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Anomaly Multiplets Regrisaruzation, And Solitons

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40 years of friendship and 17 articles with Marc

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ANOMALIES IN SUPERSYMMETRIC THEORIES

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It was pointed out some time ago by Ferrara and Zumino[1)] that in a supersymmetric theory the axial current j^5_μ, the improved energy-momentum tensor $\theta_{\mu\nu}$ and the improved supersymmetry current S_μ can be identified with the components of a superfield V_μ, so that an intimate relation exists between them. At the classical level this superfield satisfies certain relations reflecting the conservation and trace relations (modulo mass terms)

$$\partial_\mu S^\mu = \partial_\mu \theta^{\mu\nu} = 0 \tag{1}$$

$$\partial_\mu j^{5\mu} = 0 \tag{2a}$$

$$\theta_\mu^\mu = \partial_\mu (x_\nu \theta^{\mu\nu}) = 0 \tag{2b}$$

$$\gamma^\mu S_\mu = \partial_\mu (\gamma . x \, S^\mu) = 0 \tag{2c}$$

We note that the improved supersymmetry current is obtained by adding terms to the Noether current so as to satisfy Eq. (2c) while maintaining Eq. (1)[1)].

$$j_\mu^5 = A \overleftrightarrow{D}_\mu B - \frac{i}{4} \bar{\chi} \gamma_5 \gamma_\mu \chi \tag{8c}$$

The form of the chiral current follows from the chiral properties implicit in Eqs. (4).

We look at one-loop matrix elements of the above quantities between the vacuum and states of the vector multiplet (λ, v_μ). The anomaly of j_μ^5 is the usual chiral anomaly[6], as obtained from the diagram of Fig. 1a. Only the spinor part of the current contributes.

The spinor current anomaly can be calculated from the diagrams in Figs. (1b), (1c) (1d)[7], while the trace anomaly is known also[5]. One finds, with the external fields on shell:

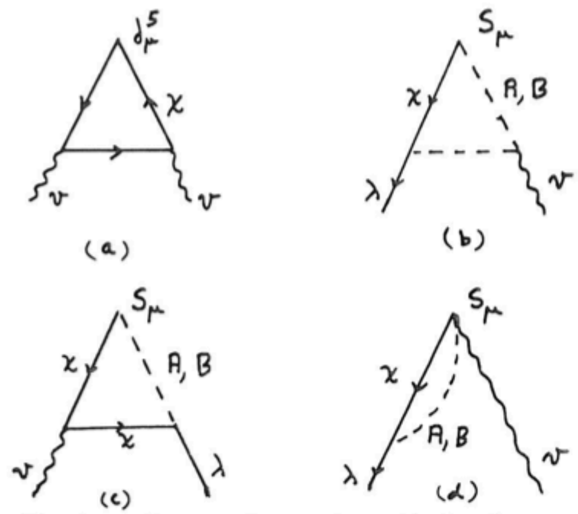


Fig. 1 : Diagrams for one-loop chiral and supercurrent anomalies.

Marc has clarified several times long-standing problems and confusion by using new (and old) methods of regularization and computation. One such problem is the "anomaly puzzle".

Toni Rebhan, Robert Wimmer and I have studied the past 15 years the one-loop quantum corrections to the mass M and the central charge(s) Z of solitons: susy kinks, vortices, and N=2 and N=4 monopoles.

We have constructed a new regularization scheme for these systems (and more general systems) which gives BPS saturation ($M^{(1)}=Z^{(1)}$). The corrections to $Z^{(1)}$ looked like an anomaly, but the usual susy current, and thus also the central charge current, is conserved (free from anomalies).

For solitons, problems with R_ξ gauge in superspace

$$\mathcal{L}_{\text{fix}} = \left(\bar{D}^2 v + \bar{D}^2 \frac{1}{\Box} \bar{\psi}\, \phi_{\text{vor}}\right) \left(D^2 v + D^2 \frac{1}{\Box} \psi\, \bar{\phi}_{\text{vor}}\right)$$

Following FZ who found for the N=1 WZ model

$$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_{\alpha} S$$

one can begin with an "early easy" anomaly (the chiral anomaly in their case), and get the other anomalies from susy. Since $\not{x} j_{\mu}$ is the conformal susy current, there is also a conformal central charge current, and the question arose:

Is $Z^{(1)}$ equal to the anomaly in a conformal central charge current?

