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Ultra-cold Trapped Gas of Neutral Atoms with Tunable Interaction

December 4, 2004

1. Quantum atomic gas
2. Interaction effects in a dilute gas.
3. Feshbach Resonance.
4. Stability of a Fermi gas
5. Universality near resonance.
6. BCS-BEC cross-over
   using Feshbach Resonance.

For Quantum Effects at temperature T,

: de Broglie thermal wave length $\lambda \equiv \frac{h}{\sqrt{2\pi M k_B T}} \geq$ average distance between atoms, $\approx n^{-1/3}$,

$$n^{1/3}\lambda \geq 1 \qquad (1)$$

For a dilute gas of Rb^{87} with number density $n = 10^{13}/\mathrm{cc}$, the temperature has to be lowered to 100 nK to obtain $\lambda \approx 6 \times 10^{-5}$ cm, so that $\lambda n^{1/3} = 1.3$

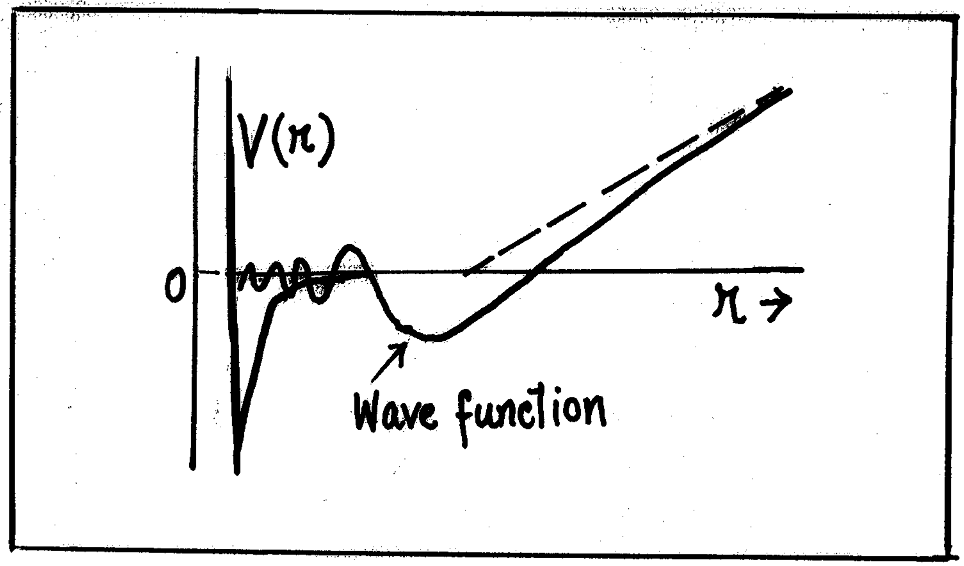
Diluteness ensures less recombination & less three-body effects.

Interaction Effects:

(in Dilute Gas)

For $n = 10^{12}/\text{cc}$

$\bar{l} \simeq 10^{-4}\ \text{cm} \gg \text{Range } R$



$$H = \sum_{i=1}^{N}\left(-\frac{\hbar^2}{2M}\nabla_i^2 + \frac{1}{2}M\omega_i^2 r_i^2\right) + \frac{4\pi\hbar^2 a}{M}\sum_{i<j}\delta(\mathbf{r}_i - \mathbf{r}_j) \qquad (2)$$

