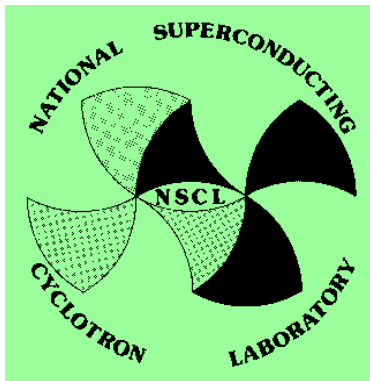


*Betty Tsang*

*Subal Das Gupta Festschrift  
McGill University, Montreal  
Dec 4, 2004*

The National Superconducting  
Cyclotron Laboratory  
Michigan State University

*Subal Das Gupta Festschrift  
McGill University, Montreal  
Dec 4, 2004*



The National Superconducting  
Cyclotron Laboratory  
Michigan State University

# Isoscaling in Nuclear Reactions

*Betty Tsang*

*Subal Das Gupta Festschrift*

*McGill University, Montreal*

*Dec 4, 2004*



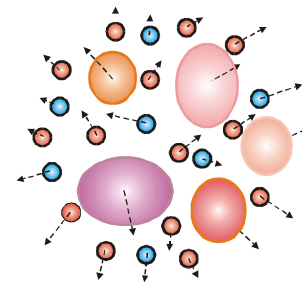
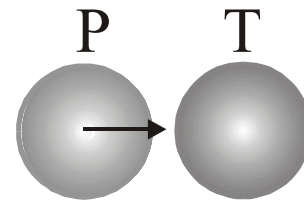
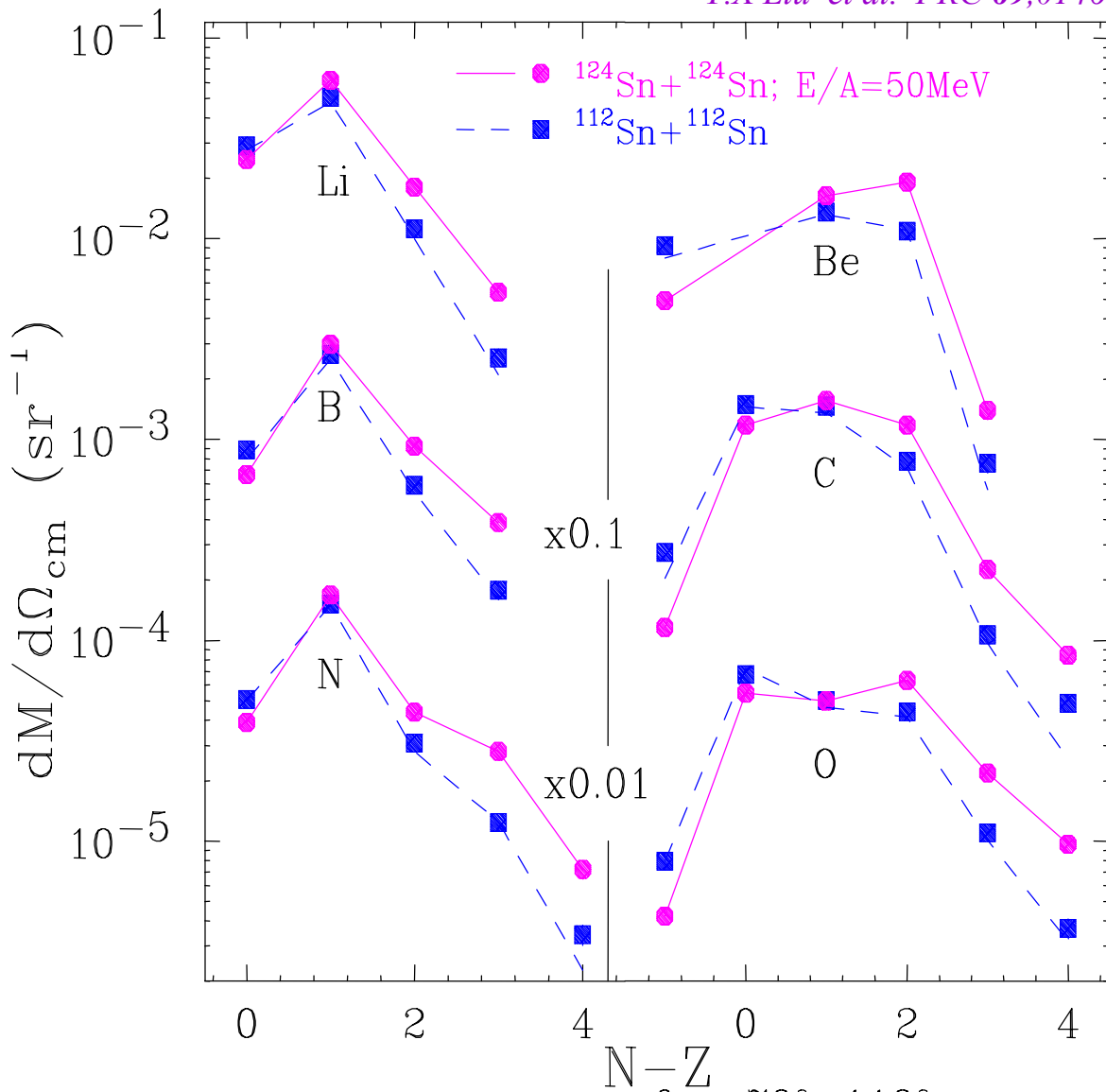
The National Superconducting  
Cyclotron Laboratory  
Michigan State University

# Outline

- What is isoscaling?
- Where is it observed?
  - From multifragmentation to binary reactions
- What is the physics of isoscaling? – Subal D.
- What can we learn from it?
  - Density dependence of symmetry energy.

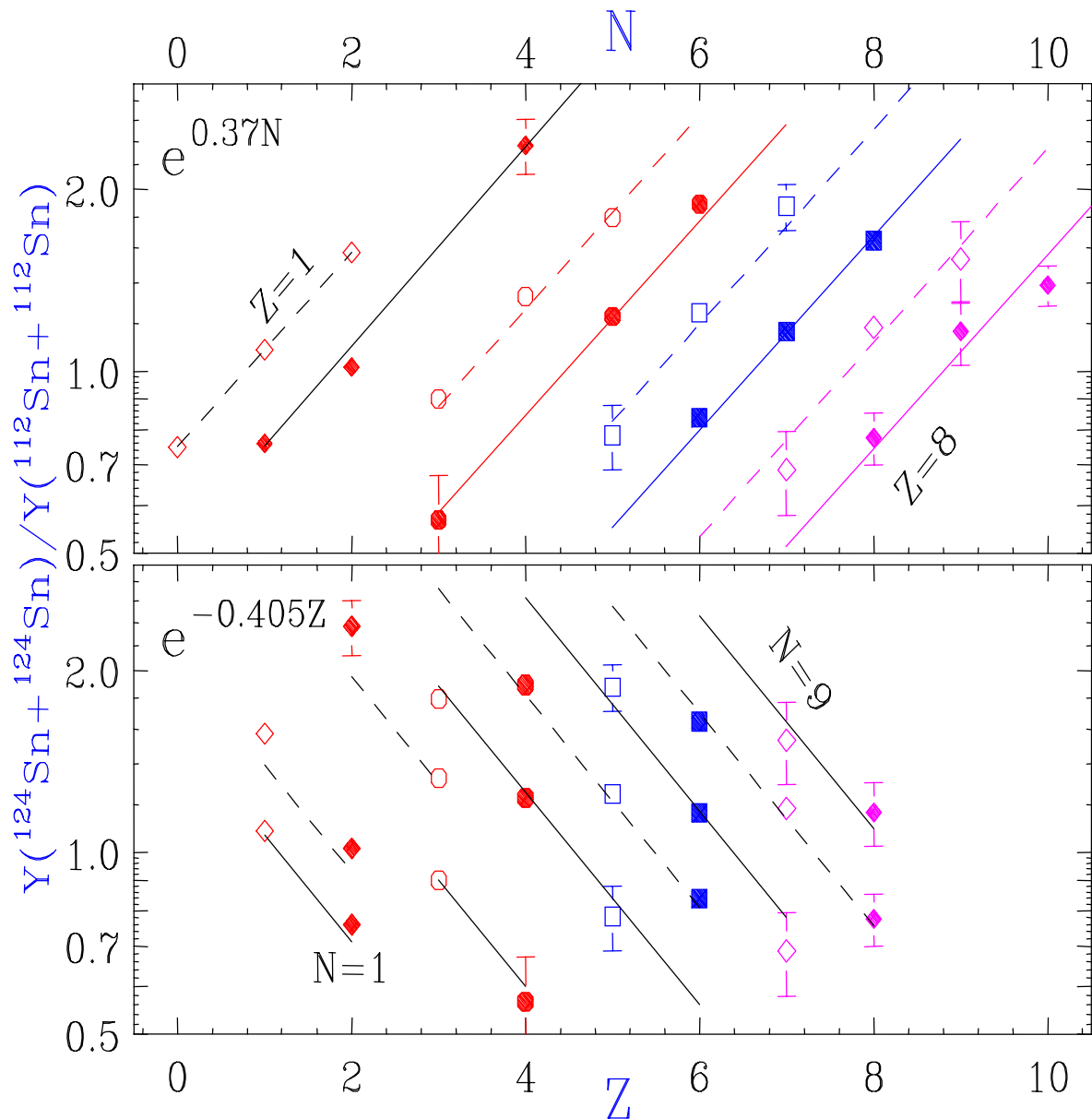
# Isoscaling constructed from Measured Isotopic yields

