) \times \Big(s\, (\text{Tr}_{1324} + \text{Tr}_{1423}) + t\, (\text{Tr}_{1243} + \text{Tr}_{1342}) + u\, (\text{Tr}_{1234} + \text{Tr}_{1432}) \Big)$$

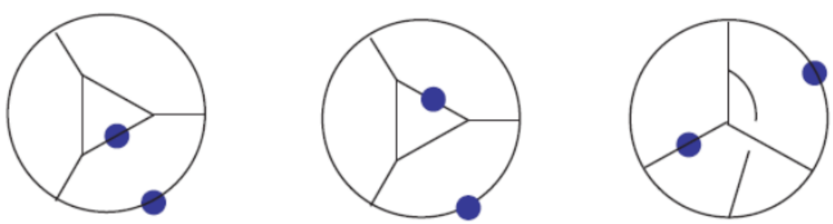
4 loop UV pole in $D = 11/2$

D=4+6/L
rule so far works for N=4 SYM and N=8 supergravity

- Reduce integrals to basis $\{V_1, V_2, V_8\}$
- Final answer is remarkably simple:

$$\mathcal{M}_4^{(4)}\Big|_{\text{pole}} = -\frac{23}{8}\left(\frac{\kappa}{2}\right)^{10} stu\,(s^2+t^2+u^2)^2\, M_4^{\text{tree}}\, (V_1 + 2V_2 + V_8)$$

- Again, same linear combination as in N_c^2 part of N=4 SYM pole!



V_1

Deep and still unexplained relation to N=4 SYM

- **Gravity UV divergence is directly proportional to subleading color single-trace divergence of $N = 4$ super-Yang-Mills theory.**
- **Same happens at 1-3 loops.**

What about $L = 5$?

N=4 SYM: Bern, Carrasco, Johansson, Roiban, 1207.6666

- Motivation: Various arguments point to 7 loops as the possible first divergence for N=8 SUGRA in D=4, associated with a D^8R^4 counterterm:

Howe, Lindstrom, NPB181, 487 (1981); Bossard, Howe, Stelle, 0908.3883; Kallosh, 0903.4630; Green, Russo, Vanhove, 1002.3805; Bjornsson, Green, 1004.2692; Bossard, Howe, Stelle, 1009.0743; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger,1009.1643

- Same D^8R^4 counterterm shows up at $L = 4$ in $D = 5.5$
- Does 5 loops → $D^{10}R^4$ (same UV as N=4 SYM)?
  or → D^8R^4 (worse UV as N=4 SYM)?
- 5 loops would be a very strong indicator for 7 loops
- Now 100s of nonvanishing cubic 4-point graphs!

The problem is to bring the SYM answer to the form were color-kinematic duality can be used.

Lance Dixon's outlook on N=8, as in his talk at Hermann's meeting, valid now

Outlook

- Through 4 loops, the 4-graviton scattering amplitude of N=8 supergravity has UV behavior no worse than the corresponding 4-gluon amplitude of N=4 SYM.
- Finite remainder also remarkably simple (at 2 loops).
- Precise pole for N=8 supergravity bears a remarkable relation with subleading-color single trace pole in N=4 SYM in the same critical dimension, at 2, 3 and 4 loops.
- Is this an accident, or could it foreshadow equal critical dimensions $D_c = 26/5$ also at 5 loops? Which in turn would suggest that 7 loops is not where N=8 supergravity first diverges… If not there, where? L = 8? L = ∞ ?

UV Surprises in Half-Maximal Supergravity

Using Bern-Carrasco-Johansson color-kinematic duality: double-copy method

Gravity From Gauge Theory

$$-i\left(\frac{2}{\kappa}\right)^{(n-2)} \mathcal{M}_n^{\text{tree}}(1,2,\ldots,n) = \sum_i \frac{n_i\,\tilde{n}_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

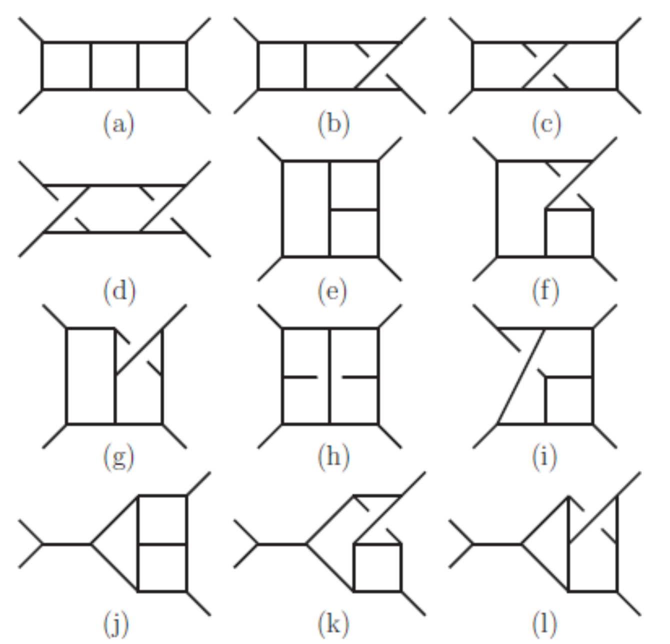
	n	$\tilde{n}$
$N = 8$ sugra:	$(N = 4$ sYM$)\times$	$(N = 4$ sYM$)$
$N = 4$ sugra:	$(N = 4$ sYM$)\times$	$(N = 0$ sYM$)$
$N = 0$ sugra:	$(N = 0$ sYM$)\times$	$(N = 0$ sYM$)$