For attractive interaction (negative a), a Bose gas becomes unstable for

$$\kappa = \frac{N|a|}{b} = 0.46 \quad (0.459 \pm 0.012 \pm 0.054)$$

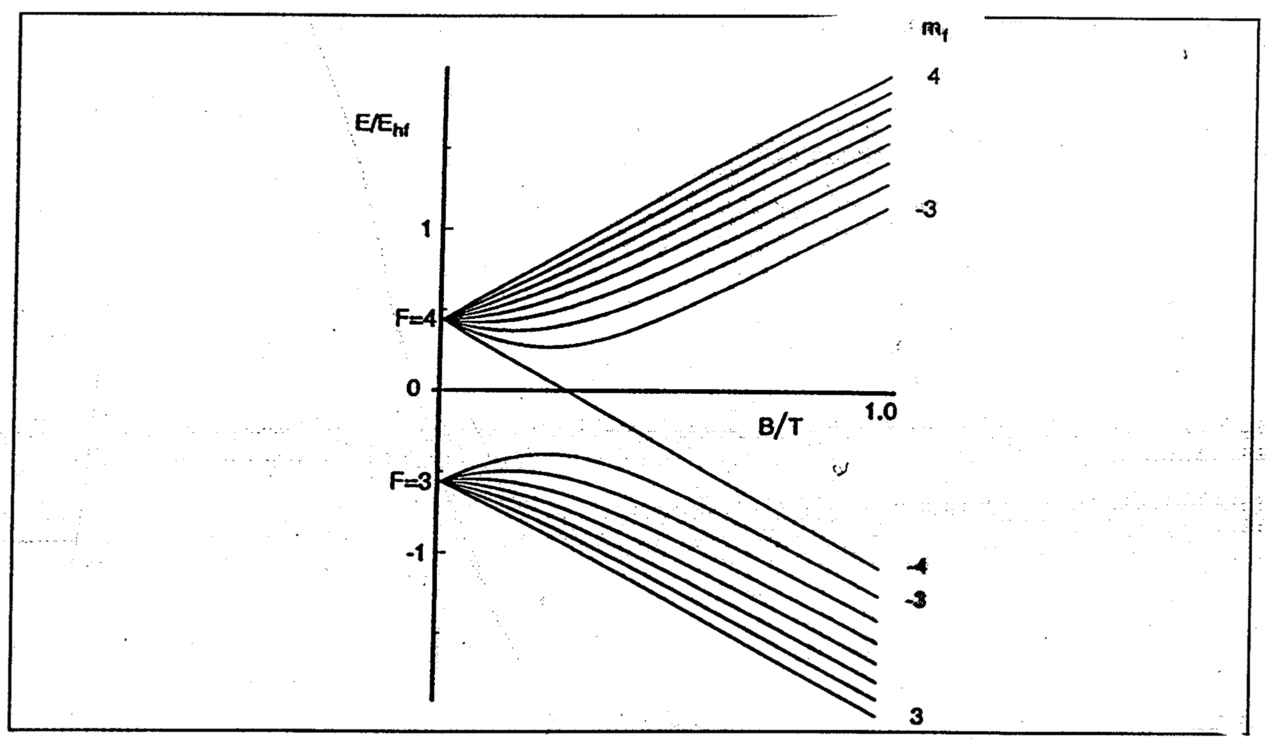
for ^{85}Rb

Roberts et. al, PRL 86, 4211 (2001)

What about a Fermi gas ?