*T.X Liu et al. PRC 69,014603*



**Multifragmentation**

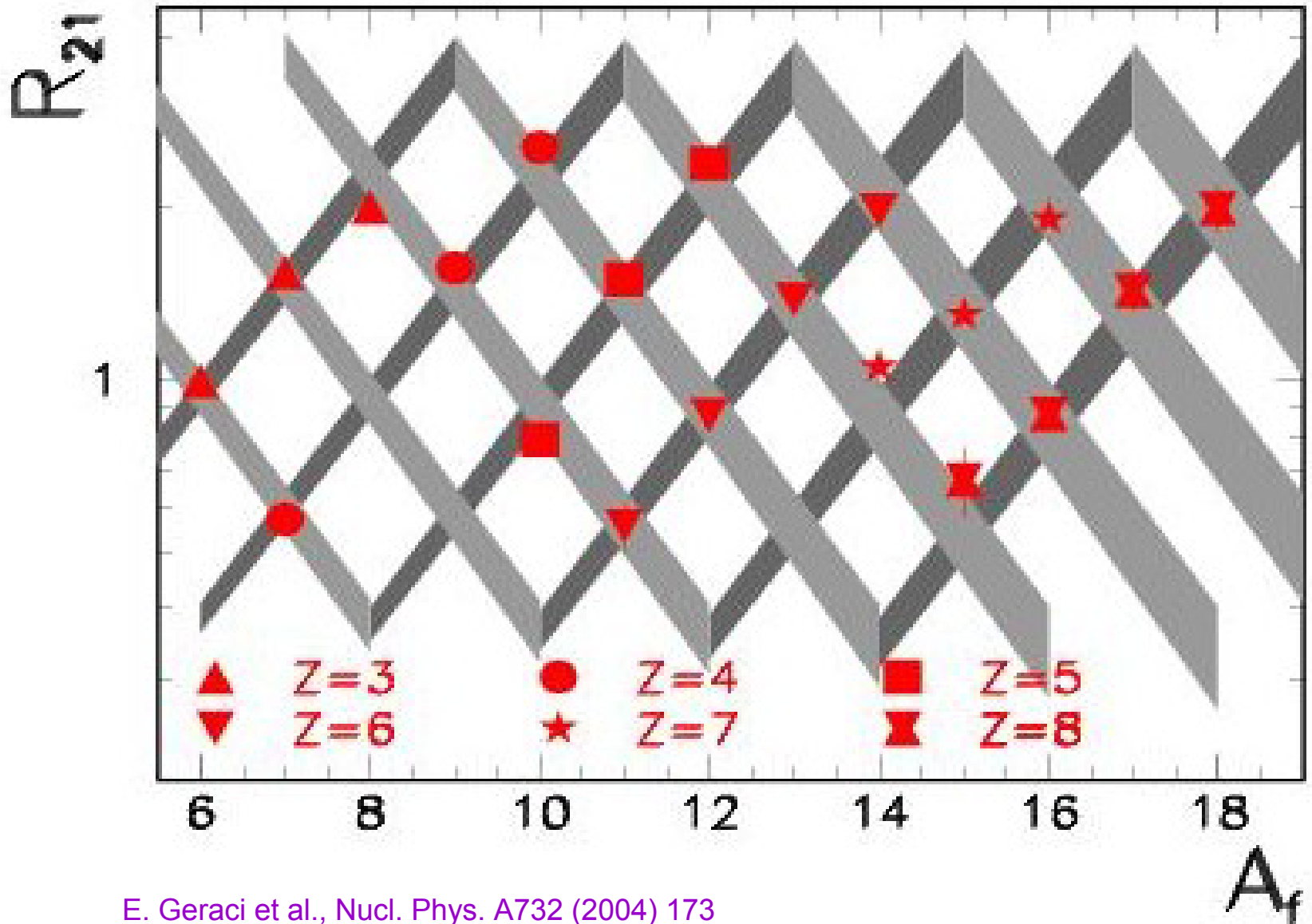
# Isoscaling from Relative Isotope Ratios



$$R_{21} = Y_2 / Y_1$$

$$\propto e^{N\Delta\mu_n / T + Z\Delta\mu_p / T}$$

$^{112}\text{Sn}+^{58}\text{Ni}$  and  $^{124}\text{Sn}+^{64}\text{Ni}$  at 35 A MeV; Central collisions, CHIMERA-REVERSE experiment



# Simple derivation of the isoscaling law

- Basic trends from Grand Canonical ensemble:
  - Yields  $\propto$  term with exponential dependence on the chemical potentials.

$$Y(N, Z)_{HOT} \propto \exp\left(\left[\mu_n N + \mu_p Z + B(N, Z)\right] / T\right) \cdot Z_{int}(N, Z)$$

$$\text{where } Z_{int} = \sum_i (2J_i + 1) \exp(-E_i^* / T)$$

$$Y(N, Z)_{COLD} = Y(N, Z)_{HOT} * f(N, Z)$$

feeding correction

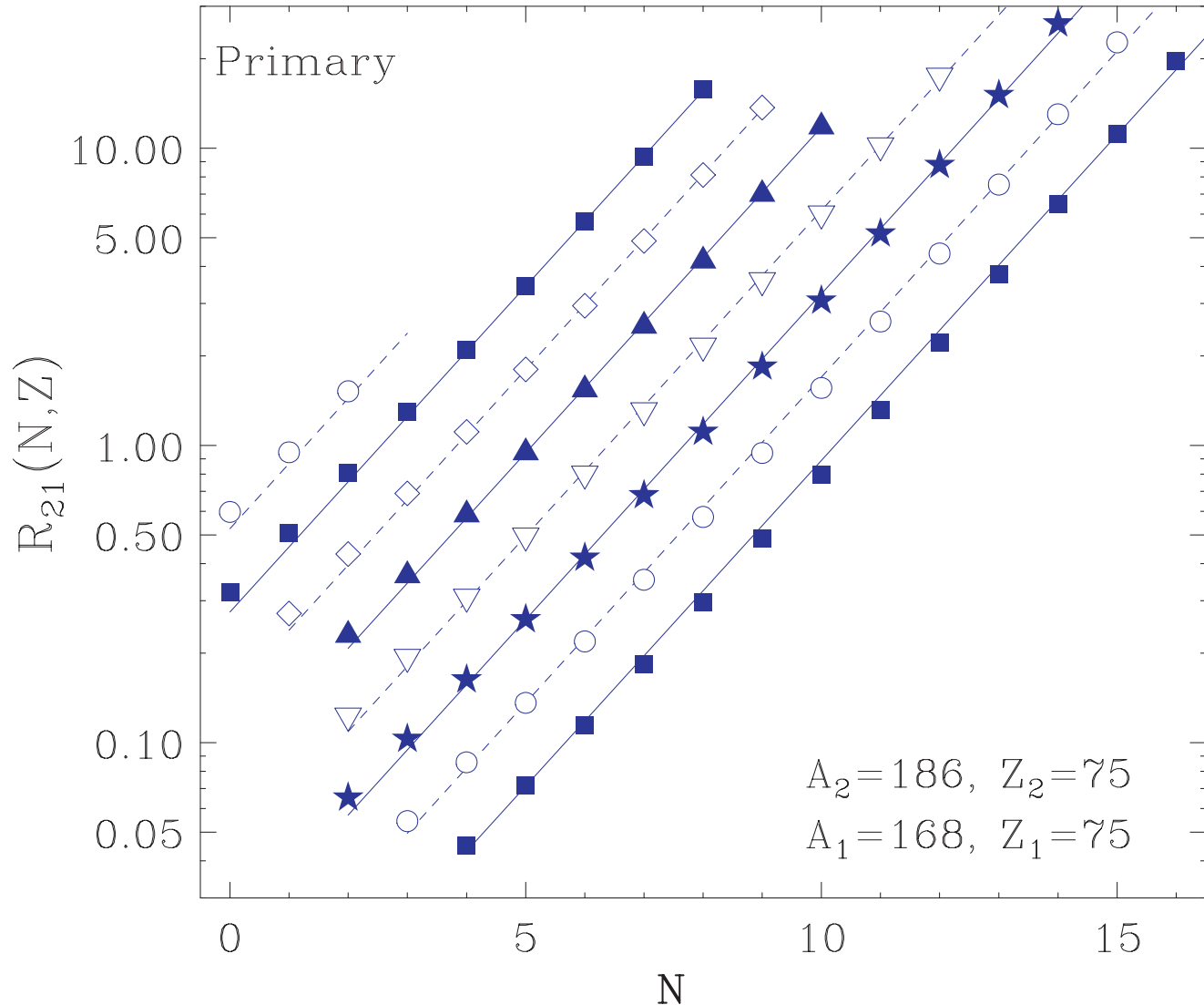
- Ratios to reduce sensitivity to secondary decays:

$$R_{21}(N, Z) = \frac{Y_2(N, Z)}{Y_1(N, Z)} \approx C \cdot e^{N \Delta \mu_n / T + Z \Delta \mu_p / T}$$

- Scaling parameters  $C$ ,  $\alpha = \Delta \mu_n / T$ ,  $\beta = \Delta \mu_p / T$



Isoscaling in statistical models : PRC 64, 054615 (2001)



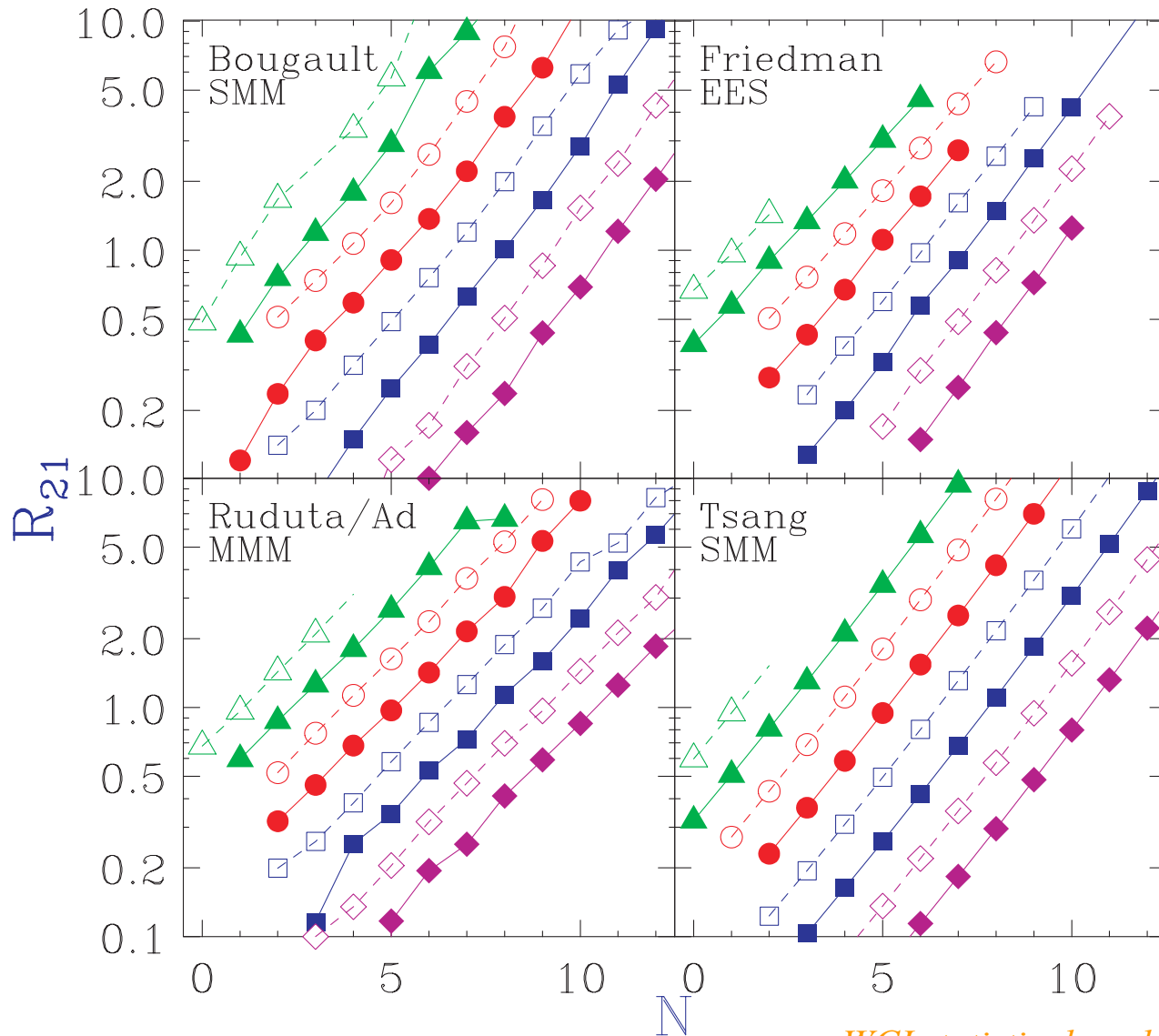
1. Temperature dependence
2. Density dependence
3. Size and charge dependence
4.  $\alpha \propto \delta_1 - \delta_2$   
 $\delta = (N-Z)/(N+Z)$