$N = 0$ sugra: graviton + antisym tensor + dilaton

N=4 Three Loop Result

- ✓ Series expand the integrand and select the logarithmic terms
- ✓ Reduce all the tensors in the integrand
- ✓ Regulate infrared divergences
- ✓ Subtract subdivergences
- ✓ Evaluate vacuum integrals

Graph	$(\text{divergence})/(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3}+\frac{205}{27648}\frac{1}{\epsilon^2}+\left(-\frac{5551}{768}\zeta_3+\frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3}-\frac{1}{4}\frac{1}{\epsilon^2}+\left(\frac{593}{288}\zeta_3-\frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3}+\frac{2057}{6912}\frac{1}{\epsilon^2}+\left(\frac{10769}{2304}\zeta_3-\frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3}-\frac{41}{1536}\frac{1}{\epsilon^2}+\left(\frac{3227}{2304}\zeta_3-\frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3}-\frac{29}{1024}\frac{1}{\epsilon^2}+\left(-\frac{2087}{2304}\zeta_3-\frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3}+\frac{9}{64}\frac{1}{\epsilon^2}+\left(\frac{101}{12}\zeta_3-\frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3}+\frac{89}{1152}\frac{1}{\epsilon^2}+\left(-\frac{377}{144}\zeta_3+\frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3}-\frac{251}{1152}\frac{1}{\epsilon^2}+\left(-\frac{835}{144}\zeta_3+\frac{7385}{3456}\right)\frac{1}{\epsilon}$



✓ The sum of all 12 graphs is finite!

Opinions on the Result

- If R^4 counterterm is allowed by supersymmetry, why is it not present?
  - R^4 nonrenormalization from heterotic string

    Tourkine, Vanhove (2012)
  - Violates Noether-Gaillard-Zumino current conservation

    Kallosh (2012)
  - Hidden superconformal symmetry

    Kallosh, Ferrara, Van Proeyen (2012)
  - Existence of off-shell N=4 superspace formalism

    Bossard, Howe, Stelle (2012)
- Different proposals lead to different expectations for four loops

N=4 Supergravity Four Loop Status

- ✓ Series expand the integrand and select the logarithmic terms
- ✓ Reduce all the tensors in the integrand
- ✓ Regulate infrared divergences
- ➢ Subtract subdivergences
- ✓ Evaluate vacuum integrals

- Calculation nearly done
  - Overall cancellation of ϵ^{-4} and ϵ^{-3}
  - Now working on showing the necessary cancellation of ϵ^{-2}
- Stay tuned! ϵ^{-1} ?

COUNTERTERMS IN EXTENDED SUPERGRAVITIES

1981

R.E. KALLOSH

Geometrical invariants respecting all necessary symmetries of the theory are shown to exist, starting from the 8th (4th) loop approximation in $N = 8$ ($N = 4$) on-shell supergravity. 3-loop counterterms are presented on a linearized level for $N = 4$ and $N = 8$ theories. The corresponding 3-loop non-linear invariants are discussed.

$$N \geq 4 \qquad L = N$$

1981

HIGHER ORDER INVARIANTS IN EXTENDED SUPERGRAVITY

P. HOWE   U. LINDSTRÖM

On-shell linearized extended supergravity is presented in superspace for all N. The formalism is then used in the construction of higher order invariants which may serve as counterterm lagrangians. It is shown that three-loop counterterms exist for $N \leqslant 3$ and $(N-1)$-loop counterterms for $N \geqslant 4$. In the full non-linear theory, the presence of a global non-compact symmetry group for $N \geqslant 4$ does not allow a simple extension of the $(N-1)$-loop term, but N-loop counterterms may be constructed.

N=4, L=4 and N=8, L=8 were expected as full superspace invariants in 1981

However, now we came to conclusion that all of our superspace invariants from 1981 have a problem, they are supersymmetric only due to classical field eqs. This we knew before. However, we have shown now using Bernard's et al recent N=2 genuine off shell R^4 construction that the superspace needs a deformation, which has not been done so far for N=4 and N=8.

