Improved methods for hypergraphs

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[arXiv:1302.3277]

Earlier work:

"Hypersymmetry": N=2 supersymmetry — Fayet ('76)

⋮ (cf. "Bambi Meets Godzilla")

Hypergraphs: Ivanov, Galperin, Ogievetsky, Sokatchev ('85)

Gonzalez-Rey, Roček, Wiles, Lindström, von Unge ('97-8)

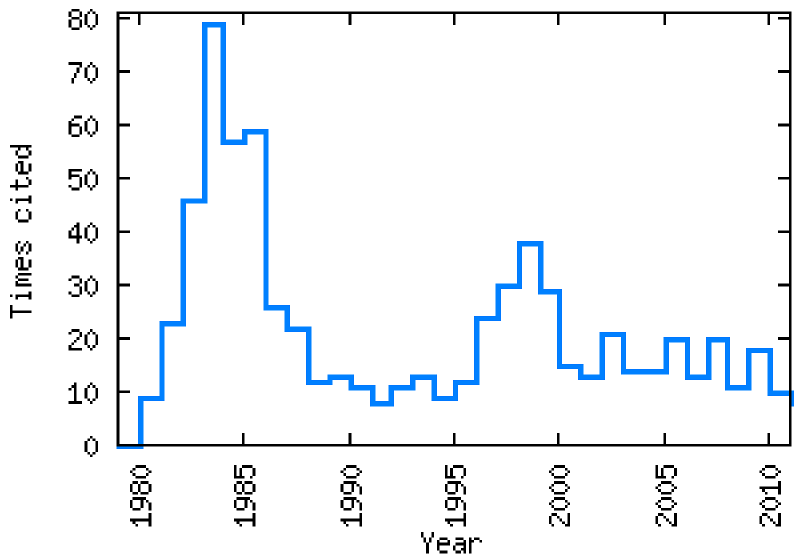
Jain, Siegel ('09-12)

Background hyperfields: Buchbinder^2, Ivanov, Kuzenko, Ovrut, McArthur, Petrov ('97-'02)

This paper does for N=2 supergraphs what was done for N=1 by ...

Improved methods for supergraphs

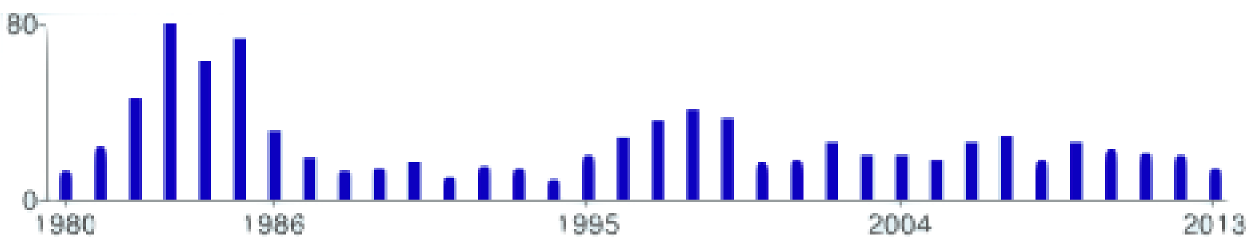
Marcus T. Grisaru, W. Siegel (Brandeis U.), M. Roček (Cambridge U.). Jun 1979. 32 pp.
Published in Nucl.Phys. B159 (1979) 429
Cited by 742 records (INSPIRE)



⇐ **Actual experimental data related to supersymmetry**

⇓

Cited by 910 (Google scholar)



Background field formalism

	N = 1	N = 2 (6D N=1)
quantum superfield	scalar	scalar
superspace	x, full θ	x, analytic ($\frac{1}{2}$) θ, (internal) y
representation	chiral	analytic
background superfield	spinor	spinor
superspace	x, full θ	x, full θ, no y
representation	real	real
nonrenormalization	obvious*	obvious*
effective action	x, θ	x, θ (no internal)

*As for N=1,

Quantum superfield is scalar prepotential of dimension 0;

background superfield is spinor (or maybe vector) potential with dimension > 0.

N=4 Yang-Mills

1-loop cancelations in N=4 Yang-Mills as formulated in N=1 or N=2 superspace:

	N = 1	N = 2
scalar multiplets	3	1
Faddeev-Popov ghosts	−2	−2
Nielsen-Kallosh ghosts	−1	1
total	0	0
vector multiplets	1	1
“extra” ghosts	0	1−2
total	1	0*

***Same propagator, different vertex $\Rightarrow$ cancels only y-divergence $\delta(0)$.**

In both cases, vector multiplets etc. contribute only @ 4-point & higher, scalar multiplets etc. also @ lower-point.

Equations

In case there's too much time left, some actual equations:

scalar/FP/NK propagator: $\frac{1}{y_{12}^3}\nabla_{1\vartheta}^4\nabla_{2\vartheta}^4\delta^8(\theta_{12})\frac{1}{\frac{1}{2}k^2}$

vector/XR propagator: $\frac{\delta(y_{12})}{y_1}\nabla_{1\vartheta}^4\delta^8(\theta_{12})\frac{1}{\frac{1}{2}k^2}$

scalar/FP/NK vertex: $\int d^4\theta\, dy\, (\hat{\Box} - \Box_0)$

vector vertex: $\int d^4\theta\, dy\, y(\hat{\Box} - \Box_0)$

XR vertex: $\int d^4\theta\, d^2y\, [-1 + y_1\delta(y_{12})](\hat{\Box} - \Box_0)$

Above are for just 1 loop (free quantum in background).

For vertices, use ∇_ϑ^4 from propagator to make $\int d^4\theta\, \nabla_\vartheta^4 = \int d^8\theta$.

Conclusions

(1) Same kind of simplifications for N=2 as for N=1 (1 loop & higher)

(2) Quantum field $V(x, \theta, y)$, where $A_\vartheta = 0$;

background fields A_θ, A_ϑ, where $A_y = 0$, trivial dependence on y

(3) Classical action in analytic superspace $d^4x\, d^4\theta\, dy$, nonlocal in y;

effective action in "full" superspace $d^4x\, d^4\theta\, d^4\vartheta$, no y

(4) N=3 supergraphs (for N=4 Yang-Mills): in progress

(5) Supergravity

(6) 1st-quantization?

That's a good question!

Quantum vertices (background appears only through ∇_ϑ^4):

scalar: $$-\int d^4\theta\, \bar{\Upsilon}(e^V - 1)\Upsilon$$

vector: $$\int d^4\theta\, d^4\vartheta\, d^n y\, \frac{(-1)^n}{n} \frac{(e^{V_1} - 1)\cdots(e^{V_n} - 1)}{y_{12}y_{23}\cdots y_{n1}}$$

FP: $$-\int d^4\theta\, (yb + \bar{b})\mathcal{L}_{V/2}\left[coth(\mathcal{L}_{V/2})\left(c - \frac{\bar{c}}{y}\right) + \left(c + \frac{\bar{c}}{y}\right)\right]$$

Nonlocality in y gets no background covariantization, since $A_y = 0$.