*Aspects of statistical model for multifragmentation*  
*P. Bhattacharyya, S. Das Gupta, and A. Z. Mekjian*  
*PRC 60, 064625 (1999)*

# Isoscaling in statistical models

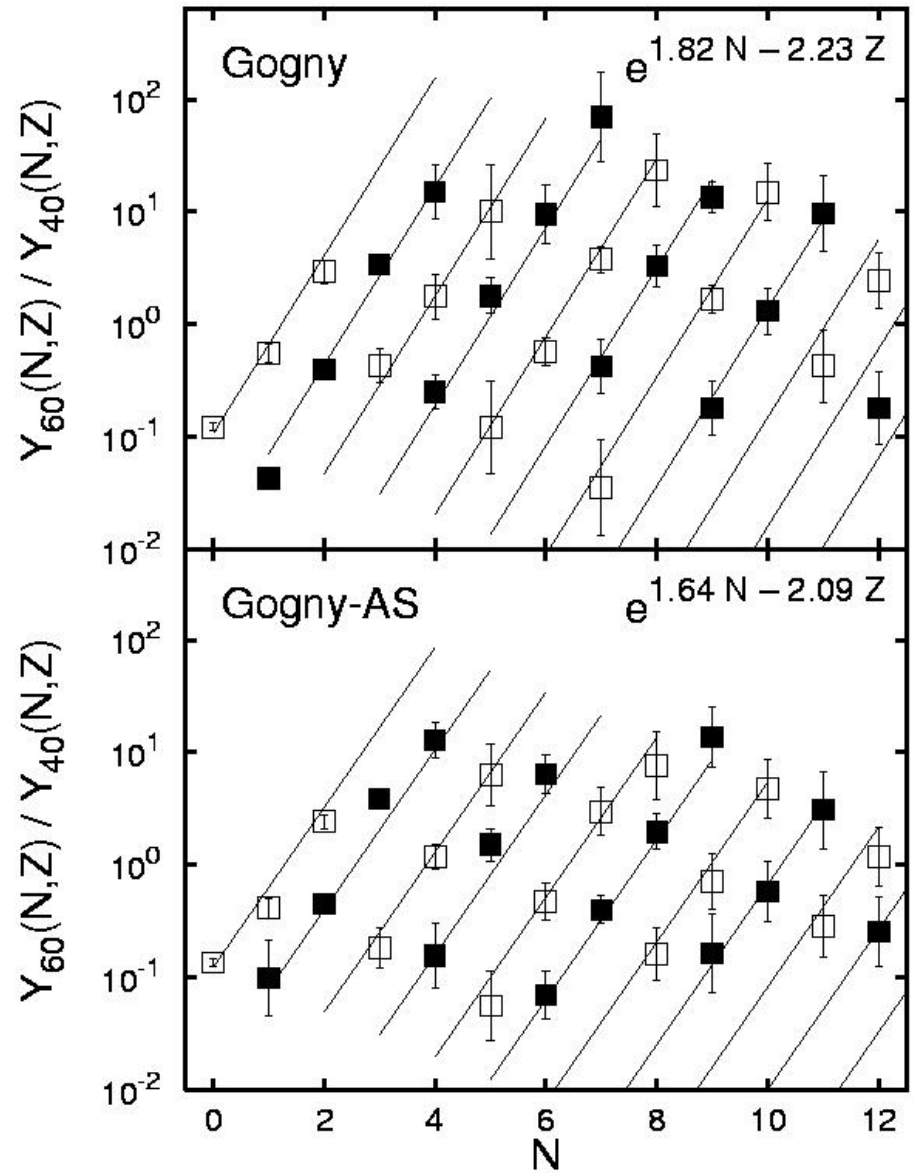
Primary distributions show good isoscaling

$A_2=186, Z_2=75; A_1=168, Z_1=75$





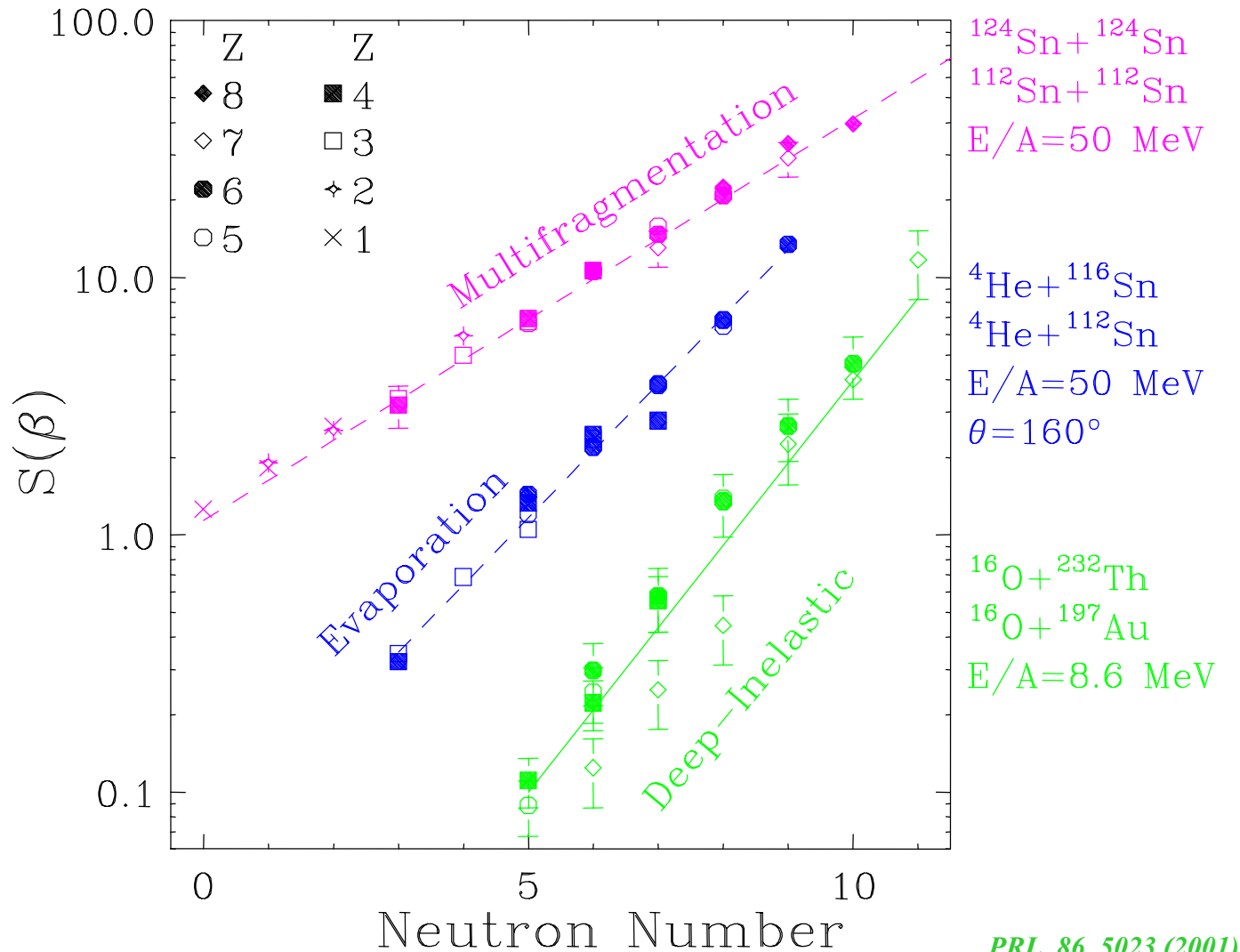
# Isoscaling in Antisymmetrized Molecular Dynamical model



# Isoscaling observed in many reactions

$$Y_2 / Y_1 \propto e^{(N\Delta\mu_n + Z\Delta\mu_p) / T}$$

More Data

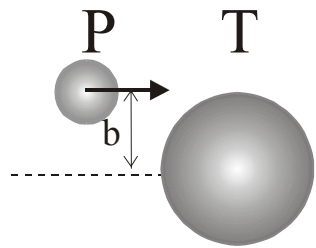


$^{58}\text{Ni} + ^{58}\text{Ni}$   
 $^{58}\text{Fe} + ^{58}\text{Fe}$   
 $E/A = 30, 40, 47$   
*Shetty et al (2003)*

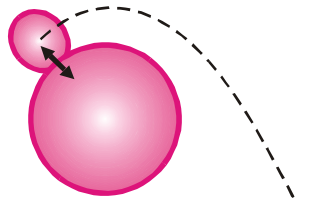
$p, ^4\text{He} + ^{116}\text{Sn}$   
 $p, ^4\text{He} + ^{124}\text{Sn}$   
 $E/A > 1$  GeV  
*Botvina, Trautmann (2002)*

$^{16}\text{O} + ^{232}\text{Th}$   
 $^{16}\text{O} + ^{197}\text{Au}$   
 $E/A = 8.6$  MeV

$^{86}\text{Kr} + ^{116}\text{Sn}, ^{124}\text{Sn}$   
 $^{86}\text{Kr} + ^{58}\text{Ni}, ^{64}\text{Ni}$   
 $E/A = 35$  MeV  
*Souliotis et al (2003)*



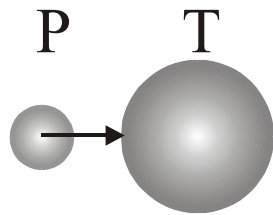
## DIC



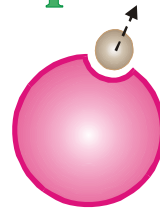
Q Value, Sep. E

$E_{\text{Coul}}$   $E_{\text{sym}}$

$$R_{21} \propto \exp[(-\Delta S_n \cdot N - \Delta S_p \cdot Z)/T]$$



## Evaporation

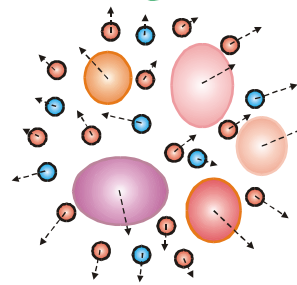
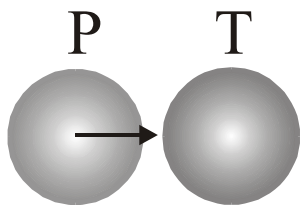