Therefore we may expect to learn more either from explicit N=4, L=4, (N=8,L=8) computations, or by studies of extended supergravity superspace deformations.

Whichever will come first…

We have recently studied the (unknown previously) deformation of the **globally supersymmetric** theories

Dirac-Born-Infeld-Volkov-Akulov and Deformation of Supersymmetry
Eric Bergshoeff, Frederik Coomans, Renata Kallosh, C.S. Shahbazi, Antoine Van Proeyen
2013

Deformation of N=4 and N=8 supergravities in presence of higher derivatives ?

Superconformal Symmetry, NMSSM, and Inflation, 2010
Jordan Frame Supergravity and Inflation in NMSSM, 2010
Ferrara, Kallosh, Linde, Marrani, Van Proeyen

Superconformal symmetry, supergravity and cosmology
Kallosh, Kofman, Linde, Van Proeyen, 2000

Work in Progress, 2013

Textbook version:

Einstein frame

Kaku, Townsend and van Nieuwenhuizen 1977
Cremmer, Julia, Scherk, Ferrara, Girardello and van Nieuwenhuizen 1979
Barbieri, Ferrara, Nanopoulos and Stelle 1982
Cremmer, Ferrara, Girardello and Van Proeyen 1983
Girardi, Grimm, Muller and Wess 1984
Weinberg, QFT Volume III, 2000

Einstein frame, by gauge fixing of the conformal compensator

RK, Kofman, Linde and Van Proeyen 2000

$$-\frac{1}{6}\mathcal{N}(X,\bar{X})R \quad \Longrightarrow \quad \frac{1}{2}M_p^2 R$$

New version:

Jordan frame

Ferrara, Kallosh, A.L., Marrani and Van Proeyen, 1004.0712

$$-\frac{1}{6}\mathcal{N}(X,\bar{X})\,R \quad \Longrightarrow \quad -\frac{1}{6}\Phi(z,\bar{z})\,R$$

Jordan frame

CSS

Ferrara, Kallosh, A.L., Marrani and Van Proeyen, 1008.2942

CSS (canonical superconformal supergravity) is a special class of SUGRA models with canonical kinetic terms and a potential as in a global SUSY

Canonical Superconformal Supergravity

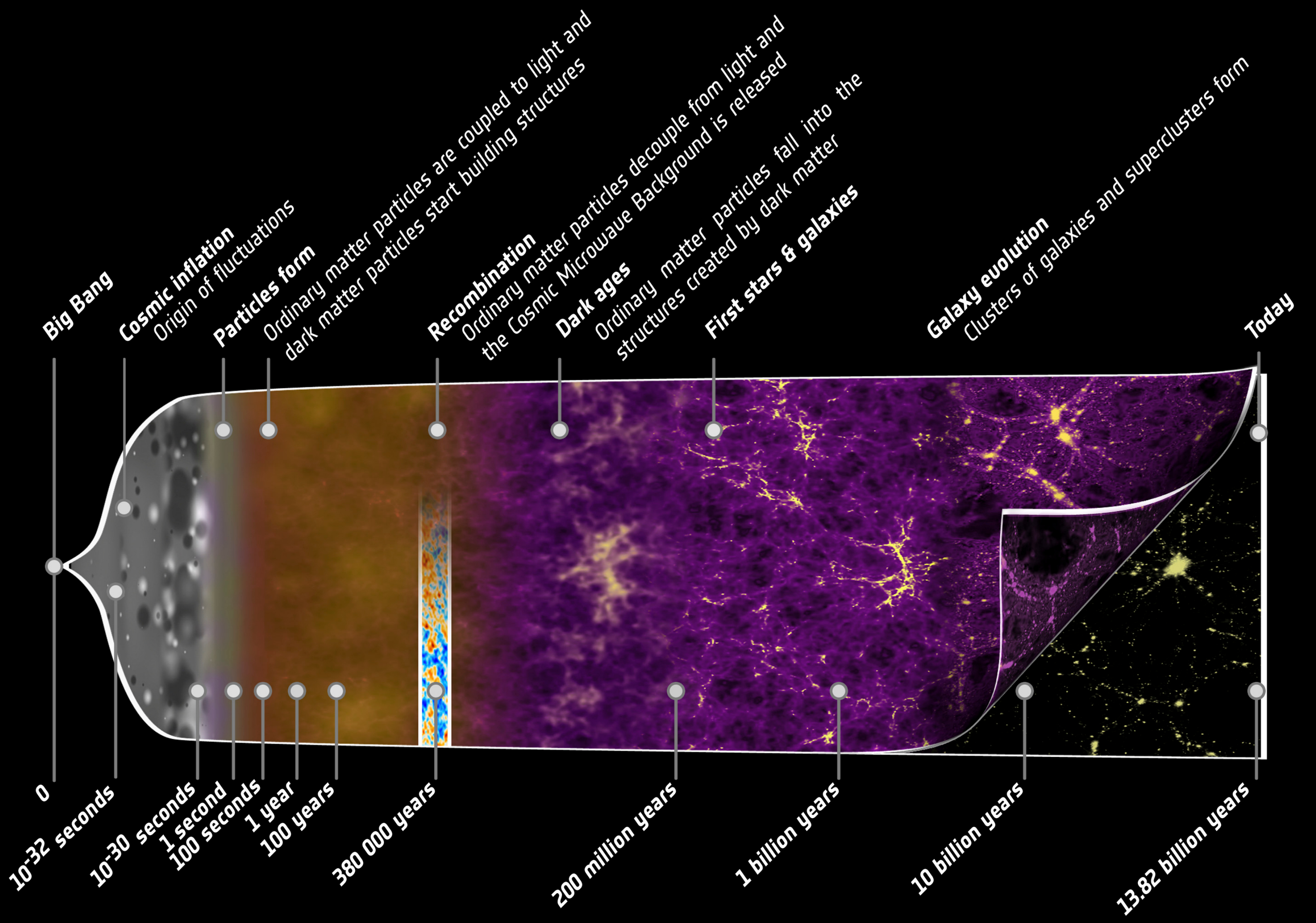
Top – down approach: We found a class of supergravity models in the Jordan frame, in which, under certain conditions, superconformal invariance of the matter part of the action remains unbroken. Such theories are very simple: kinetic terms are canonical, and scalar potential is the same as in global SUSY: $V = |\partial W|^2$

