PHYS 514 GENERAL RELATIVITY AND COSMOLOGY 2018
READING and PROBLEM SET 4

READING: Textbook, Sections 4.1 - 4.5

PROBLEMS, due February 8 2018 (in class):

1. In class I defined the covariant derivative of a vector field. Prove that the coefficients of the covariant derivative transform as a tensor of type (1,1) under a coordinate transformation. Show that the partial derivatives of such a vector field do not transform as a tensor.

2. In class I showed that the parallel transport along a closed curve spanned by two coordinate vector fields X and Y is given by the operator

$$R(X,Y) = \nabla_X \nabla_Y - \nabla_Y \nabla_X$$

. Show that this leads to, for general vector fields X and Y, to the result

$$R(X,Y) \nabla_X \nabla_Y - \nabla_Y \nabla_X - \nabla_{[X,Y]}$$

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3. Consider a two dimensional cone. Use polar coordinates with origin taken to be the tip of the cone (the singular point of this "manifold" - it has manifold structure everywhere except at the tip of the cone). Write down the metric on the surface of the cone induced by the Euclidean metric of the three-dimensional space in which the cone lives. Find the geodesics between two points an angle $\delta\phi$ apart at the same radius.

4. Consider the metric

$$ds^2 = dt^2 - a(t)^2 [dx^2 + dy^2 + dz^2] \,,$$

where $a(t)$ in an increasing function of time (and x, y, z are Euclidean spatial coordinates), which describes a homogeneous and isotropic expanding universe.
a) Compute the Christoffel symbols (check your answers with those in the text).
b) Write down the equation of motion of a point particle in this metric in the absence of external forces and derive the time dependence of the physical velocity. Comment on your result.

5/6. Compute the Riemann tensor elements of the metric of the previous problem.