We began with the susy kink in d=2. But then we moved to N=2 monopoles in d=4 and here we found problems with the N=2 anomaly multiplet. Our explicit calculations for various anomalies (chiral anomaly, susy anomaly) did not agree with what the currents of the N=1 formulation (Grisaru-West and Grisaru-Milevski-Zanon) predicted. We are led to the following conjectures:

Conjecture 1:The N=2 current multiplet and anomaly multiplet contain extra terms beyond those provided by the corresponding manifestly N=1 susy formulation.

Conjecture 2:The N=1 superspace results as they now stand, cannot be extended to a full N=2 superspace formulation.

BPS and anomalies for the susy kink.

$$\mathcal{L} = -\tfrac{1}{2}\left[\,(\partial_\mu\varphi)^2 + \bar\psi\partial\!\!\!/\psi + U^2 + U'\,\bar\psi\psi\,\right]$$

The susy current $j_\mu = -(\,\partial\!\!\!/\varphi + U\,)\gamma_\mu\psi$ transforms into the stress tensor and the central charge current

$$\delta j_\mu = -2\,T_\mu{}^\nu\gamma_\nu\epsilon - 2\,\zeta_\mu\gamma_3\epsilon \quad \text{with} \quad \zeta_\mu = \epsilon_{\mu\nu}\partial^\nu U$$

where $\tfrac{1}{2}U^2 = \dfrac{\lambda}{4}\left(\varphi^2 - \dfrac{\mu^2+\Delta\mu^2}{\lambda}\right)^2$

$M^{(1)}\neq 0$ from $H = \int \tfrac{1}{2}(\partial_x\varphi)^2 + \ldots = \int \partial_x(\tfrac{1}{2}\,\varphi\partial_x\varphi) - \tfrac{1}{2}\,\varphi\,\partial_x^2\,\varphi + \ldots$

About the 1-loop quantum corrections to ζ_μ:

- Zero because total derivative? No.
- Zero after detailed calculation? Yes.

But $M^{(1)} \neq 0$. So where is $Z^{(1)}$?

Answer: the (1,1) susy WZ model in d = 1 + 1 is also a susy model in d = 2 + 1.

Go to 2 + 1 dimensions, keep the soliton in one dimension, and do dimensional regularization in the other dimension. (In dimensional reduction there is no space for the soliton). This preserves susy, but no need for evanescent terms etc…

$$\{Q^{\pm}, Q^{\pm}\} = H \pm (Z_x - P_y) \quad , \quad \{Q^{+}, Q^{-}\} = P_x + Z_y$$

$$P_m = \dot{\varphi}\partial_m\varphi - \tfrac{1}{2}\bar{\psi}\gamma^0\partial_m\psi \quad , \quad Z_m = U(\varphi)\partial_m\varphi$$

So $Z_{reg} = Z_x - P_y$. Then < solitons | P_y | solitons > gives $Z_{reg}^{(1)}$!

Details:

$$(\partial_x + U_K')\psi_+ = (\partial_0 - \partial_y)\psi_-$$

$$(\partial_x - U_K')\psi_- = (\partial_0 + \partial_y)\psi_+$$

$$\psi = \psi_0 + \int \frac{d^\epsilon \ell}{(2\pi)^{\frac{\epsilon}{2}}} \int \frac{dk}{2\pi} \frac{1}{\sqrt{2\omega_{k,\ell}}} \left[b_{k,\ell} \begin{pmatrix} \sqrt{\omega+\ell}\ \phi_k(x) \\ \sqrt{\omega-\ell}\ i s_k(x) \end{pmatrix} e^{-i(\omega t - \ell y)} + b_{k,\ell}^\dagger\ (c.c.) \right],$$

$$\psi_0 = \int \frac{d^\epsilon \ell}{(2\pi)^{\frac{\epsilon}{2}}} \frac{1}{2|\ell|} \left[b_{0,\ell} \begin{pmatrix} \sqrt{|\ell| + \ell}\, \phi_0(x) \\ 0 \end{pmatrix} e^{i\ell y - i|\ell| t} + b_{0,\ell}^\dagger \begin{pmatrix} \sqrt{|\ell| + \ell}\, \phi_0(x) \\ 0 \end{pmatrix} e^{-i\ell y + i|\ell| t} \right]$$

where $\quad s_k(x) = \dfrac{(\partial_x + U_K')\,\phi_k(x)}{\omega_k}$

The zero mode $\phi_0(x)$ becomes massless right-moving chiral domain wall fermions!

$$\langle P_y \rangle_{\text{fermions}} =$$

$$\frac{1}{2}\int \frac{d^\epsilon \ell}{(2\pi)^{\frac{\epsilon}{2}}} \int \frac{dk}{2\pi} \frac{\ell^2}{\sqrt{2\omega_{k,\ell}}} \underbrace{\left(|\phi_k(x)|^2 - |s_k(x)|^2 \right) dx}\, d^\epsilon y$$

from index theorem or directly : $\frac{2m}{k^2+m^2}$

$$= -\frac{m}{2\pi}$$

Dimensional reduction and the kink

Cf Grisaru, Zanon, Cambridge 1985

j^μ = as before. Problem: $\gamma^\mu j_\mu = 0$ (+ explicit breaking).
Solution: renormalization requires an evanescent counterterm for j^μ, which in turn gives the anomaly.

