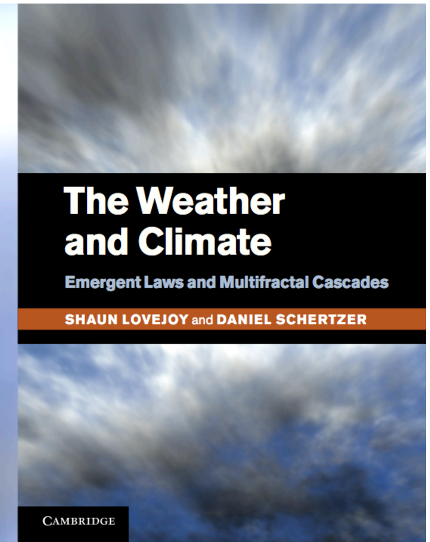
Emergent atmospheric laws: Why the weather and the climate are not what you expect



S. Lovejoy, McGill

**University of Reading,
15 April, 12:00-13:00, 2013**

The Emergence of physical laws

Quantum mechanics → Classical Mechanics

Large scales (usually)

Statistical mechanics → Continuum mechanics, thermodynamics

Large numbers of particles

General Relativity → Special Relativity

Low energy mass density

Special Relativity → Classical (Galilean) Relativity

Velocities << speed of light

Low level (fundamental) — high level

The emergence of atmospheric dynamics

(Classical)

Continuum mechanics

Low level (fundamental)

Large Re

Laws of turbulence

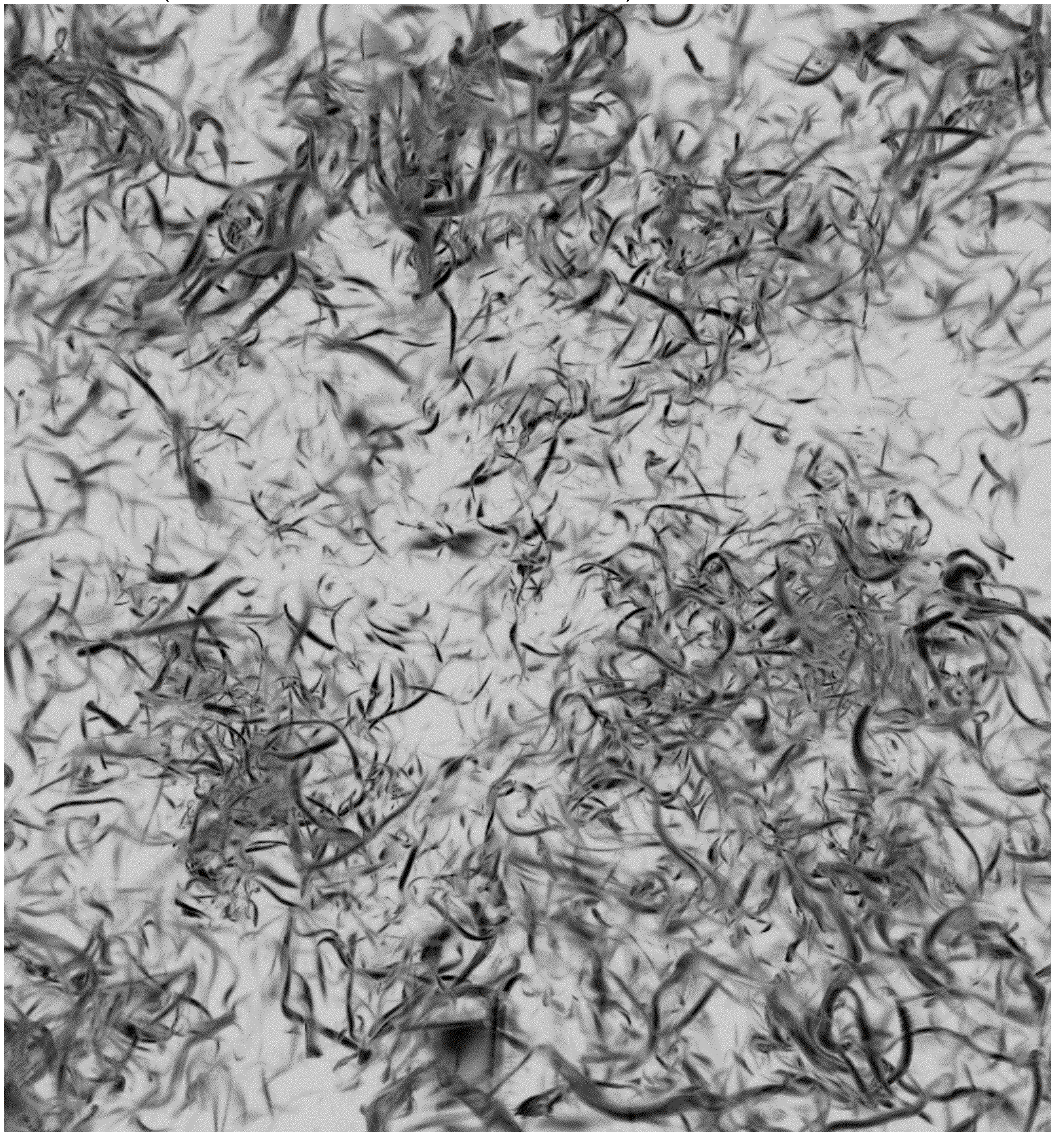
Classical:

Richardson, Kolmogorov, Corrsin, Obukhov, Bolgiano

High level

Vortices in strongly turbulent fluid

(M. Wiczek, numerical simulation, 2010)



$$\Delta v(\underline{\Delta r}) = \varphi |\underline{\Delta r}|^{H}$$

e.g. Kolmogorov $\varphi = \varepsilon^{1/3}$, $H = 1/3$

a) $|\underline{\Delta r}| <\approx 100m$ b) isotropic

c) $\varphi \approx$ constant, quasi Gaussian

Emergence of Atmospheric laws

(Modern)

Fluctuations ≈ (turbulent flux) x (scale)^H

Differences, tendencies, wavelet coefficients

Cascading Turbulent flux

Anisotropic Space-time Scale function

Fluctuation /conservation exponent

Fourier domain:

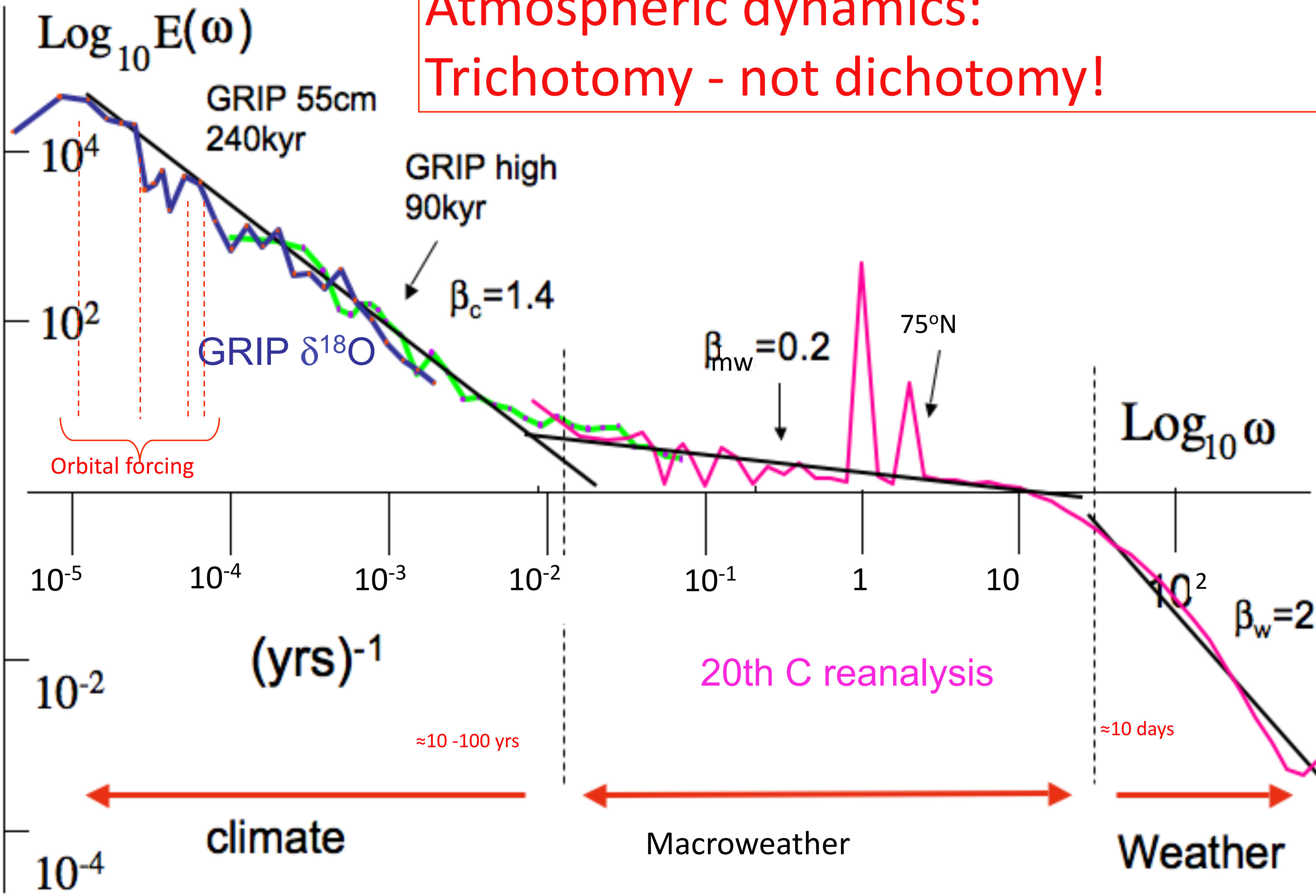
$$\left(\frac{Variance_{observables}}{wavenumber}\right)=\left(\frac{Variance_{flux}}{wavenumber}\right)\left(wavenumber\right)^{-2H}$$

