Complex Geometry and Sigma Models

U. Lindström[1]

[1] Department of Physics and Astronomy
Division of Theoretical Physics
University of Uppsala

Marc Celebration

UPPSALA UNIVERSITET

History

Sigma models in d=2

The (1,1) analysis by Gates Hull and Roček gives:

Susy	(0,0) (1,1)	(2,2)	(2,2)	(4,4)	(4,4)
Bgd	G, B	G	G, B	G	G, B
Geom	Riem.	Kähler	biherm.	hyperk.	bihyperc.

Bihermitean Geometry

$$(M, g, J_{(\pm)}, H)$$

$$J^2_{(\pm)} = -\mathbf{1} \;, \quad J^t_{(\pm)} g J_{(\pm)} = g \;, \quad \nabla^{(\pm)} J_{(\pm)} = 0$$

$$\Gamma^{(\pm)} = \Gamma^0 \pm \tfrac{1}{2} g^{-1} H \;, \quad H = dB \;.$$

$$E := g + B$$

Generalized Complex Geometry

Complex structure:

$$\mathcal{J} \in End(TM \oplus T^*M), \qquad \mathcal{J}^2 = -1$$

$$\Pi_\pm := \tfrac{1}{2}(\mathbf{1} \pm \mathcal{J})$$

"Nijenhuis":

$$\mathcal{N}_C(\mathcal{J}) = 0 \iff \Pi_\mp[\Pi_\pm U, \Pi_\pm V]_C = 0$$

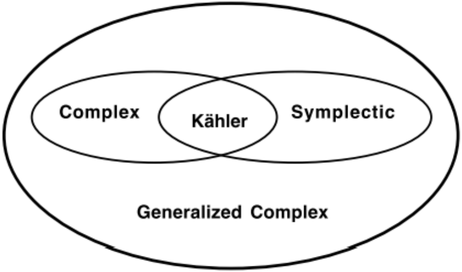
where

$$U = (u, \xi)\ , \quad V = (v, \rho)$$

$$[U, V]_C = [u, v] + \mathcal{L}_u\rho - \mathcal{L}_v\xi - \tfrac{1}{2}d(\imath_u\rho - \imath_v\xi)$$

The automorphisms of this courant bracket are diffeomorphisms and *b*-transforms:

$$e^b(u, \xi) = (u, \xi + \imath_u b)\ , \quad db = 0\ .$$


Complex
Kähler
Symplectic
Generalized Complex

Bihermitean=Generalized Kähler

Description on $T \oplus T^*$

$$\mathcal{J}^2_{(1,2)} = -\mathbf{1}\ , \quad [\mathcal{J}_{(1)}, \mathcal{J}_{(2)}] = 0\ , \quad \mathcal{J}^t_{(1,2)}\mathcal{I}\mathcal{J}_{(1,2)} = \mathcal{I}\ , \quad \mathcal{G} := -\mathcal{J}_{(1)}\mathcal{J}_{(2)}$$

$$\mathcal{J}_{(1,2)} =$$

$$\begin{pmatrix} 1 & 0 \\ B & 1 \end{pmatrix} \begin{pmatrix} J_{(+)} \pm J_{(-)} & -(\omega^{-1}_{(+)} \mp \omega^{-1}_{(-)}) \\ \omega_{(+)} \mp \omega_{(-)} & -(J^t_{(+)} \pm J^t_{(-)}) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -B & 1 \end{pmatrix}$$

Collaboration


Generalized Kähler Potential

Geometric data: $(M, g, H, J_{(\pm)})$ or $(M, g, J_{(\pm)})$ or $(M, \mathcal{F}_{(\pm)}, J_{(\pm)})$. In each case, there is a complete description in terms of a Generalized Kähler potential K. Unlike the Kähler case, the expressions are non-linear in second derivatives of K. E.g.,

$$J_{(+)} = \begin{pmatrix} J & 0 \\ (K_{LR})^{-1}[J, K_{LL}] & (K_{LR})^{-1} J K_{LR} \end{pmatrix}$$

$$g = \Omega[J_{(+)}, J_{(-)}]$$

$$\mathcal{F}_{(+)} = d\lambda_{(+)} \ , \quad \lambda_{(+)\ell} = i K_R J (K_{LR})^{-1} K_{L\ell} \ , \ldots$$