$j^\mu =$  $+$  = 0 (From no tadpoles $\mu_0^2 = \mu^2 + \Delta\mu^2$)

But  $= \dfrac{U''(\varphi)}{2\pi} \, \dfrac{\hat{\hat{\delta}}_\mu{}^\lambda}{\epsilon} \, \gamma_\lambda \, \psi = j_\mu^{\rm div}$

(From $\not{k}\gamma_\mu\not{k} = -\kappa^2\gamma_\mu + \dfrac{2}{n}\kappa^2\hat{\delta}_\mu{}^\lambda\gamma_\lambda$)

$$j^\mu_{\rm ren} = j^\mu - j^\mu_{\rm div} \quad \Rightarrow \quad -\gamma_\mu \, j^\mu_{\rm div} = -\frac{U''(\varphi)}{2\pi}$$

Now use susy to get corrections to central charge:

$$\zeta_\mu^{\text{div}} = \frac{\hat{\hat{\delta}}_\mu{}^\lambda}{\epsilon}\, \varepsilon_{\lambda\nu}\, \frac{\partial^\nu U'}{4\pi}$$

But this is finite!! Because

$$\hat{\hat{\delta}}_\mu{}^\lambda \varepsilon_{\lambda\nu} = \epsilon\, \varepsilon_{\mu\nu} + \hat{\hat{\delta}}_\nu{}^\lambda \varepsilon_{\lambda\mu} \quad \text{and} \quad \hat{\hat{\delta}}_\mu{}^\lambda \partial^\nu U' = 0$$

Lesson: begin with "early easy" current. Then use susy to get corrections to higher currents.

Variation of $\not\partial j_\mu$ gives $T_{\mu\nu}x^\nu$ and

$j^{(\zeta)}{}_\nu{}^\mu = x^\rho \varepsilon_{\rho\nu}\zeta^\mu$: the conformal central charge current.

Since $\partial_\mu j^{(\zeta)}{}_\nu{}^\mu = \varepsilon_{\mu\nu}\zeta^\mu$ we see that:

The anomaly in the conformal central charge current is equal to the finite one-loop correction to the ordinary central charge current.

Results for vortices:

$N = 2$ susy vortex in $d = 2 + 1$ from $N = 1$ in $d = 3 + 1$
No divs in odd dims in dim. reg.

•Vanishing tadpole condition yields finite renormalization

δv^2 of v_{Higgs}:  = 0

•Difference of spectral densities for bosons and fermions $\rho(k^2)$ vanishes in this case, so $M^{(1)}$ due to δv^2.

•Again $Z_{ren} = Z + P_y$, but now $\langle P_y \rangle = 0$ due to $\rho(k^2) = 0$.

•But the modes for the Higgs fluctuations get an extra phase factor $e^{in\theta}$ from the long-range massless QED background gauge field for the vortex solution. And $Z = \int \partial_k \left(\varepsilon_{k\ell} \zeta_\ell \right)$

with $\zeta_\ell = \ldots + \phi^\dagger D_\ell \phi$

yields $$\int_0^{2\pi} d\theta \eta^\dagger \partial_\theta \eta = 2\pi n \langle \eta^\dagger \eta \rangle_{r \to \infty} = 2\pi n \, \delta v^2$$

So: no anomalies, but still $M^{(1)} = Z^{(1)} \neq 0$

Results for monopoles:

N=2 case: as kinks

• $M^{(1)}$ = zero point energies + boundary terms

⇧ ⇧

$\Delta\rho(k^2) \neq 0$ from index theorem zero

• Z factors for $M_{cl} = 4\pi v_0/g_0$. Use renormalization theory for background field method.

• Again $Z^{(1)}$ from $\langle P_y \rangle$. BPS holds

N = 4 case: expect $M^{(1)} = Z^{(1)} = 0$. Correct, but…
Zero point energies = 0. But boundary terms ≠ 0 and divergent!
Ordinary renorm. of no help (all Z = 1), but extra-ordinary ren. does help:

$$j^\mu = (j^\mu_{\text{imp}} - \Delta j^\mu_{\text{imp}}) \mp (Z_{\text{imp}} - 1)\Delta j^\mu_{\text{imp}}$$

The improved currents are finite, but the improvement terms renormalize muliplicatively as composite operators.