No such terms as e^K, $-3|W|^2$, $K^{i\bar{k}}$, $K_{,i}W$ **they all cancel!**

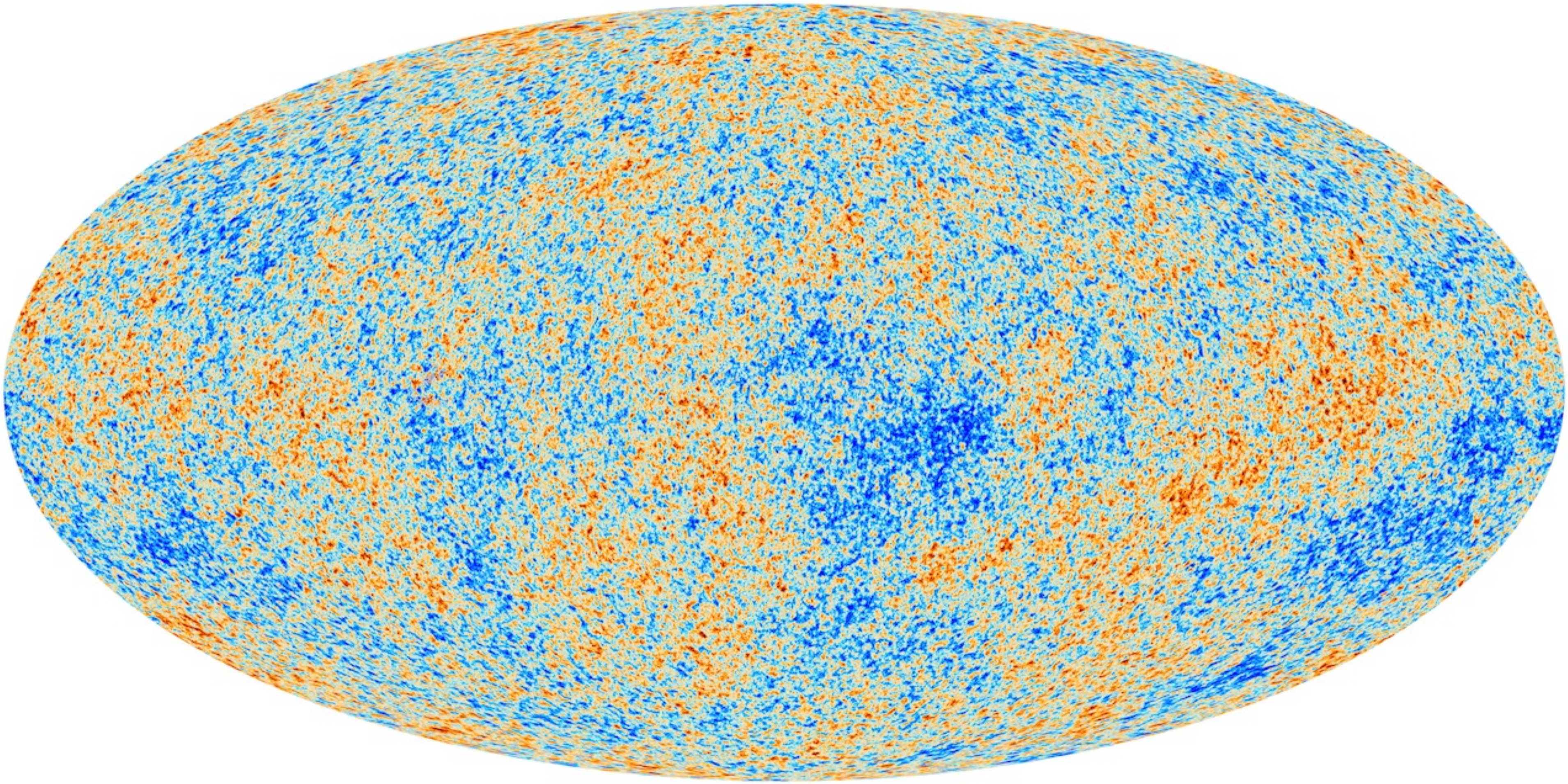
We call it **Canonical Superconformal Supergravity (CSS)**.

