3D strings: an open and shut case?

Paul K. Townsend

DAMTP, Cambridge University, UK

Based on work with Luca Mezincescu
arXiv: 1008.2334, 1008.2775, 1011.5049, 1106.1374, 1111.3384

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with Luca Mezincescu and Alasdair Routh
to appear

Grisaru-fest, McGill, April 2013

Closed NG string

- Closed NG string Lagrangian is

$$L = \oint d\sigma \left\{ \dot{X} \cdot P - \tfrac{1}{2} e \left[P^2 + (TX')^2 \right] - uX' \cdot P \right\}$$

- Light-cone gauge: $(X^+)' = P'_- = 0$. Eliminate auxiliary fields to get action for zero modes (x, p) and transverse variables $(\mathbf{X}, \mathbf{P})$

$$L = \dot{x} \cdot p + \oint d\sigma \dot{\mathbf{X}} \cdot \mathbf{P} - \tfrac{1}{2} e_0 \left(p^2 + M^2 \right) - u_0 \oint d\sigma \mathbf{X}' \cdot \mathbf{P}$$

where $M^2 = \oint d\sigma [\mathbf{P}^2 + (T\mathbf{X}')^2]$.

- Quantize to get spectrum $M^2 = (4\pi T)[N + \tilde{N} - a]$ subject to level-matching constraint $N = \tilde{N}$.

Lorentz anomalies

➳ Lorentz invariance is not manifest so there is possible Lorentz anomaly. In fact (GGRT)

$$\left[J^{i-}, J^{j-}\right] = \sum_m (\cdots)^{ij}_m \Delta_m$$

Must be zero, but there are **two** ways this can happen:

➳ Standard way: $\Delta_m = 0$. Satisfied iff $D = 26$ and $a = 2$. Leads to critical string.

➳ Non-standard way: $(\cdots)^{ij}_m = 0$. Satisfied if $D = 3$. But spectrum contains particles of **irrational spin**.

Open NG string

- $\delta S|_{\text{on-shell}} = 0 \quad \Rightarrow \quad$ Neumann or Dirichlet bcs

Dirichlet $\Rightarrow$ Dp-branes. e.g. D0-branes:

- can't use light-cone gauge (X^0, X^1 mode expansions differ).

Use Arvis gauge: $P_0 + TX_1' = p_0$ & $P_1 + TX_0' = 0$, to get

$$L = \dot{x}^0 p_0 + \oint d\sigma\, \dot{\mathbf{X}} \cdot \mathbf{P} + \frac{1}{2} e_0 \left(p_0^2 - M^2 \right)$$

where $M^2 = \oint d\sigma [\mathbf{P}^2 + (T\mathbf{X}')^2]$, as for closed string (but open string mode expansion).

- Rotation anomaly unless $\begin{cases} \text{either} & \text{D=26 \& a=1} \quad \text{(Arvis,'83)} \\ \text{or} & \text{D=3} \end{cases}$

3D Poincaré UIRs (Binegar '81)

- Poincaré group generated by 3-vectors $\mathcal{P}_\mu$ and $\mathcal{J}_\mu$. Massive UIRs classified by Casimirs

$$-\mathcal{P}^2 \equiv M^2 , \qquad \mathcal{P} \cdot \mathcal{J} \equiv Mh$$

M is mass and h is "relativistic helicity". Define $|h|$ to be "spin".

- $2h \notin \mathbb{Z} \quad \Rightarrow \quad$ Anyon (by 3D spin/statistics theorem)

- $2h \notin \mathbb{Z} \quad$ **but** $\quad 4h \in \mathbb{Z} \quad \Rightarrow \quad$ Semion

- Spin not defined if $M^2 = 0$, but still 3 UIRs: Boson & Fermion, and "infinite spin" (analog of 4D "continuous spin")

Covariant quantization?

➽ Covariant action for particle of helicity h is

$$I = \int dt \left\{(\dot{X}^\mu P_\mu - \tfrac{1}{2}e\left(P^2 + M^2\right)\right\} + h\, I_{LWZ}$$

Lorentz-Wess-Zumino term constructed from the closed super-Poincaré invariant 2-form $\left(P^2\right)^{-\frac{3}{2}} \varepsilon^{\mu\nu\rho} P_\mu \, dP_\nu \wedge dP_\rho$ (Shonfeld '81)

➽ Light-cone gauge quantization $\rightarrow$ **one-component** KG-equation but in terms of coordinates that are *non-local* functions of X

➽ Covariant equation for $h \neq 0$ requires an **infinite comnponent** field (Jackiw & Nair '91, Plyuschay, '91). How do we find it by covariant quantization of the particle? Obviously harder for string!