Separation Energy

$E_{\text{Coul}}$   $E_{\text{sym}}$

$$R_{21} \propto \exp[((-\Delta S_n + \Delta f_n^*) \cdot N + (-\Delta S_p + \Delta f_p^* + \Delta \Phi) \cdot Z)/T]$$

## Multifragmentation

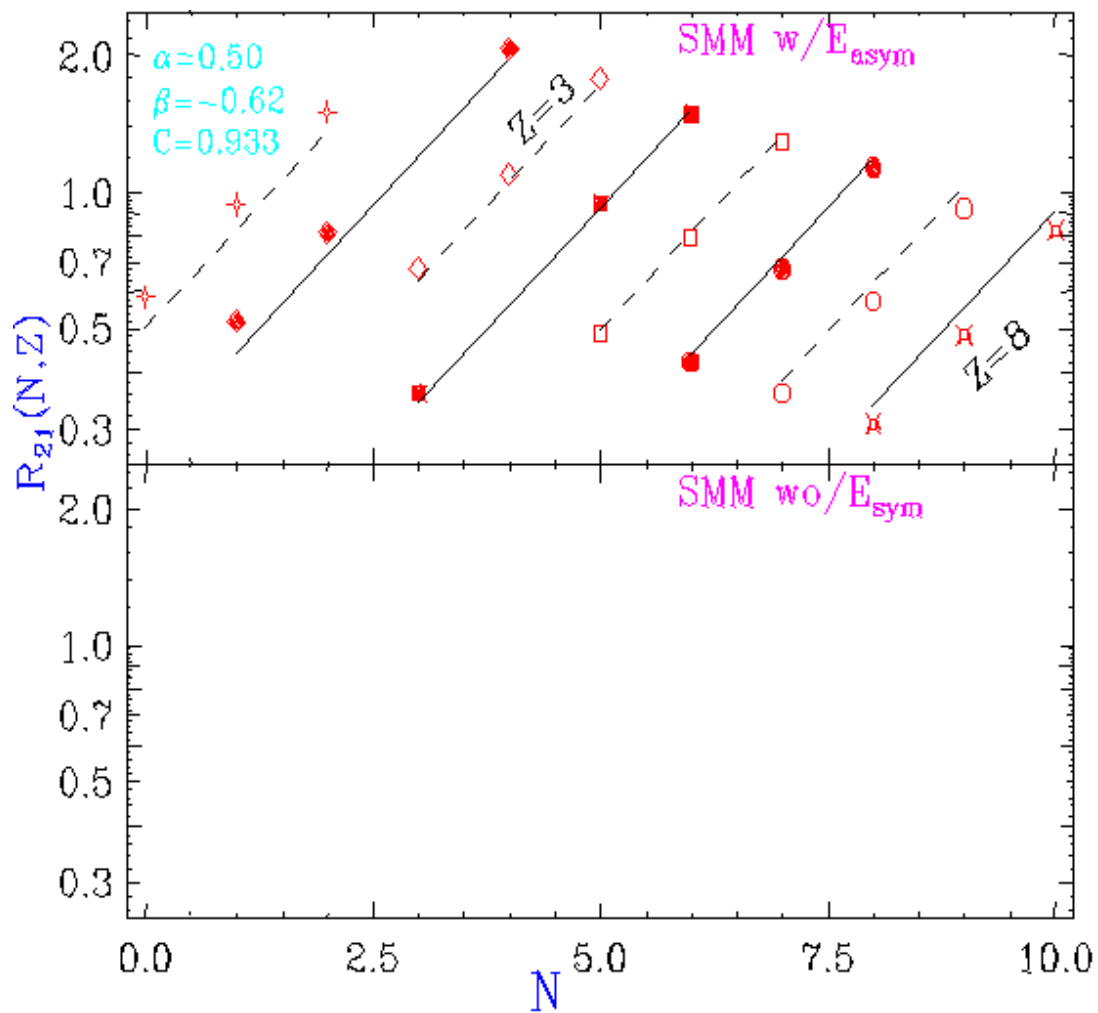


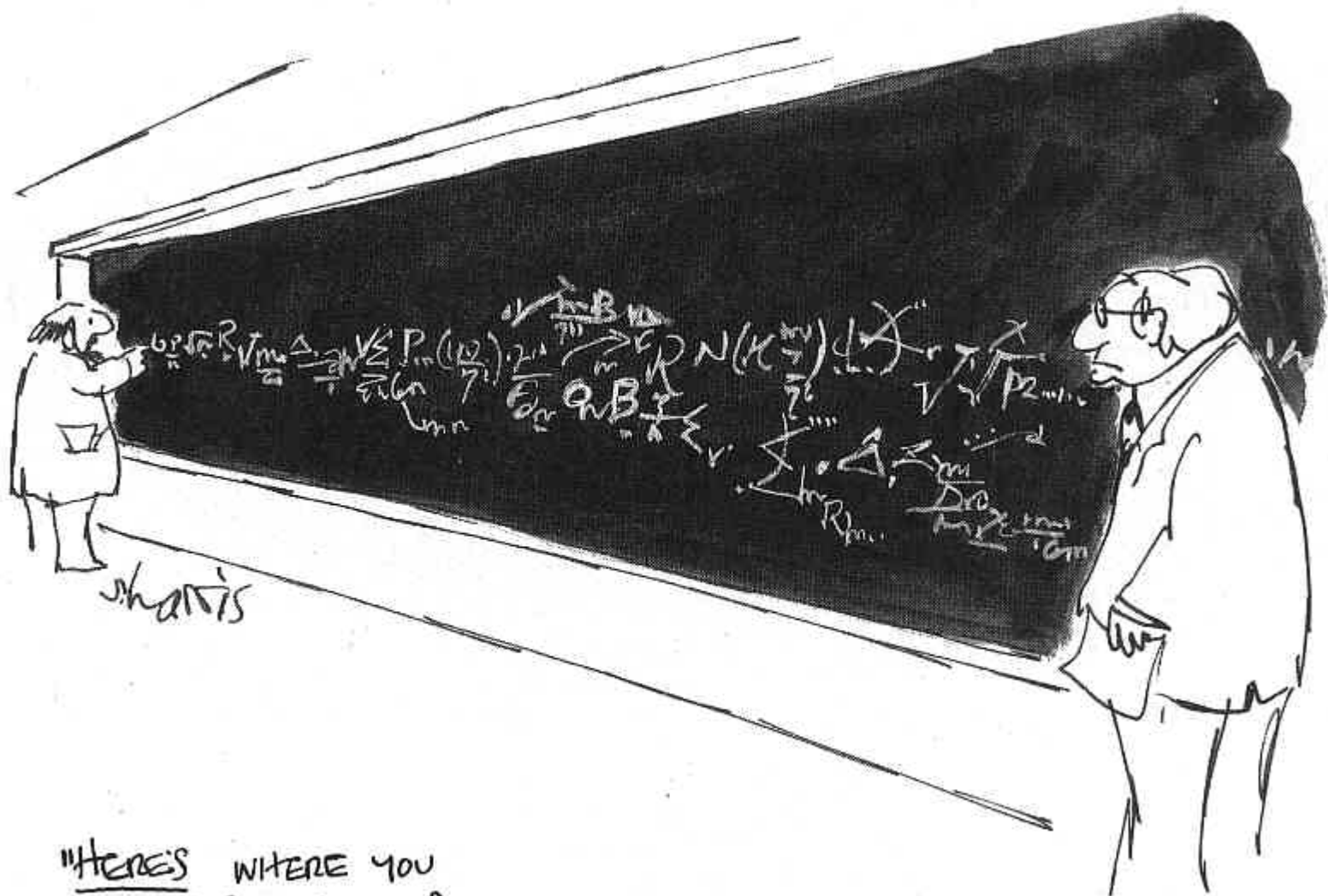
Chemical Potentials

$E_{\text{Coul}}$   $E_{\text{sym}}$   $\rho_p$   $\rho_n$

$$R_{21} \propto \exp[(-\Delta \mu_n \cdot N - \Delta \mu_p \cdot Z)/T]$$

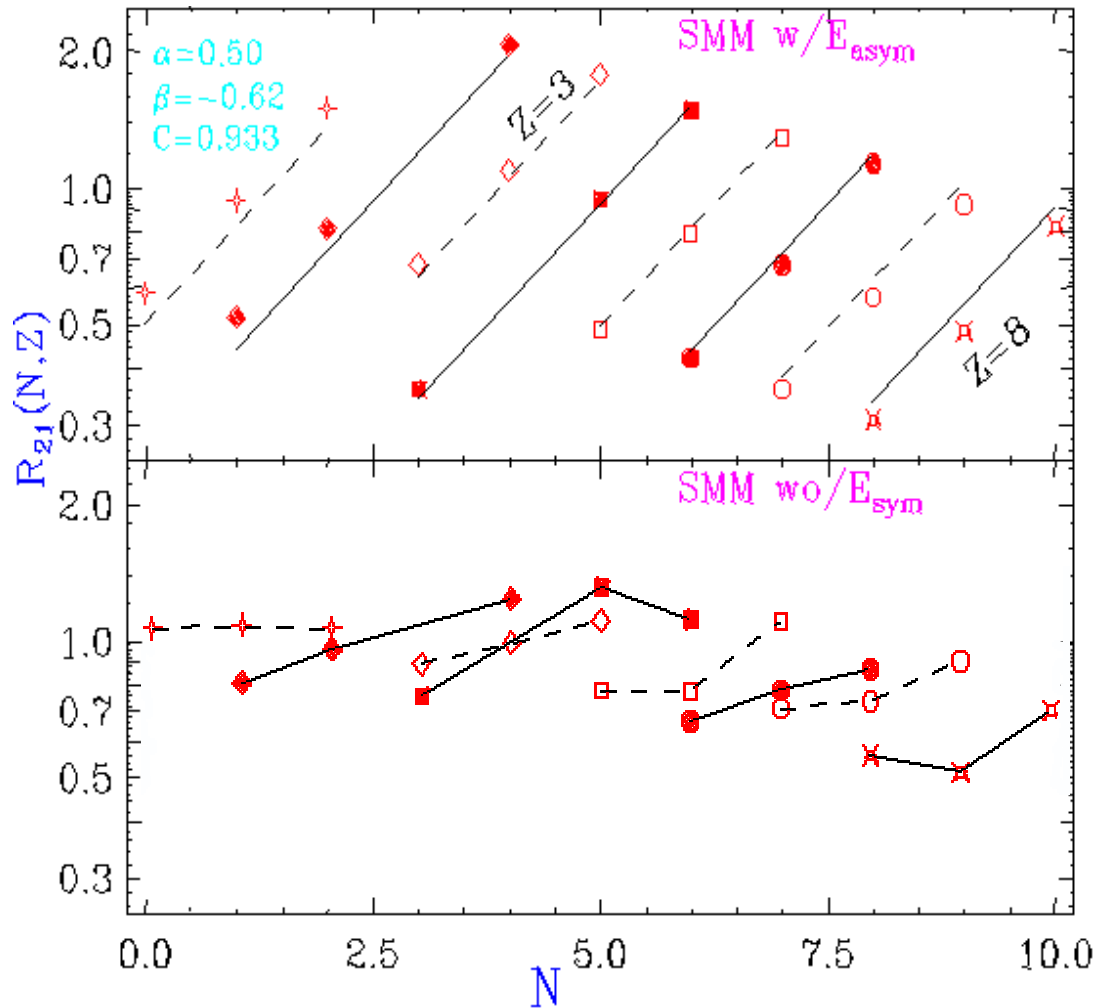
# Origin of isoscaling





"HERE'S WHERE YOU MADE YOUR MISTAKE."