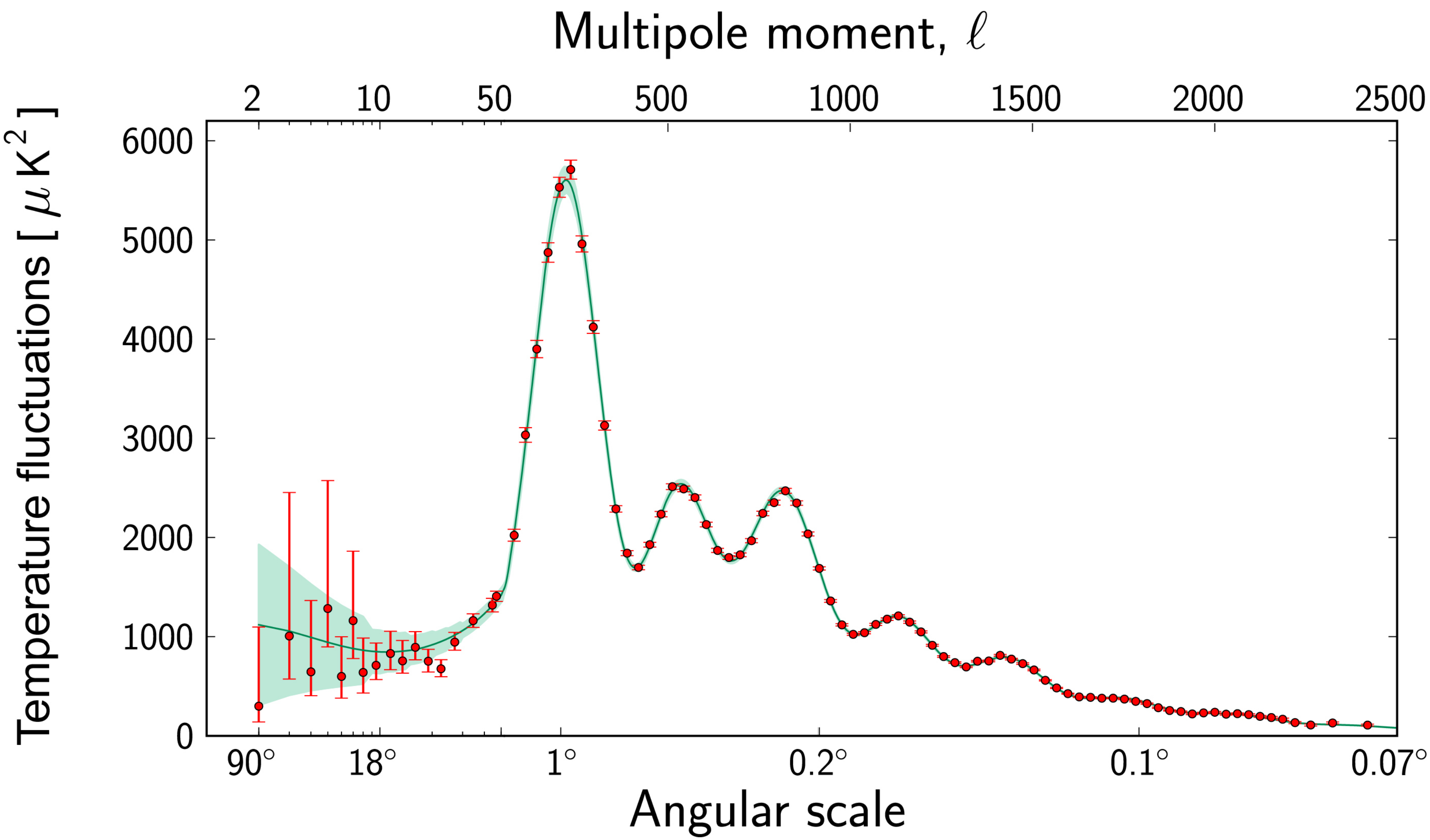
The main condition: matter fields are decoupled from conformal compensator.