Generating function

There are two special sets of Darboux coordinates for the symplectic form Ω. One set, $(\mathbb{X}^L, \mathbb{Y}_L)$, is also canonical coordinates for $J_{(+)}$ and the other set, $(\mathbb{X}^R, \mathbb{Y}_R)$ is canonical coordinates for $J_{(-)}$. The symplectomorphism that relates the two sets of coordinates has thus a generating function. This generating function is in fact the generalized Kähler-potential $K(\mathbb{X}^L, \mathbb{X}^R)$.

$(\mathbb{X}^L, \mathbb{Y}_L)$	$\leftarrow K(\mathbb{X}^L, \mathbb{X}^R) \rightarrow$	$(\mathbb{X}^R, \mathbb{Y}_R)$
$J_{(+)} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$ $d\Omega = d\mathbb{X}^\ell \wedge d\mathbb{Y}_\ell + c.c.$		$J_{(-)} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$ $d\Omega = \mathbb{X}^r \wedge \mathbb{Y}_r + c.c$

This fact is a key ingredient in the proof that we have a complete description or GKG.

$d = 2\ ,\ \ N = (2,2)$

$$S = \int \mathbb{D}_+\bar{\mathbb{D}}_+\mathbb{D}_-\bar{\mathbb{D}}_- K(\phi^c, \chi^t, \mathbb{X}^L, \mathbb{X}^R)$$

Constrained superfields:

$$\begin{aligned} \bar{\mathbb{D}}_\pm \phi^a &= 0\ , \\ \bar{\mathbb{D}}_+ \chi^{a'} = \mathbb{D}_- \chi^{a'} &= 0\ , \\ \bar{\mathbb{D}}_+ \mathbb{X}^\ell &= 0\ , \\ \bar{\mathbb{D}}_- \mathbb{X}^r &= 0\ . \end{aligned}$$

Notation: $c := a, \bar{a}\ ,\quad t := a', \bar{a}'\ ,\quad L := \ell, \bar{\ell}\ ,\quad R := r, \bar{r}\ .$

Superspace encodes and dictates all the geometric formulations of Generalized Kähler Geometry

Modulo Irregular Points!!!

Some Results

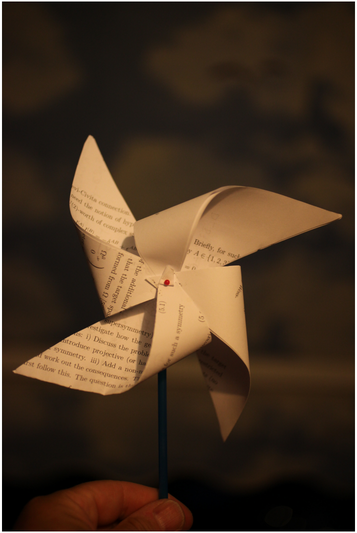
- A complete coordinatization of GKG away from irregular points.
- GKG has a Generalized Kähler potential K.
- The non-linearities in the description of the geometry has found an interpretation as a quotient construction from an auxiliary higher dimensional space with Kac-Moody symmetries.
- We introduced and studied the notion of a biholomorphic gerbe with connection.

- We studied the local conditions for a generalized Kähler manifold to be a generalized Calabi-Yau manifold and we derived a generalization of the complex Monge-Ampère equation to describe this. Its solutions give solutions of type II supergravity with metric, dilaton and H-field. This result also relates the pure spinor formulation of GKG to the generalized Kähler potential.

" The Quantum geometry of N=(2,2) nonlinear sigma models" Marcus T. Grisaru, M. Massar, A. Sevrin, J. Troost. Phys.Lett. B412 (1997) 53-58.

- The appropriate connections for gauging sigma models describing GKG have been constructed. Ingredients in T-duality.
- The various definitions of GKG corresponding sigma model formulations. This is seen, e.g., in the $(2,1)$ formulation we presented.

?



ALL THE BEST MARC!!