# Origin of isoscaling



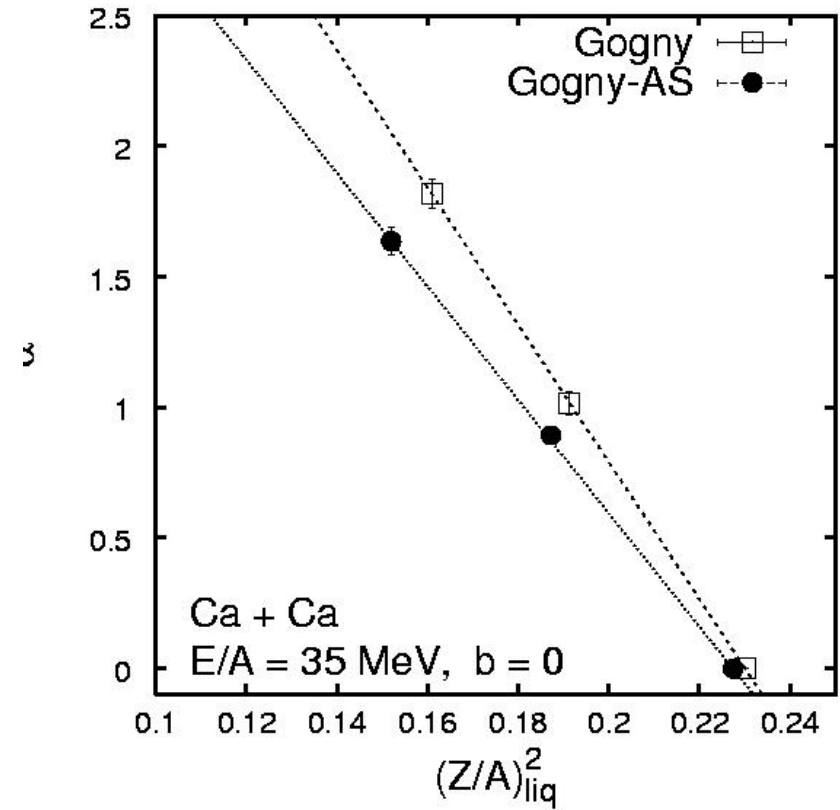
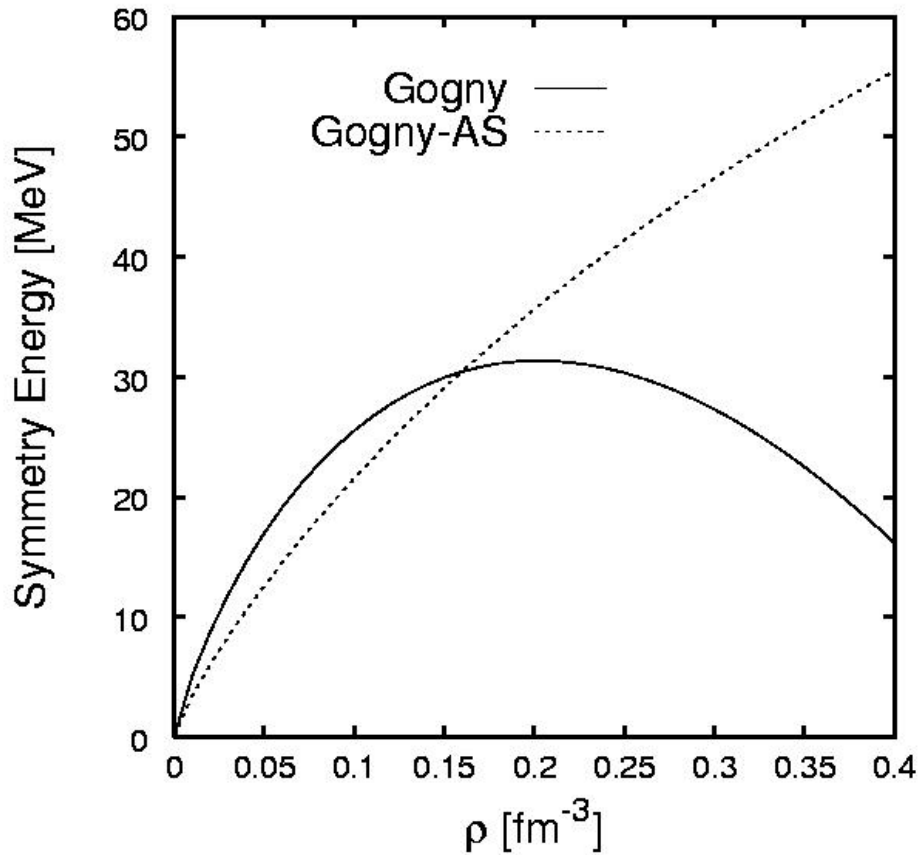
➤ *Isoscaling disappears when the symmetry energy is set to zero*

➤ *Provides an observable to study symmetry energy*



# Symmetry energy from AMD

*A. Ono et al. PRC 68,051601 (2003)*



$\alpha$  depends on symmetry term interactions

# Boltzmann equation for heavy ion collisions

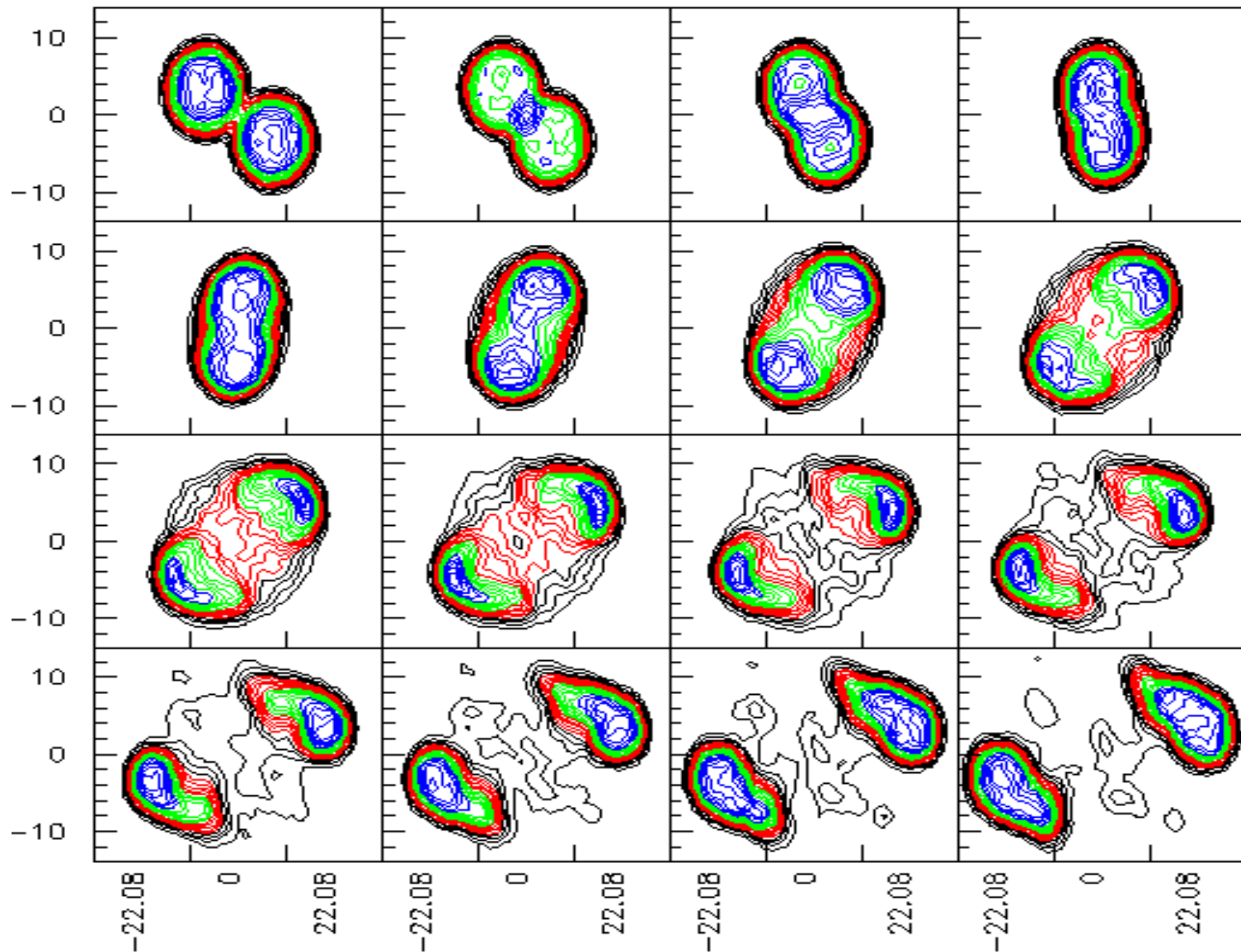
G. F. Bertsch and H. Kruse, S. Das Gupta

PRC 29, 673 (1984)