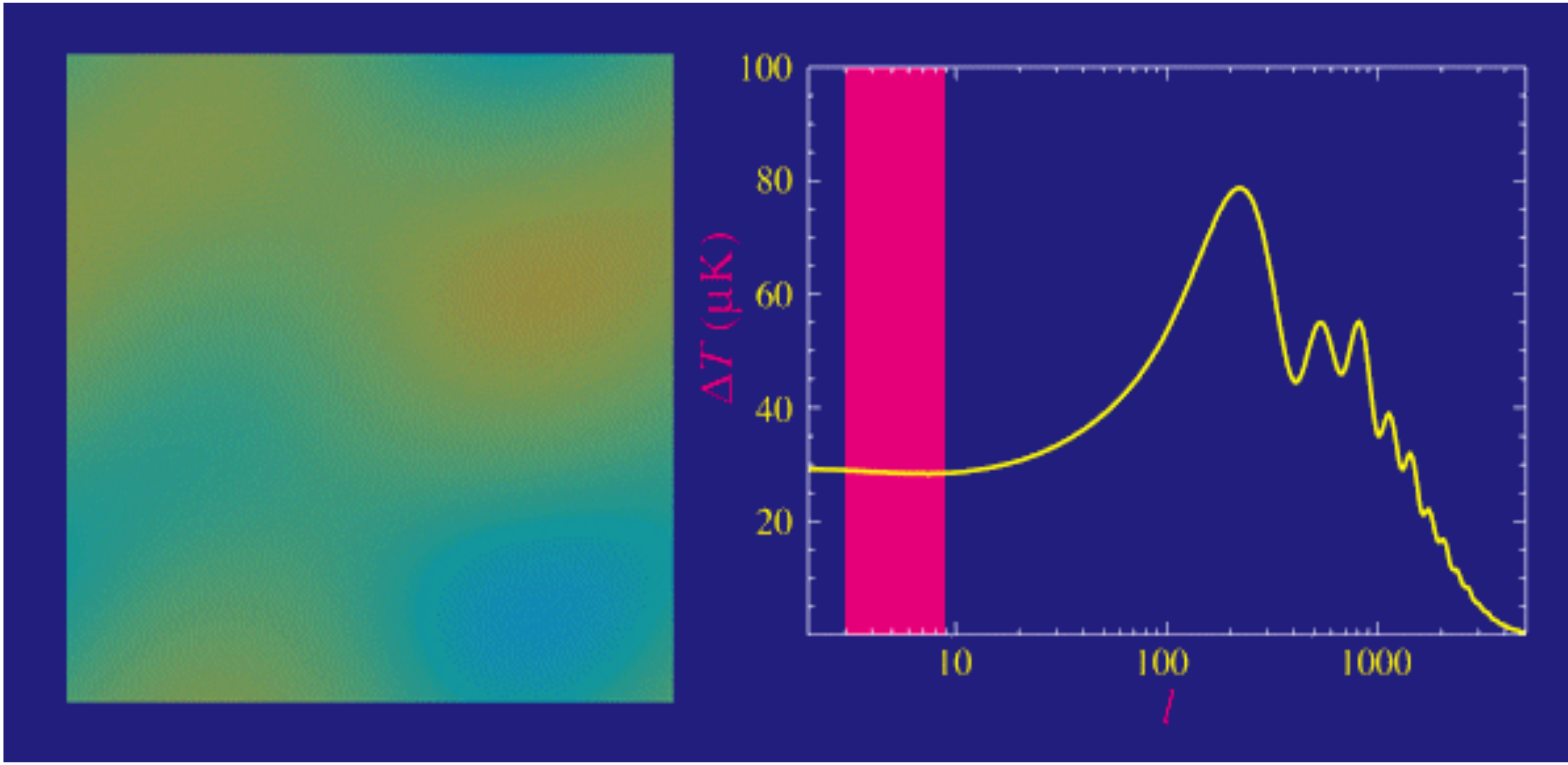
Bottom - up approach: One can embed SUSY to SUGRA in many different ways. For theories with scale-invariant superpotentials and canonical kinetic terms, such as the scale-invariant NMSSM, there is a special choice of the Kahler potential which allows to embed SUSY to the CSS. This embedding is trivial: One simply adds to supergravity the action of the global SUSY conformally coupled to gravity. The scalar potential and kinetic terms remain the same as in the global SUSY.