Magnetic Properties of Cs^{133}

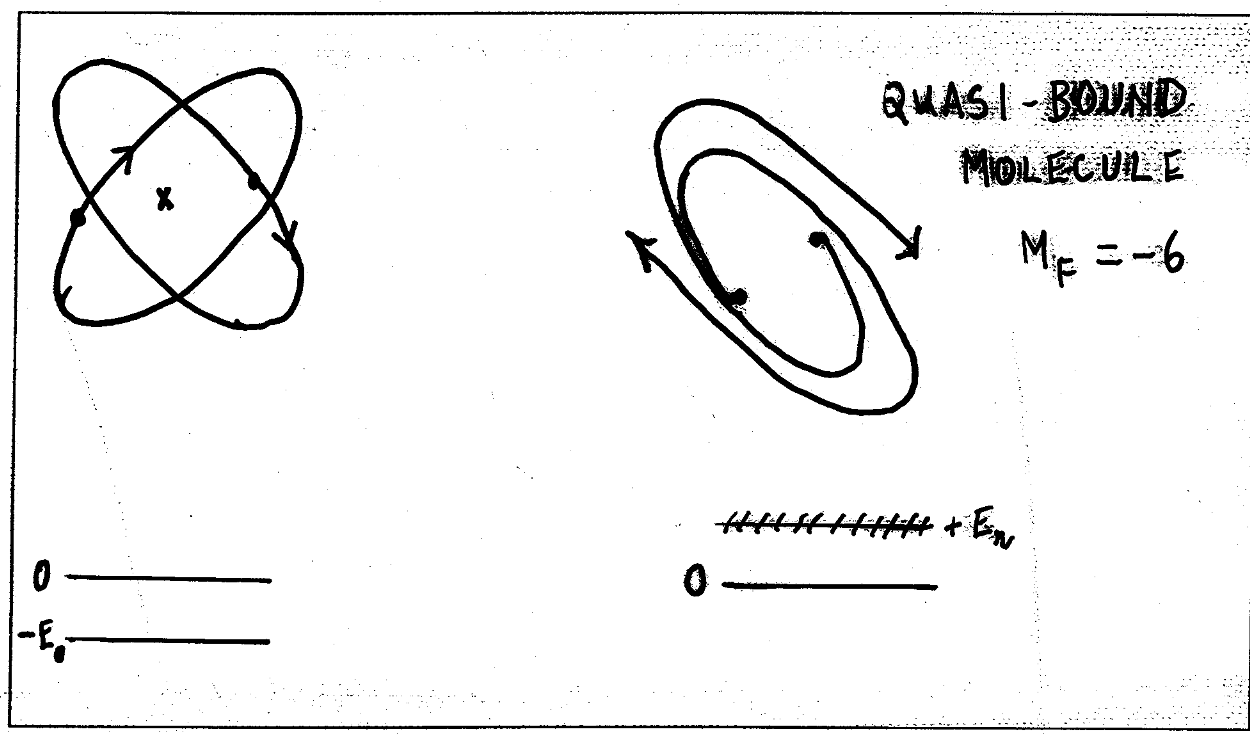
It has $(133-55)=78$ neutrons. Nuclear spin $I=7/2$, electronic spin $S=1/2$. Total spin $f=4$ or 3. The $z-$ component of the spin, m_f, can take $(2f+1)$ discrete values, from $f, f-1, ..-f$. In a magnetic field, these states split up in energy.



Cs atoms polarized in the state $(f=3, m_f=-3)$ at low magnetic fields will prefer to collect there.

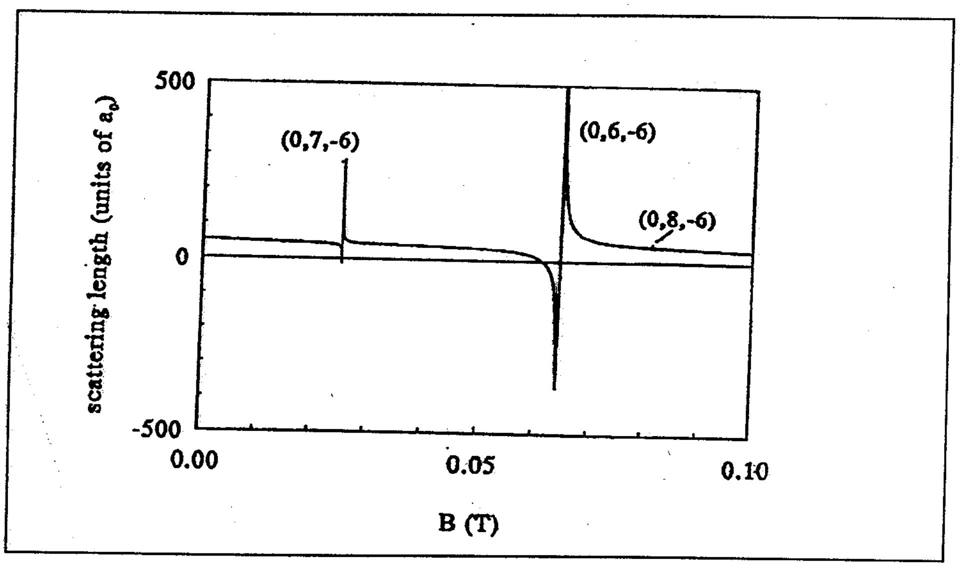
Feshbach Resonance and the tuning of interatomic interaction

- Bound States and Resonances



- Feshbach Resonance

Consider the elastic collision of two Cs atoms, each in the $|3,-3>$ state. when the energy of relative motion of the two atoms is close to one of the "resonance" energies the diatomic molecule, the scattering length changes drastically as shown below.