Sn+Sn; E/A=50 MeV

b=6.6 fm

*Micha Kilburn, REU 2003*

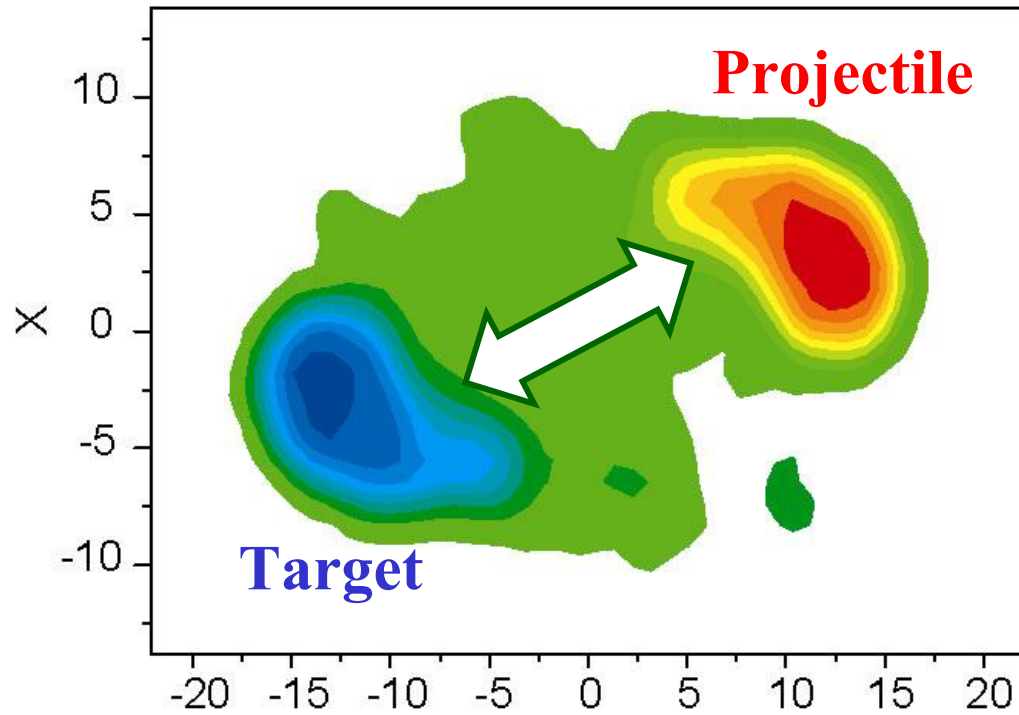


# Isospin diffusion in the projectile-like region

## *Basic ideas:*

- *Peripheral reactions*
- *Asymmetric collisions*  
 $^{124}\text{Sn} + ^{112}\text{Sn}$ ,  $^{112}\text{Sn} + ^{124}\text{Sn}$   
-- diffusion

Lijun Shi

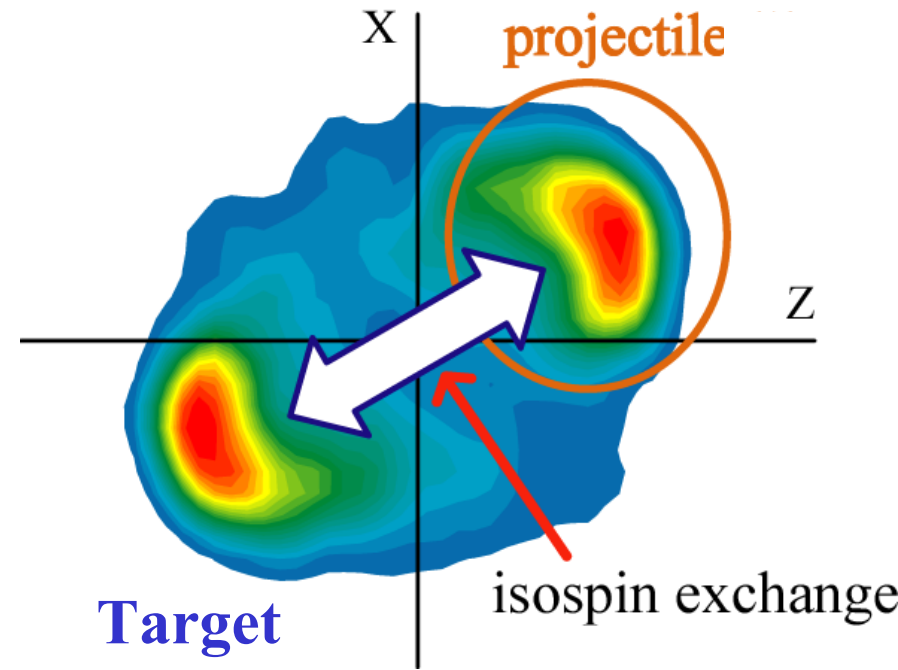


$$\delta = (N - Z) / (N + Z)$$

# Isospin diffusion in the projectile-like region

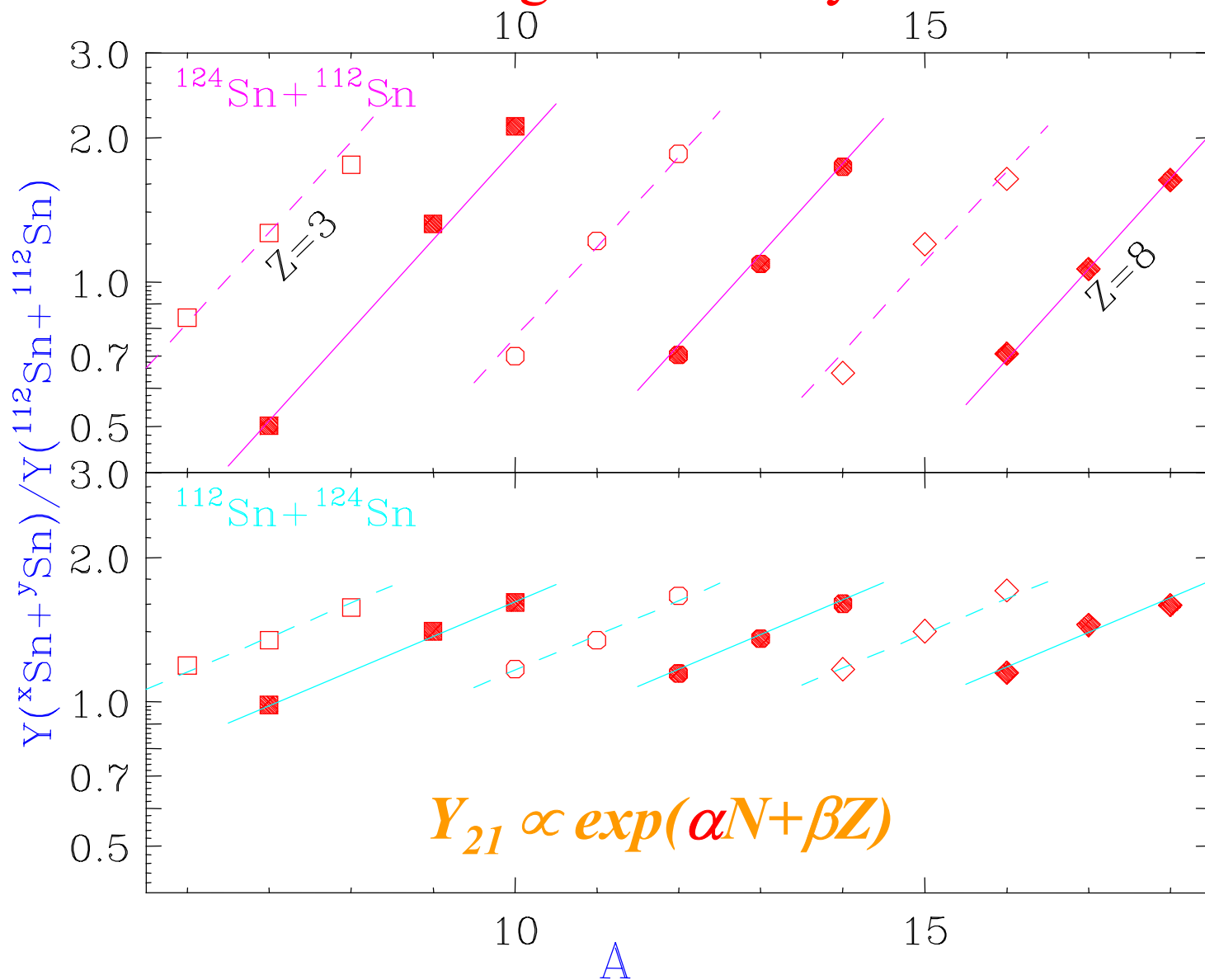
## *Basic ideas:*

- *Peripheral reactions*
- *Asymmetric collisions*  
 $^{124}\text{Sn} + ^{112}\text{Sn}$ ,  $^{112}\text{Sn} + ^{124}\text{Sn}$   
-- diffusion
- **Symmetric Collisions**  
 $^{124}\text{Sn} + ^{124}\text{Sn}$ ,  $^{112}\text{Sn} + ^{112}\text{Sn}$   
-- no diffusion
- *Relative change between target and projectile is the diffusion effect*



$$\delta = (N - Z) / (N + Z)$$

# Isoscaling of mixed systems



*Experimental: isoscaling;  $Y_{21} \propto \exp(\alpha N + \beta Z)$*

*Theoretical :  $\delta = (N-Z)/(N+Z)$*

*$\alpha \propto \delta_1 - \delta_2$  (Subal's SMM)*

## Isospin Transport Ratio

$$R_i = \frac{2x - x_{124+124} - x_{112+112}}{x_{124+124} - x_{112+112}}$$

Rami et al., PRL, 84, 1120 (2000)

