PHYS 514 GENERAL RELATIVITY AND COSMOLOGY 2018
MIDTERM EXAM, Tuesday, February 27, 2018

This is an in-class open-book exam. You are allowed to use the textbook and your notes. NO CELL PHONES! I am giving more problems than I expect anyone to be able to solve in the given time. The idea is to give you some choice. N.B. **Please describe in words how you plan to answer the questions. Do not start immediately by writing down equations**.

1. (5 pts) Consider a variant of the Pound-Rebka experiment. An experimenter on a tower of height h above the surface of the Earth lets a ball of mass m fall to the ground from rest. At the surface, the total energy is “magically” transformed into energy of a single photon which travels back up to the experimenter, where the energy is transformed back to rest mass. Show that if not for the gravitational redshift, you could construct a perpetual motion machine (a machine from which you could extract an arbitrary amount of energy). What happens if you take the gravitational redshift into account?

2. (5 pts) Consider the surface of the earth (radius R_E) and write down the metric of the surface in some convenient system of coordinates.

a) Determine the distance between two points both at latitude 45^o and with longitudes 0 and 12 hours, respectively (the range of longitudes is from 0 to 24 hrs.).

b) Do the same calculation if the first point is at latitude 45^0 and longitude 0 hrs. and the second point lies on the equator at longitude 6 hrs..

N.B. No calculus is required to solve this problem!

3. (15pts) Consider the flow on a two dimensional plane which for $y < 0$ is uniform in y direction. Between $y = 0$ and $y = 1$ the flow is forced to converge towards the y-axis until the density has doubled. The convergence is smooth. For $y > 1$ the flow is once again uniform along in y direction, maintaining the larger density. Consider the coordinate vector fields ∂_x and ∂_y

a) Sketch the flow lines.

b) Write down a vector field X which generates this flow.

c) What are the covariant derivatives of ∂_x and ∂_y with respect to X? Give a geometrical justification of your answer.

d) What are the Lie derivatives of ∂_x and ∂_y with respect to X? Give a geometrical justification of your answer.

4. (15pts) For the 2-dimensional metric

$$ds^2 = r^2(d\theta^2 + sin^2(\theta)d\varphi^2)\,.$$

Use the tetrad basis to compute the non-vanishing Riemann tensor elements and the Ricci scalar.

5. (10 pts) You are on a planet located at a distance of $r_1 = 100r_s$ from a black hole with Schwarzschild radius r_s. Your parents travel to another planet located at a distance of $r_2 = 9/8r_s$ from the same black hole and spend ten years there (according to their clocks). How much have you aged when your parents return? You can neglect the time it takes to travel to and from the resort. First, explain in words why the parents age more or less. Also, derive a formula which gives you the difference in ageing for general r_1 and r_2.