Cs
(Theoretical)
Tiesinga et al
PRA 47, 4114 (1993).

The resonance energy, in turn, can be tuned by sweeping the external magnetic field. This gives a practical way of tuning the interaction between two atoms. The effective interaction between two atoms can be changed from repulsive to attractive by varying the magnetic field.

Expt. ^{85}Rb $(f=2, m_f=-2)$ atoms, resonance tuning [redacted]
at $B = 164 \pm 7$ Gauss. Courteille et al, PRL 81, 69 (1998)

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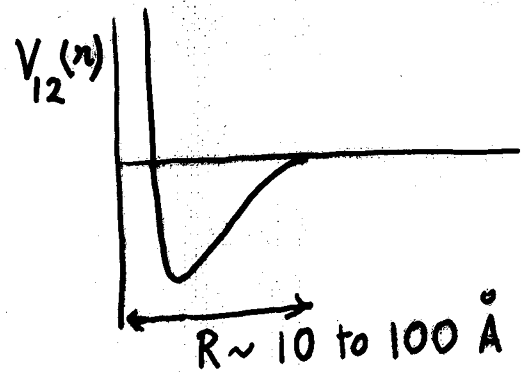
STABILITY OF A FERMI GAS WITH ATTRACTIVE INTERACTION (large -ve "a").

(Heiselberg, PRA 63 043606)



TWO OTHER LENGTH SCAES AT $T=0$

AVERAGE DISTANCE BETWEEN 2 ATOMS, $\bar{l} \propto n^{-1/3} \sim k_F^{-1}$

SCATTERING LENGTH : $|a|$

DILUTE GAS $\quad \bar{l} \gg R$

LOW DENSITY

$|a| \ll \bar{l}$

i.e., $\quad k_F|a| \ll 1$

INTERMEDIATE DENSITY

$|a| \gg \bar{l}$

ie, $\quad k_F|a| \gg 1$

FERMI GAS AT $T = 0$.

$$n = \nu \frac{k_F^3}{6\pi^2}.$$

$$\Gamma(p, p'; P) = \Gamma_0 + m \sum_k \Gamma_0 (p, k, P) [\cdots] \Gamma(k, p'; P)$$

$$\Gamma_0 = 4\pi a/m$$

Low density ($k_F |a| \ll 1$).

PAIRING GAP

$$\Delta \simeq E_F \exp\left[\frac{\pi}{2 k_F a}\right].$$

$$\text{Energy/fermion} = \frac{E}{N} = E_F \left\{ \frac{3}{5} + (\nu - 1) \frac{2}{3\pi} (k_F a) + (\nu-1) \frac{4(11 - 2\ln 2)}{35\pi^2} (k_F a)^2 + \cdots \right\}$$

SOUND SPEED $= s$

$$s^2 = \frac{1}{m} \left(\frac{\partial P}{\partial n}\right) = \frac{1}{m} \frac{\partial}{\partial n}\left[n^2 \frac{\partial}{\partial n} \left(E/N\right)\right]. = \frac{1}{3} v_F^2 \left[1 + \frac{2}{\pi}(\nu - 1) k_F a + \cdots\right]$$

What happens at $(k_F|a|) \gg 1$?

At Intermediate densities ($k_F |a| \gg 1$)

$$\frac{E}{N} \simeq E_F \left[\frac{3}{5} + \frac{(\nu-1)\frac{2}{3\pi}(k_F a)}{1 - \frac{6}{35\pi}(11 - 2\ln 2)(k_F a)} \right]$$

$$\xrightarrow{\quad} E_F \left[\frac{3}{5} - \frac{35}{9}(11 - 2\ln 2)^{-1}(\nu - 1) \right]$$

Independent of "a" & negative

remains smooth as one goes from the negative "a" side across the resonance.

$\left(\frac{E}{N}\right)$ changes sign at $\nu = \nu_c \simeq 2.5$.

Stable for $\nu = 2$.

(Gehm et al ^{6}Li atoms PR A68, 011401(R), 2003)

Universality near the unitary limit?

(Tin-Lun Ho, PRL 92, 090402; 160404 (2004).