Then BPS holds again.

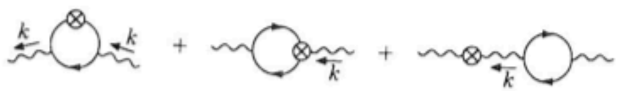
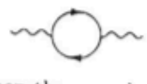


Figure 19.10. One-loop diagrams contributing to the anomalous trace of $\Theta^{\mu\nu}$.

19.10 cancel: These diagrams have the same structure, since the first has an extra propagator and an extra factor $\not p$ from the operator matrix element, and opposite overall signs.

The first term on the left-hand side of (19.159) is unexpected, since it apparently vanishes in four dimensions. However, the fermion loop diagram is divergent, and in dimensional regularization, this introduces a factor $1/(2-d/2)$. As a result, this diagram gives a nonzero contribution to the operator matrix element. In massless QED, the fermion loop diagram has the value



$$= -i(k^2 g^{\mu\nu} - k^\mu k^\nu)\frac{4}{3(4\pi)^2}\left(\Gamma(2-\tfrac{d}{2}) + (\text{finite})\right). \quad (19.160)$$

Then the complete expression for the third diagram in Fig. 19.10 is

$$\int \frac{d^4k}{(2\pi)^4} A_\mu(-k)\left(-2\frac{4-d}{4}\right)(k^2 g^{\mu\nu} - k^\mu k^\nu)\frac{-i}{k^2}\left(-ik^2\frac{4}{3(4\pi)^2}\frac{1}{2-d/2}\right)A_\nu(k). \quad (19.161)$$

This is of the form of (19.158), with

$$C = \frac{1}{12\pi^2}, \quad (19.162)$$

which is indeed the first β function coefficient in massless QED.

This discussion generalizes to QCD and other gauge theories. In a non-Abelian gauge theory, $\Theta^{\mu\nu}$ is given by the obvious generalization of (19.150) with the Abelian field-strength tensor $F_{\mu\nu}$ replaced by the non-Abelian expression $F^a_{\mu\nu}$. The trace of $\Theta^{\mu\nu}$ is again given by

$$\Theta^\mu{}_\mu = -\frac{4-d}{4}(F^a_{\lambda\sigma})^2, \quad (19.163)$$

plus terms that vanish by the equations of motion. In the background field gauge, the one-loop diagrams with the operator $\Theta^\mu{}_\mu$ inserted into the loop cancel as above. We saw in Section 16.6 that the two-point functions in this gauge sum to

$$= -i(k^2 g^{\mu\nu} - k^\mu k^\nu)\left[\frac{-b_0}{(4\pi)^2}\right]\left(\Gamma(2-\tfrac{d}{2}) + (\text{finite})\right), \quad (19.164)$$

where $\beta(g) = -b_0 g^3/(4\pi)^2$. Following through the logic of the previous paragraph, we again find the result (19.158) with the identification of C as the first β function coefficient.

Anomalies in N=2 SYM

N=2 Multiplets

$$D^{ij}\mathcal{J} = \mathcal{L}^{ij} \qquad\qquad D^{ij} = D^{i\alpha}D^j_\alpha = D^{ji}$$

$$\mathcal{J} = a\,\mathrm{tr}\,W\bar{W}$$ (real a)

$$\mathcal{L}^{ij} = \left(D^{ij}\,\mathrm{tr}\,W^2 + \bar{D}^{ij}\,\mathrm{tr}\,\bar{W}^2\right)$$

N=1 multiplets

$$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha X$$

$$J_{\alpha\dot{\alpha}} = \left(-\tfrac{1}{3}\,[\,D_\alpha, \bar{D}_{\dot{\alpha}}\,] + D^2_\alpha \bar{D}_{2\dot{\alpha}}\right)\ \mathcal{J}|$$

$$X = \mathcal{L}^{22} = D^{22}\,\mathrm{tr}\,W^2| + \bar{D}^{22}\,\mathrm{tr}\,\bar{W}^2|$$

$$= W^\alpha W_\alpha + \phi \bar{D}^2 \bar{\phi} + \bar{D}^{22}\,\mathrm{tr}\,\bar{W}^2|$$

⇧	⇧	⇧
N = 1 SYM anomaly multiplet	N=1 chiral anomaly multiplet	EXTRA!

General questions

1) Is there path integral derivation of anomalies for proper graphs?

2) What if the anomaly contains part of a field equation?

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