$$=\left(wavenumber\right)^{-\beta}$$

Space: $E(k) \approx k^{-\beta}$

Time: $E(\omega) \approx \omega^{-\beta}$

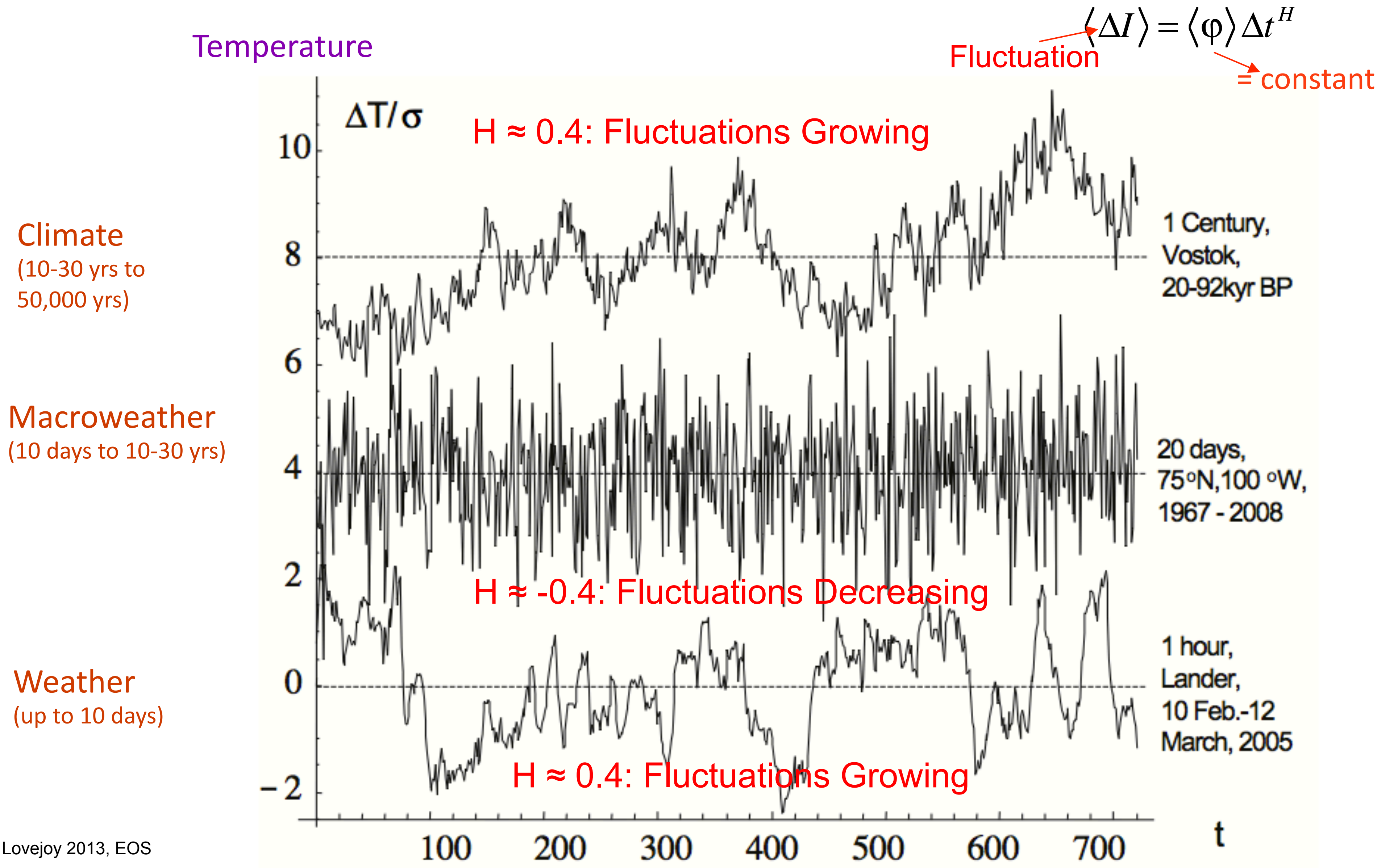
Atmospheric dynamics: Trichotomy - not dichotomy!