MECHANICAL STABILITY in TF approach

$$\mu = \frac{\hbar^2 k_F^2(\underset{\sim}{x})}{2m} + U_{MF}(\underset{\sim}{x}) + U_{trap}(\underset{\sim}{x})$$

$$U_{MF}(\underset{\sim}{x}) = \frac{4\pi\hbar^2 a_{eff}}{m}\, n(\underset{\sim}{x})$$

Gehm et al take

$$a_{eff} = \left\langle -\mathrm{Re}\left(\frac{e^{i\delta}\sin\delta}{k}\right)\right\rangle$$

$\langle\cdots\rangle$ is averaging over

$$W(k) = \frac{6}{\pi k_F^3}\left[1 - \frac{3}{2}\frac{k}{k_F} + \frac{1}{2}\left(\frac{k}{k_F}\right)^3\right]\Theta(k_F - k)$$

Taking $k\cot\delta(k) = -\dfrac{1}{a}$,

$$a_{eff} = \left\langle \frac{d\delta(k)}{dk}\right\rangle = \left\langle \frac{a}{1+k^2a^2}\right\rangle$$

$$U_{MF}(\underset{\sim}{x}) = \beta\,\frac{\hbar^2 k_F^2(\underset{\sim}{x})}{2m}$$

$\beta = -0.42$

UNIVERSAL HYPOTHESIS NEAR THE UNITARITY LIMIT
(strongly interacting)

Only relevant length scale is $n^{-1/3}$ (or k_F^{-1}), and thermodyna-mic potentials acquire universal forms (depending on the phase).

1) Superfluid Transition temp. $T_c = \gamma T_F$.
   $\gamma \approx 0.2$ to 0.5

2) $E_{int} = \beta E_F$, $\beta = -0.25$ to -0.4

3) Universal Density Profile of trapped gas.

4) Bose Gas $(E/N) \propto n^{-2/3}$ (like ~~fermions~~).

Experimental papers quoted in
Tin-Lun Ho, PRL 92, 090402 (2004).

Universality in the Boltzmann Regime ?

$(n\lambda^3 \ll 1)$

Ho + Mueller, PRL 92, 160404 (2004).

Second Virial coefficient $b_2(T)$

$$\frac{PV}{N\tau} = 1 + b_2(T)\,\lambda^3 \left(\frac{N}{V}\right) + \cdots$$

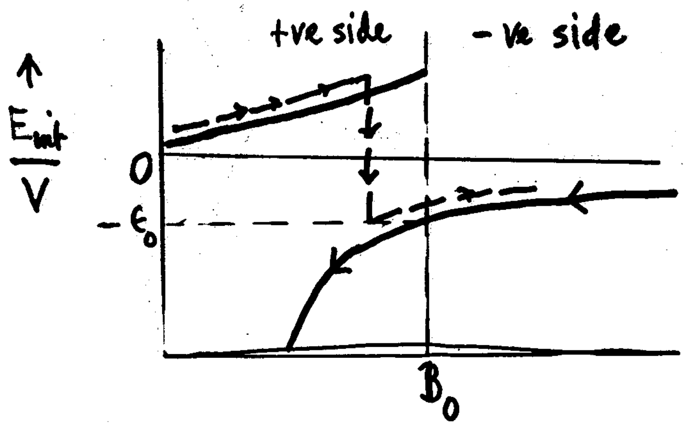
$$b_2(T) = \sum_b e^{|E_b|/\tau} + \int_0^\infty \frac{dk}{\pi}\,\frac{d\delta_0(k)}{dk}\, e^{-\frac{\hbar^2 k^2}{m\tau}}$$

$$= \sum_b e^{|E_b|/\tau} - \frac{\mathrm{sgn}(a)}{2}\left[1-\mathrm{erf}(x)\right] e^{x^2}, \quad x = \frac{\lambda}{\sqrt{2\pi}\,a}$$