$x$ =experimental or theoretical isospin observable

$x=x_{124+124} \rightarrow R_i = 1.$

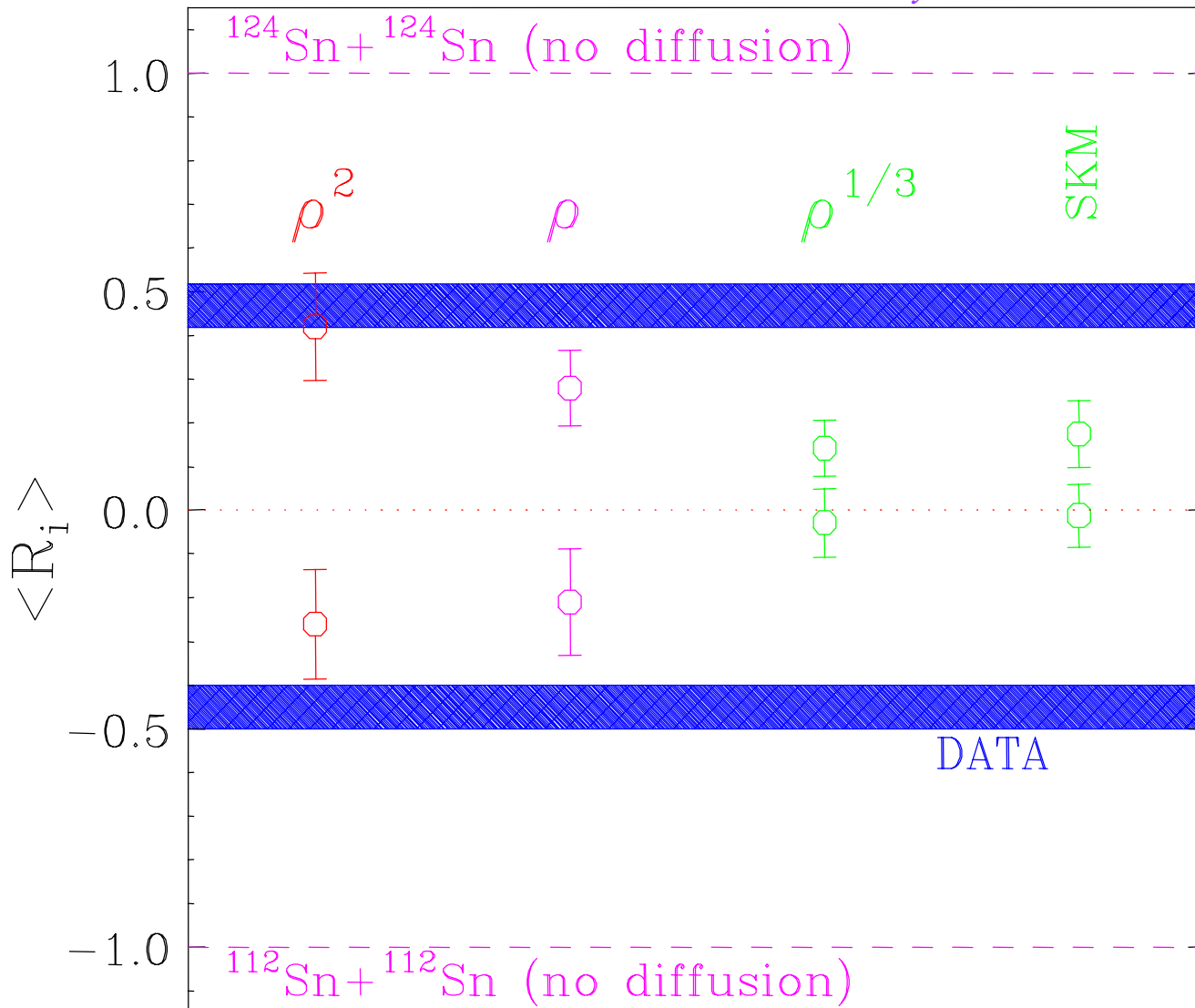
$x=x_{112+112} \rightarrow R_i = -1.$

# BUU predictions

Lijun Shi

$$E(\rho, \delta) = E(\rho, 0) + S_{\text{sym}}(\rho) \delta^2$$

$$S_{\text{sym}}(\rho) \propto \rho^\gamma$$

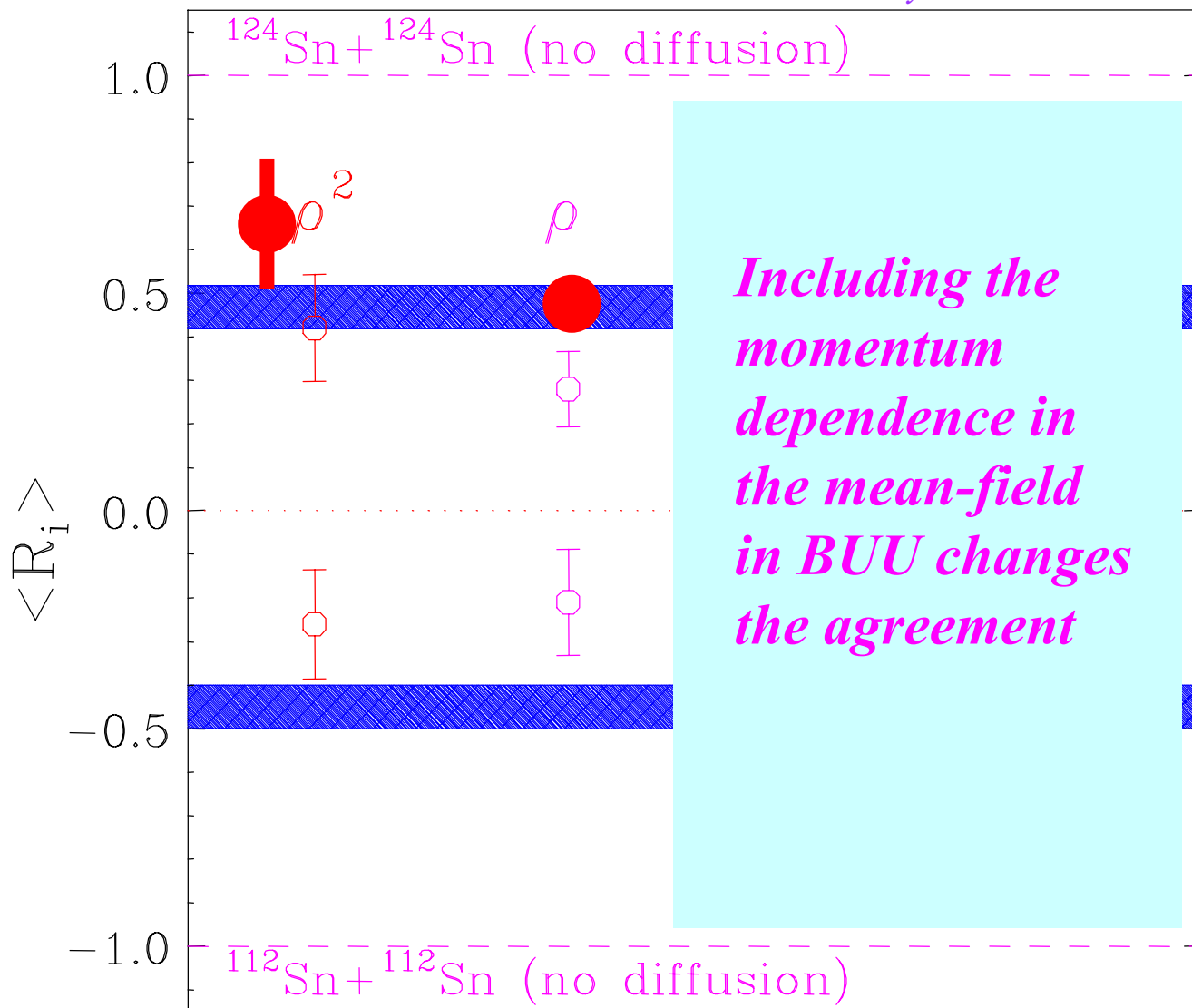


*Experimental results are in better agreement with predictions using hard symmetry terms*

# BUU predictions

$$E(\rho, \delta) = E(\rho, 0) + S_{sym}(\rho) \delta^2$$

$$S_{sym}(\rho) \propto (\rho)^\gamma$$

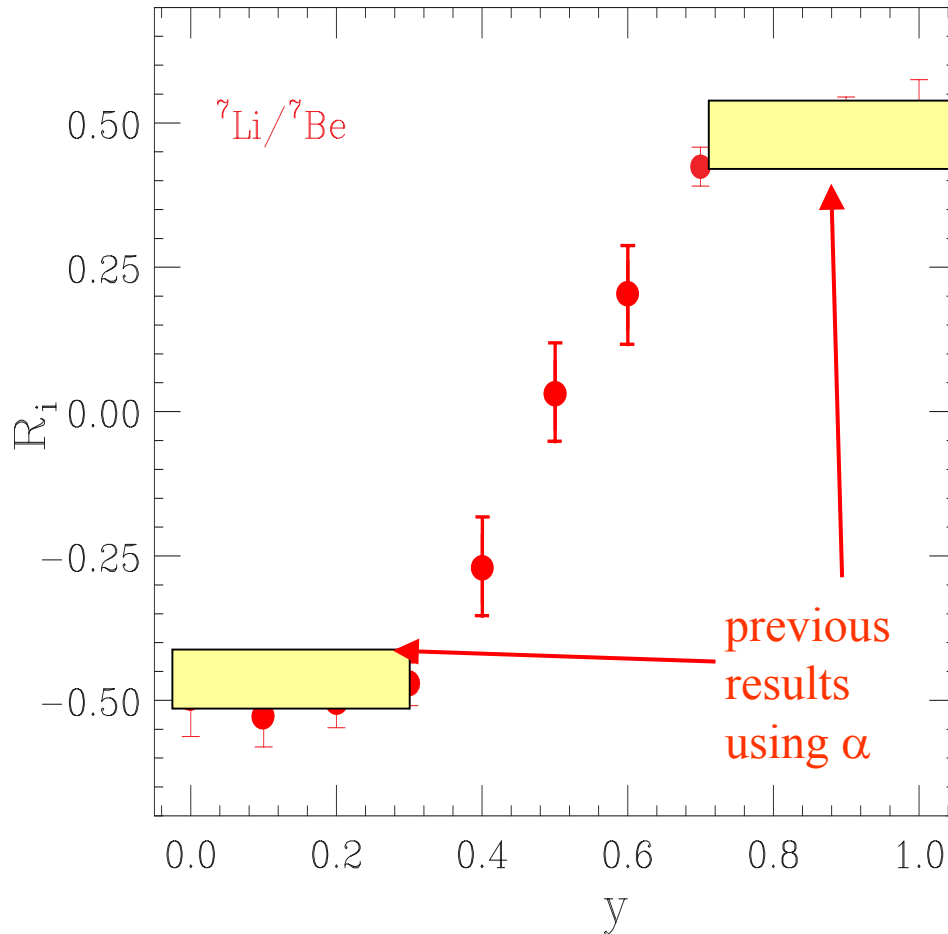


Need more  
experimental  
constraints

B.-A. Li, C. B. Das,  
S. Das Gupta, and C.  
Gale  
Phys. Rev. C **69**,  
011603 (2004)



## $\ln(Y(^7\text{Li})/Y(^7\text{Be}))$ & $R_i(^7\text{Li}, ^7\text{Be})$



- Mirror nuclei ratios provide another observable:

$$R_7 = \ln(Y_{^7\text{Be}} / Y_{^7\text{Li}}) \propto \mu_p - \mu_n$$

- Rapidity dependence is weak except at mid rapidity

May provide better constraints on the momentum dependence of the mean field

# Summary

- *A lot of work has been done on isoscaling.*
  - *Robust observable*
  - *Seen in many different reactions*
  - *Promising tool to study symmetry energy with heavy ion collisions – Isospin Diffusion*