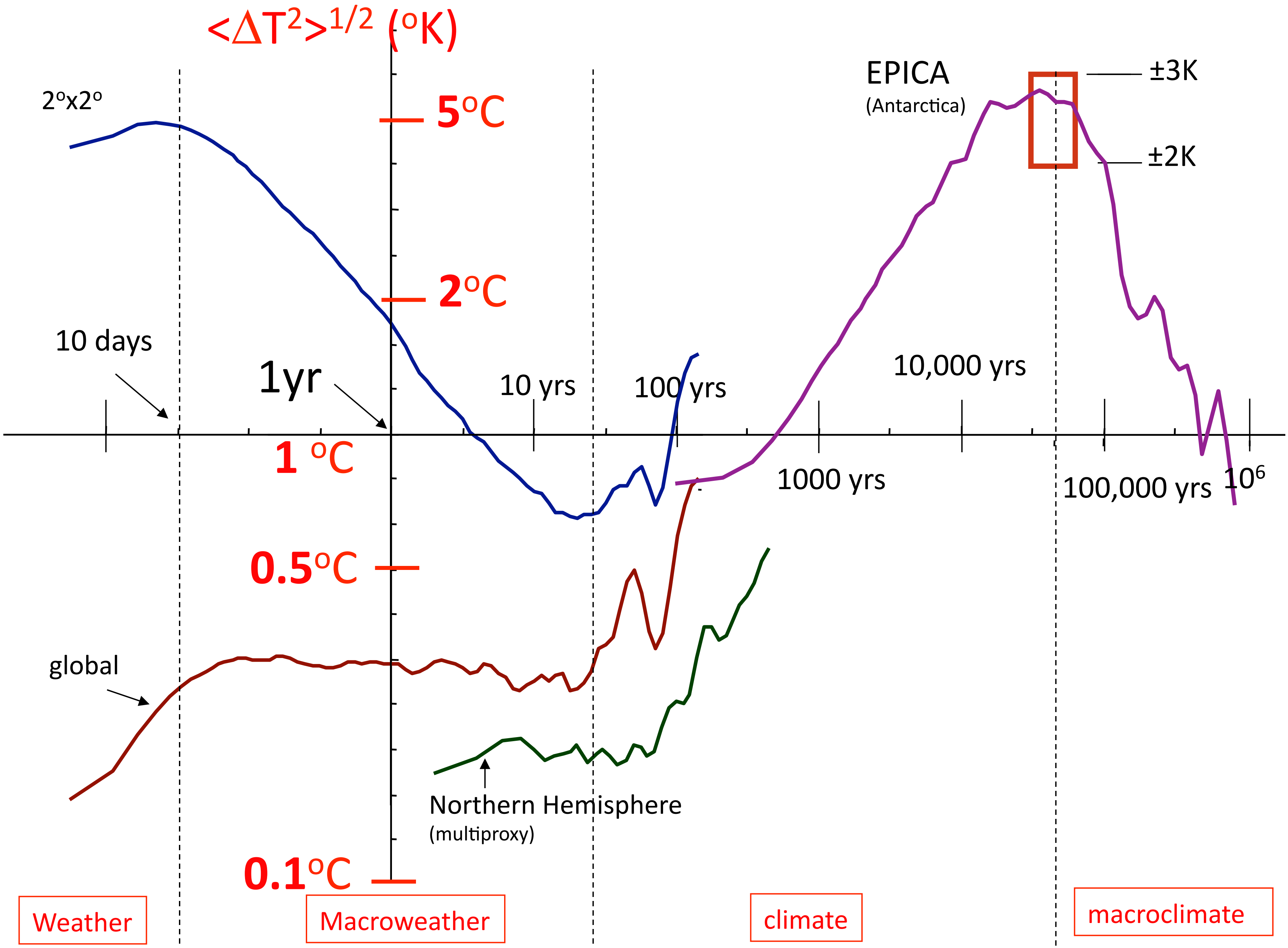
Two data sources only GRIP, 20CR

Lovejoy and Schertzer 2011

Three regimes:
three types of variability