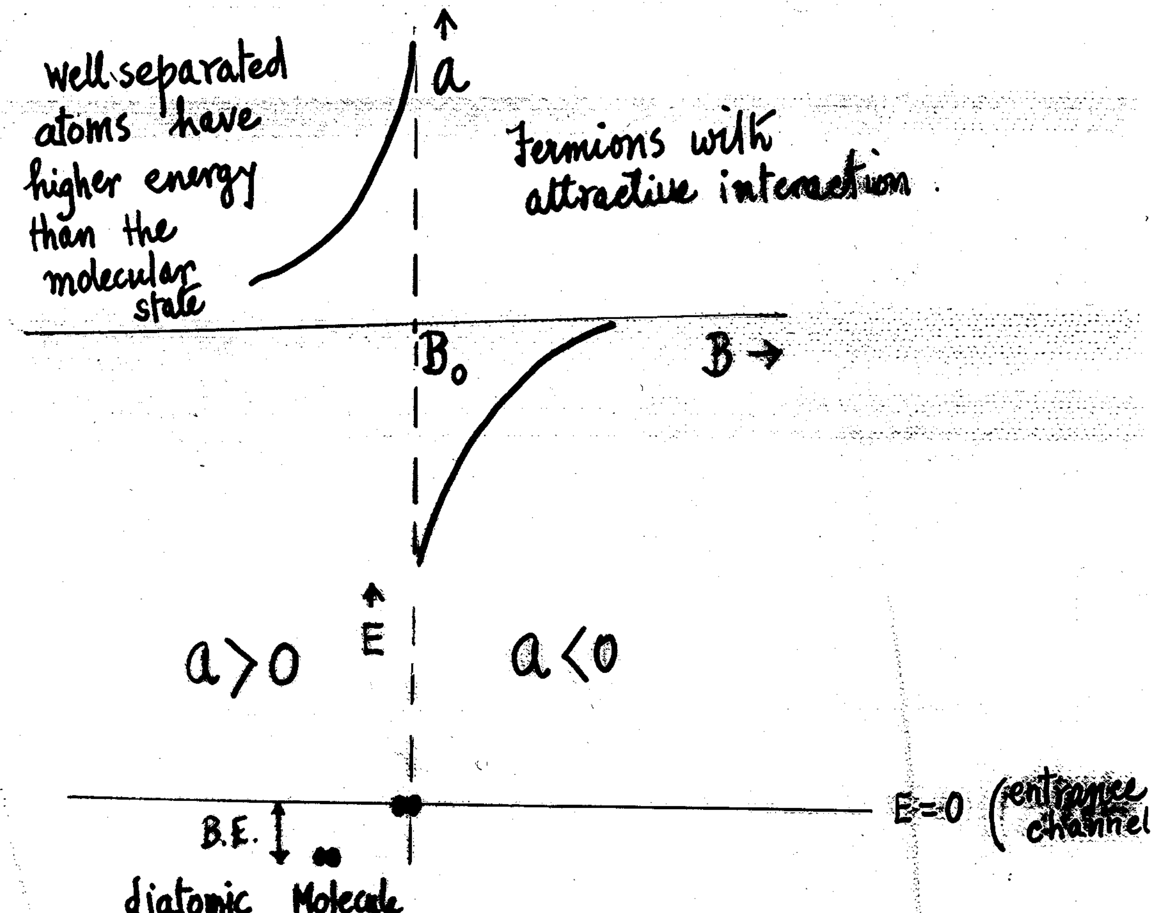
$$\frac{E}{V} = \frac{3n\tau}{2}\left(1 + \frac{n\lambda^3}{2^{7/2}}\right) + E_{int}$$

$$\frac{E_{int}}{V} = \frac{3n\tau}{2}\,(n\lambda^3)\left[-\frac{b_2}{\sqrt{2}} + \frac{\sqrt{2}}{3}\,T\,\frac{\partial b_2}{\partial T}\right] \rightarrow -\epsilon_0$$

at resonance, $b_2 = 1/2$, $\frac{\partial b_2}{\partial T} = 0$. (-ve side)



BCS - BEC CROSS-OVER USING FESHBACH RESONANCE



Regan et al
Nature 424, 47
(2003)
^{40}K atoms
$m_f = -9/2, -5/2$

BEC side

BCS (fermionic pairing)
many-body coherence

Bosonic $^{40}K_2$ molecules undergo BEC.

also $^6Li_2$
by Zweirlein et al
PRL 91, 250401 (2004)
(Bimodal velocity distribution)

Regan et al
PRL 92, 040403 (2004)