402-0389 Physics of the Very Early Universe 2019
Problem Set 3 - for Week 3

1. In class I discussed the basic equations which describe matter and gravitational potential fluctuations (Equation 4 in the reading). Verify that the resulting equation for linearlzed fluctuations about a Minkowski background is given by Equation 5.

2. Do the corresponding problem for Newtonian fluctuations about an expanding background cosmology. Namely, starting with Eqs. 4 and 11 of the reading, derive Eq. 13.

3. Consider matter fluctuations in the radiation-dominated phase of Standard Big Bang cosmology. If δ_m denotes the fractional matter density fluctuation, and δ_r the fractional radiation density fluctuations, derive the equation of motion which δ_m obeys. In this analysis, consider matter to have vanishing pressure. Study how the dominant mode of the solution scales in time.

4. When discussing classical relativistic fluctuations, I discussed the linearlized Einstein equations (57). Evaluate these equations in the case of the matter source being a real scalar field. Demonstrate how the equations can be combined to yield Eq. 62.

5. Consider large field inflation with potential

$$V(\varphi) = \tfrac{1}{2} m^2 \varphi^2 \,.$$

Compute the power spectrum of cosmological perturbations as a function of m, and determine the value of m by requiring that the amplitude of the power spectrum match the observed valuie (you can take the value to be 10^{-8}).

6. Solve the similar problem in the case of large field inflation with potential

$$V(\varphi) = \tfrac{1}{4} \lambda \varphi^4 \,.$$