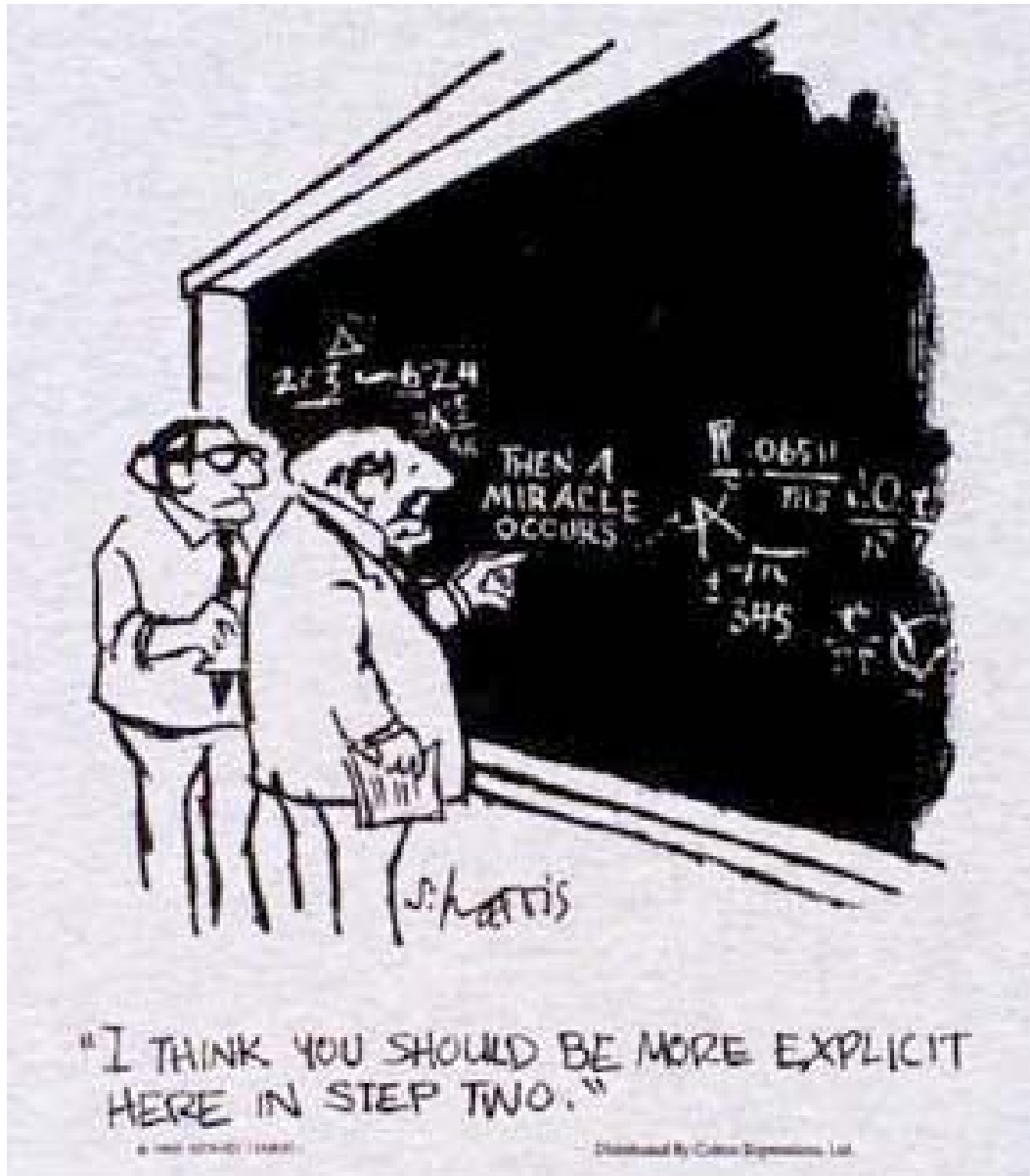
# Acknowledgements

Subal Das Gupta

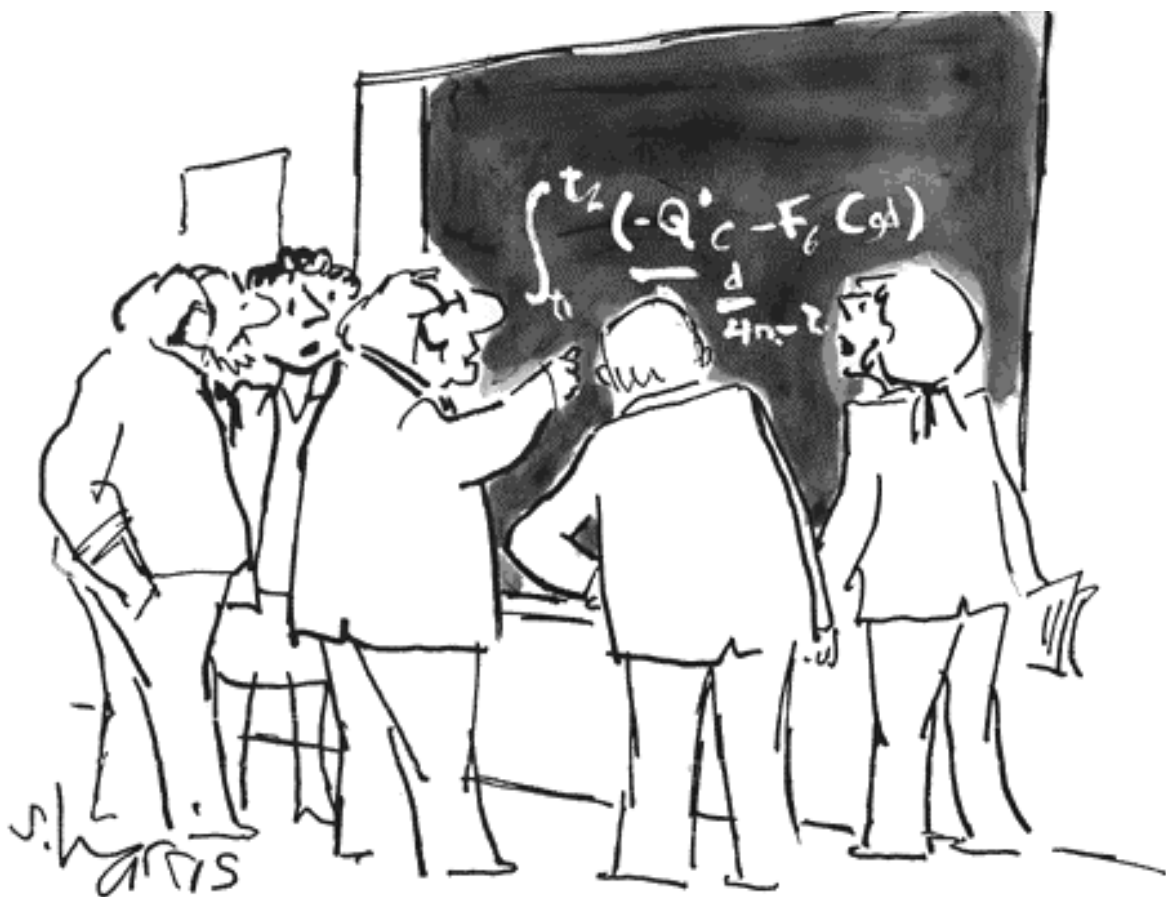
*P. Danielewicz, C.K. Gelbke, T.X. Liu, X.D. Liu, W.G. Lynch,  
L.J. Shi, R. Shomin, M.B. Tsang, W.P. Tan, M.J. Van  
Goethem, G. Verde, A. Wagner, H.F. Xi, H.S. Xu, Akira Ono,  
Bao-An Li, B. Davin, Y. Larochele, R.T. de Souza, R.J.  
Charity, L.G. Sobotka, S.R. Souza, R. Donangelo*



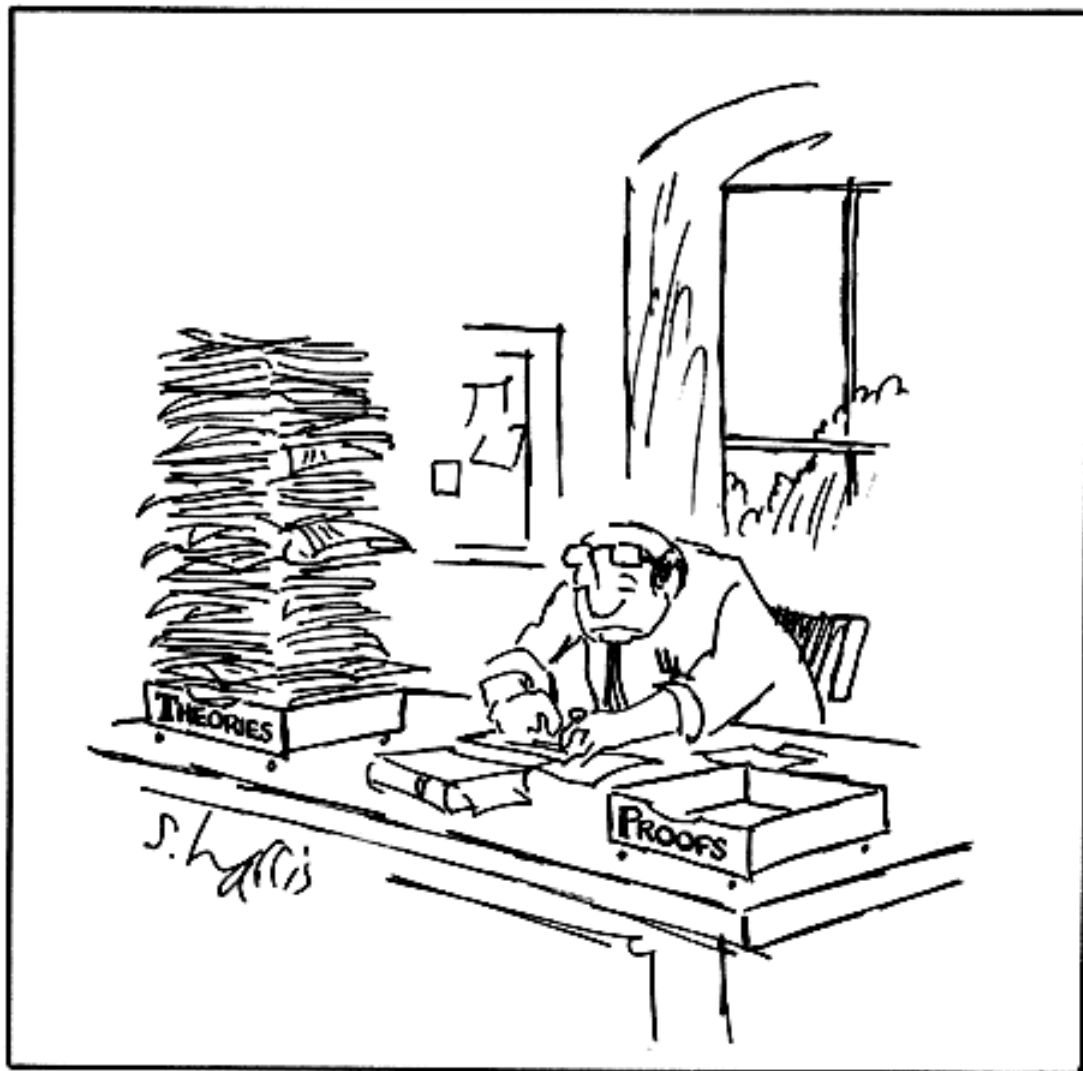
THE END



"I THINK YOU SHOULD BE MORE EXPLICIT  
HERE IN STEP TWO."



"Let's see now. Wilkes is nine years younger than Gottlieb, and Gottlieb is one-third older than me. Foster's age minus Rivera's age..."







"QUARKS. NEUTRINOS. MESONS. ALL THOSE DAMN PARTICLES  
YOU CAN'T SEE. THAT'S WHAT DROVE ME TO DRINK.  
BUT NOW I CAN SEE THEM!"