3D superstrings (M&T)

➽ Green-Schwarz superstring action exists for $D = 3, 4, 6, 10$, and $\mathcal{N} = 1, 2$. Focus on $D = 3$ and $\mathcal{N} = 2$. Quantize in light-cone gauge $\rightarrow$ bosonic annihilation operators $(a_n, \tilde{a}_n)$ and fermionic annihilation operators $(\xi_n, \tilde{\xi}_n)$.

➽ The following 'odd' operator plays a crucial role:

$$\Xi \ \propto \ \sum_n \left(a_n \xi_n^\dagger + a_n^\dagger \xi_n \right) + \sum_n \left(\tilde{a}_n \tilde{\xi}_n^\dagger + \tilde{a}_n^\dagger \tilde{\xi}_n \right) .$$

Ξ squares to the even mass-squared operator M^2 (using level-matching constraint), so it determines spectrum.

➽ Ξ commutes with super-helicity Casimir $\Rightarrow$ spectrum is super-Poincaré invariant $\Rightarrow$ no super-Poincaré anomalies.

Spectrum of closed $\mathcal{N} = 2$ 3D superstring

- 2 fermonic zero modes $\Rightarrow$ 4 massless ground states at level $N = 0$: 2 bosons and 2 fermions.

- All other states are massive. At level $N = 1$ we get 4 copies of the scalar supermultiplet with helicities $(-1/2, 0, 0, 1/2)$.

- At level $N = 2$ get 8 copies of scalar supermultiplet plus 4 copies of spin-2 supermultiplet $(1, 3/2, 3/2, 2)$ and its parity conjugate $(-2, -3/2, -3/2, -1)$.

- At level $N = 3$ get another 8 copies of the scalar supermultiplet. But remaining 28+28 supermultiplets all have irrational helicities.

Equivalence with Ramond string (RMT)

- The $D = 10$ GS string is equivalent to the RNS string with GSO projection. Proof uses light-cone gauge plus Spin(8) triality.

- The $D = 3$ GS string is equivalent to the Ramond string. Proof uses light-cone gauge plus Spin(1) triviality.

- Analog of Ξ operator is the Ramond string supercharge Q. Same mass spectrum

- Also same helicities. So 3D Ramond string has hidden 3D susy!

- Also true for open strings with free ends. Get closed string spectrum by taking $L \otimes R$ and imposing level-matching. [Other b.c.s under investigation]

$\mathcal{N} = 2$ Superparticle (M&T)

- Parity-preserving $\mathcal{N} = 2$ superparticle has action

$$I = \int dt \left\{ \left(\dot{X}^\mu + i\bar{\Theta}_a \Gamma^\mu \dot{\Theta}_a \right) P_\mu - iZ\varepsilon^{ab}\bar{\Theta}_a\dot{\Theta}_b - \frac{1}{2}e\left(P^2 + M^2\right) \right\}$$

- Z is central charge. Unitarity of quantum theory requires BPS bound $M \geq |Z|$. Saturation, $M = |Z|$, gives short BPS semion supermultiplet of helicities

$$\left(-\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$$

- Consistent with semion statistics of 3D matrix-model D0-branes (Pedder, Sonner and Tong)

A blast from the past

- $\mathcal{N} = 2$ closed strings have left-moving fermions ψ_L and right-moving fermions ψ_R. Does ψ_L commute or anti-commute with ψ_R?

- If we want no interactions between left-movers and right-movers then we want $[\psi_L, \psi_R] = 0$, i.e. $Z_2 \times Z_2$ grading. Otherwise, for $\{\psi_L, \psi_R\} = 0$ we get statistical interactions from exclusion principle.

- To get IIA string from 11D we need to put all fermions into one 32-cpt spinor. This implies $\{\psi_L, \psi_R\} = 0$ and hence Z_2 grading.

- So M-theory unification of string theory requires equivalence of two types of grading. Are they equivalent?

- Usually, $Z_2 \times Z_2$ grading gives same results as Z_2 grading (Zumino, Van Nieuwenhuizen).

- But not always! For heterotic string ghosts the two types of grading (Lorentz vs ghost statistics) give different answers for the the anomalies. Only the $Z_2 \times Z_2$ grading gives the expected results (Grisaru, Mezincescu and Townsend, 1986).

The End

- MARC, MANY THANKS FOR YOUR FRIENDSHIP OVER THE YEARS
- THANKS FOR SOME MEMORABLE COLLABORATIONS., AND
- BEST WISHES FOR THE FUTURE!