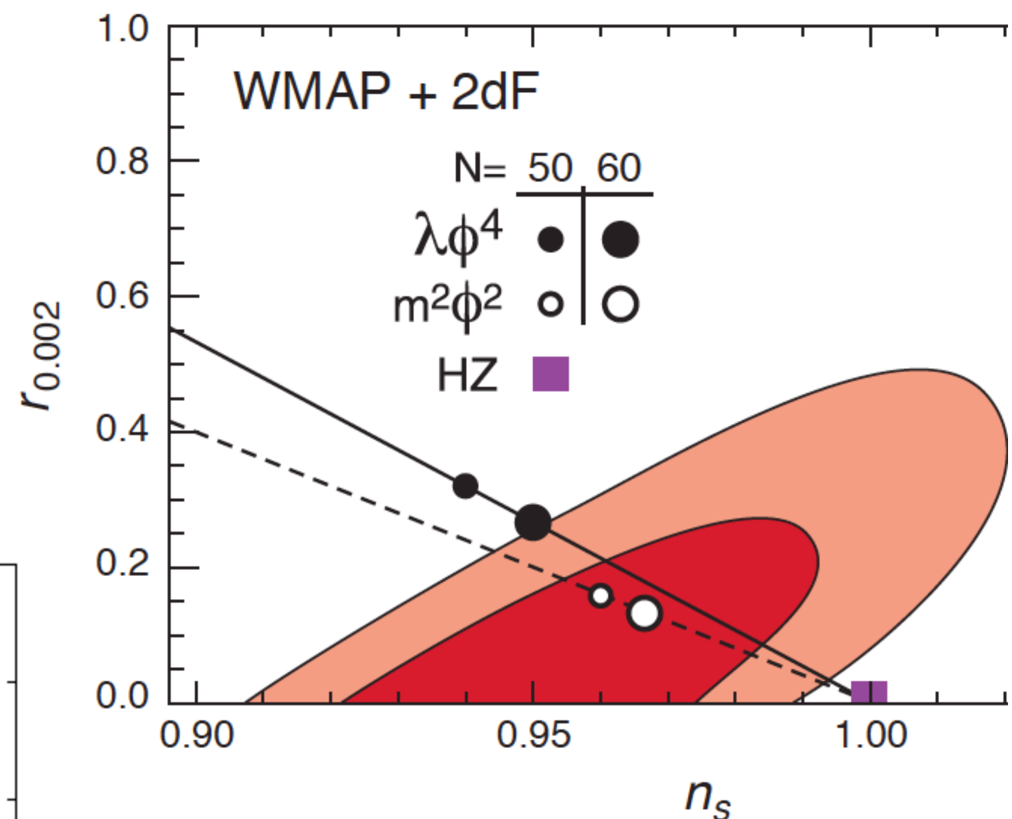
Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years



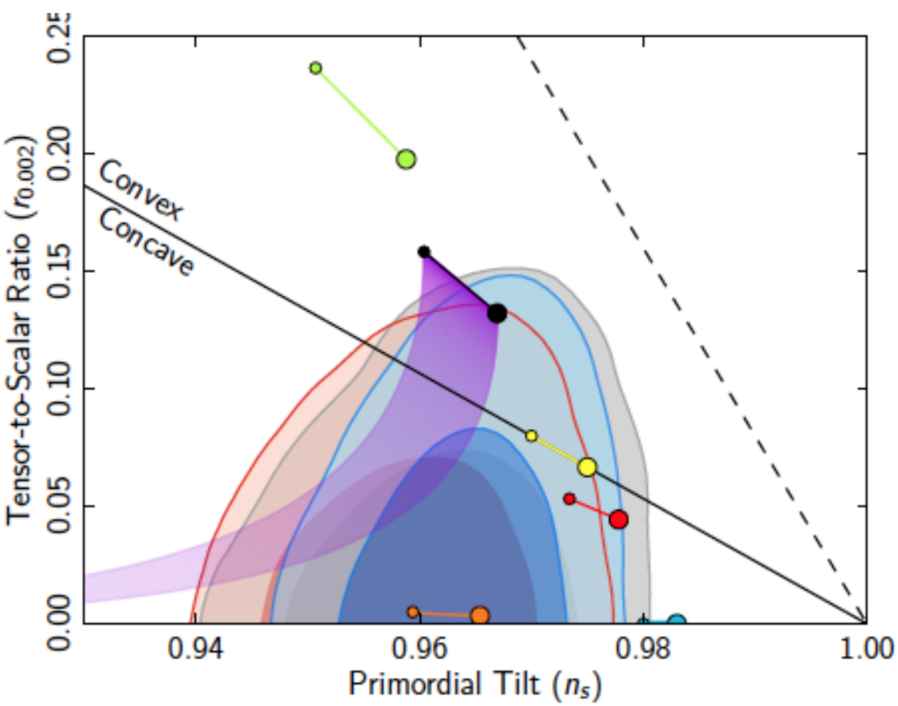

Multipole moment, ℓ
2
10
50
500
1000
1500
2000
2500
Temperature fluctuations [μK^2]
6000
5000
4000
3000
2000
1000
0
90°
18°
1°
0.2°
0.1°
0.07°
Angular scale

