PHYS 514 GENERAL RELATIVITY AND COSMOLOGY 2018
READING and PROBLEM SET 8

READING: Textbook, Sections 7.1, 7.2, 7.4 - 7.7, 8.1 - 8.4.

PROBLEMS, due March 29, 2018 (in my mail box):

1/2. Consider two inspiring black holes with mass $10M_0$, where M_0 is the mass of the sun. Assume the system is located at a distance from us which is equal to our distance from the centre of our galaxy. Assume that the initial separation is $100r_s$, where r_s is the Schwarzschild radius. In the weak field approximation, compute the gravitational wave amplitude $h(t)$ at the LIGO site as a function of time, making use of the quadrupole radiation formula. Then, using the formula for the radiated power derived in class, compute the gradual decay of the orbital radius $r(t)$ (using Newtonian physics to relate the energy density radiated to the change in the orbital radius). The approximations cease to be valid once $r(t)$ approaches r_s, so stop the calculation before that point.

3/4. In class I justified the ansatz for a cosmological metric in the form

$$ds^2 = dt^2 - a(t)^2(d\psi^2 + f_k^2(\psi)[d\theta^2 + \sin^2\theta d\varphi^2])$$

and sketched the derivation of the Einstein tensor in the tetrad basis. Perform the explicit calculation.

5. Consider the thermal equilibrium distribution for Bose and Fermi particles. For bosons this is the black body spectrum.
a) Show that if a particle species decouples at a given time t_1 with a black body distribution, it will maintain the black body distribution with a temperature which is simply redshifted by the expansion of the Universe.
b) Is the spectral form preserved for any initial distribution?
c) Is the black body spectrum the unique spectrum for which its form is preserved under expansion?

6. In class I mentioned that it is not possible in Standard Big Bang cosmology to explain the observed spatial flatness of the universe today. To understand this problem, compute the temperature evolution of $\Omega - 1$, where $\Omega = \rho/\rho_c$, and ρ_c is the energy density of a spatially flat universe. Show that this quantity decreases rapidly as the temperature increases. What does this mean for the initial conditions in the very early universe if these are to reproduce what we see today?