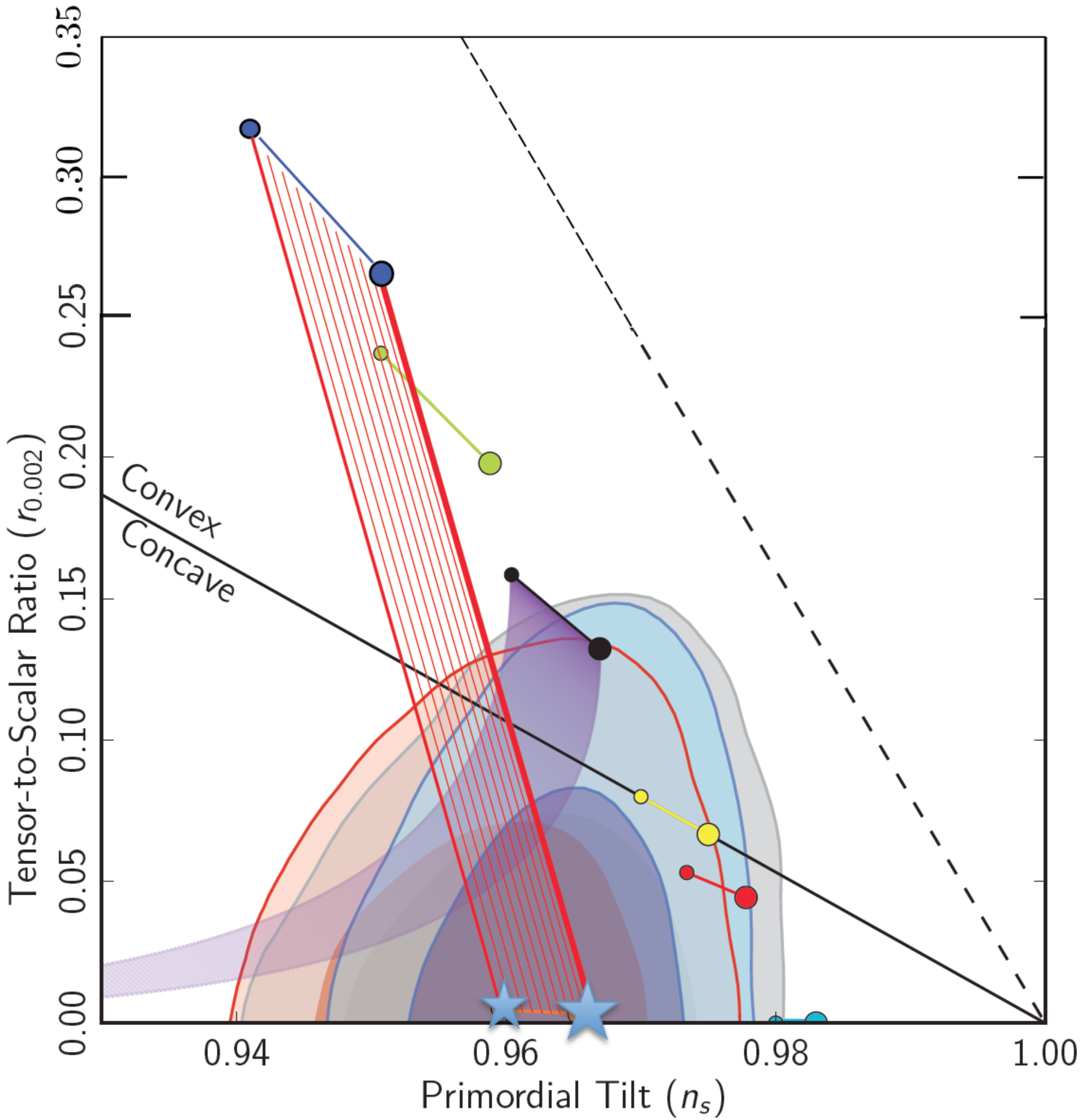

100
80
60
40
20
ΔT (μK)
10
100
1000
l

2007


WMAP + 2dF
N= 50 60
λφ4
m2φ2
HZ
r0.002
ns
1.0
0.8
0.6
0.4
0.2
0.0
0.90
0.95
1.00

2013


Convex
Concave
Tensor-to-Scalar Ratio (r0.002)
Primordial Tilt (ns)
0.94
0.96
0.98
1.00
0.00
0.05
0.10
0.15
0.20



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Tensor-to-Scalar Ratio ($r_{0.002}$)
Convex
Concave
0.94
0.96
0.98
1.00
Primordial Tilt (n_s)

Participatory Universe

John Wheeler