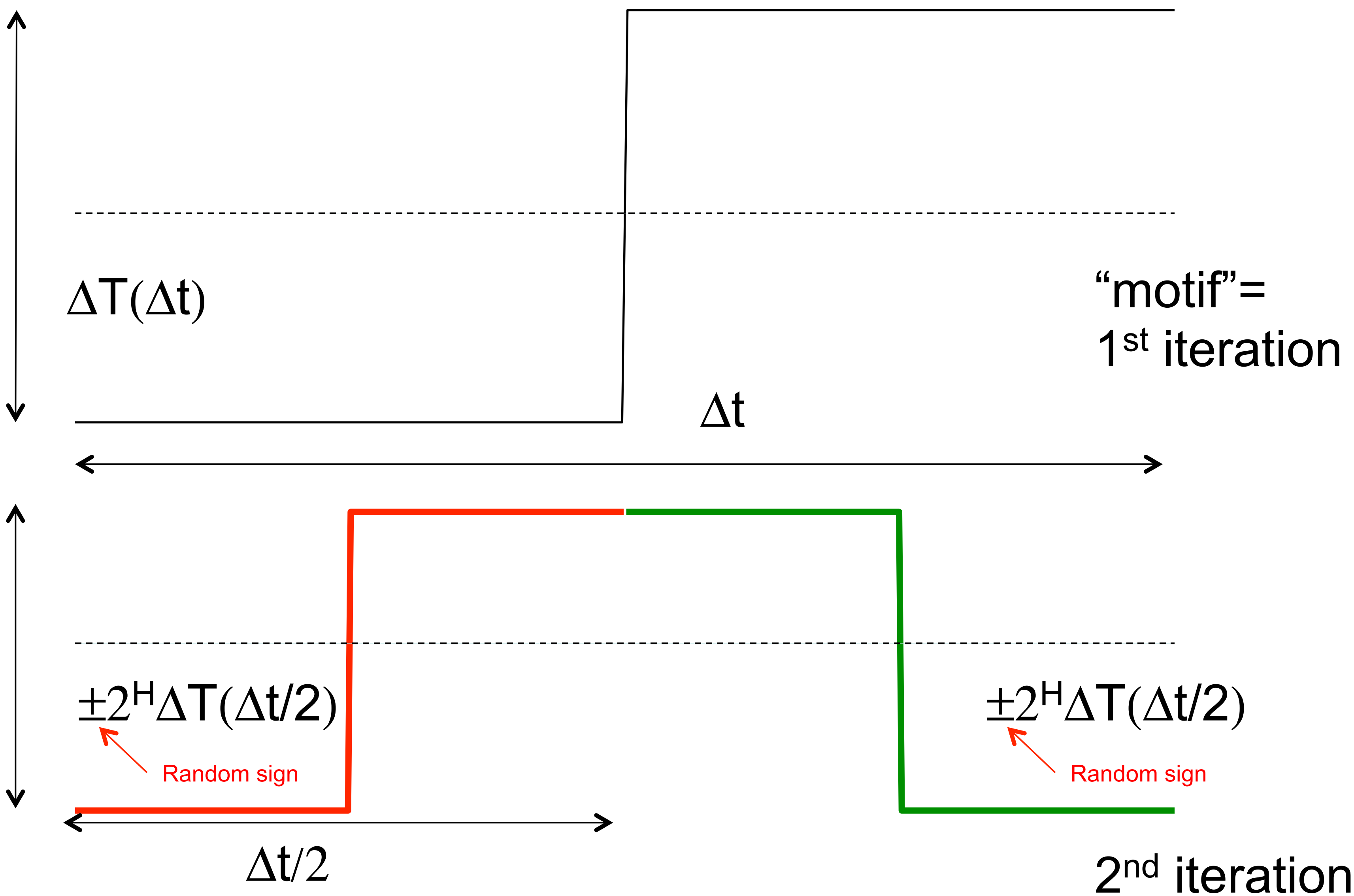


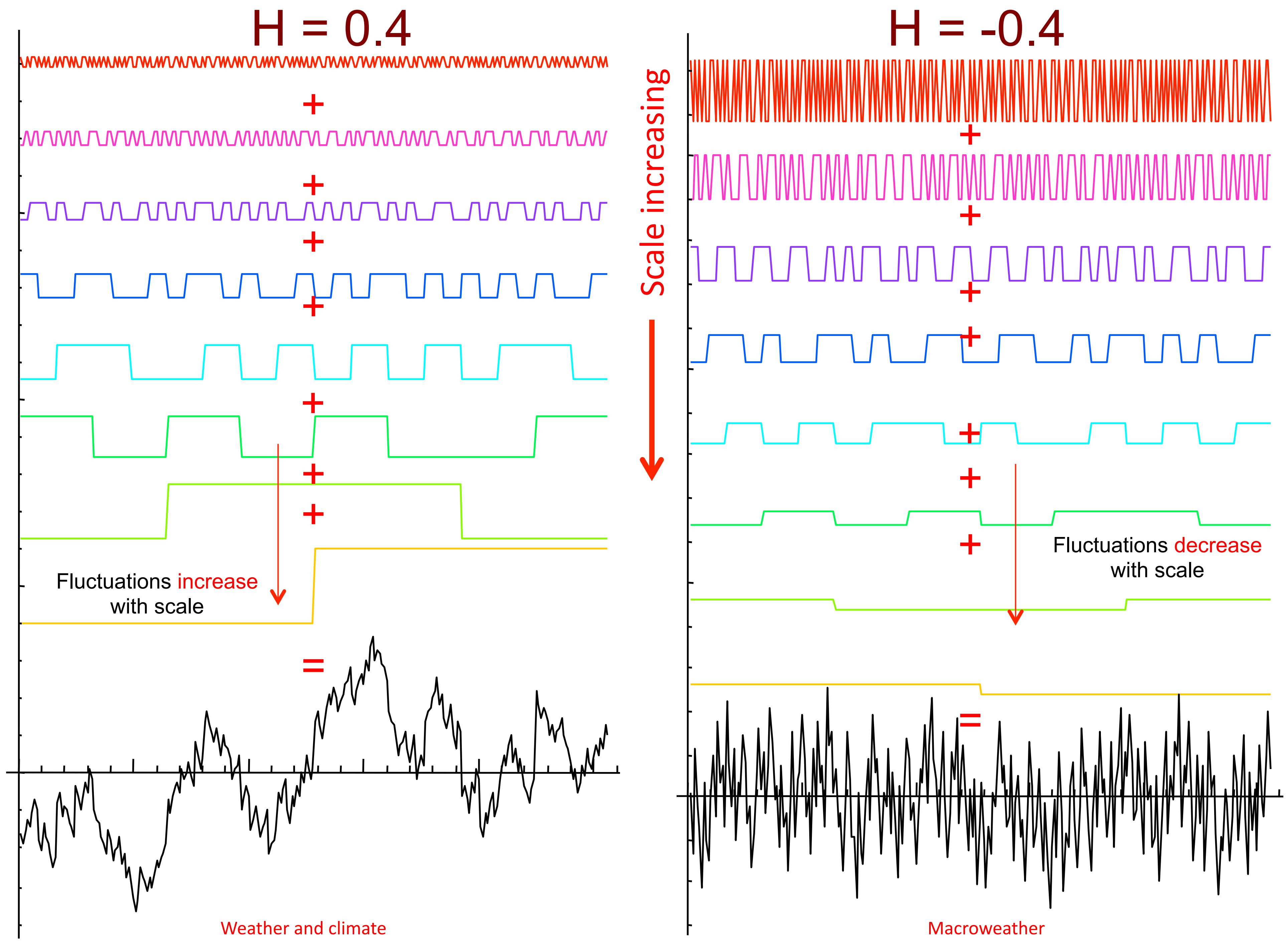
Lovejoy 2013, EOS



<ΔT²>^1/2 (°K)
2°x2°
5°C
2°C
10 days
1yr
10 yrs
100 yrs
1 °C
0.5°C
global
Northern Hemisphere
(multiproxy)
0.1°C
EPICA
(Antarctica)
±3K
±2K
10,000 yrs
1000 yrs
100,000 yrs
10^6
Weather
Macroweather
climate
macroclimate

Additive, fractal "H model"

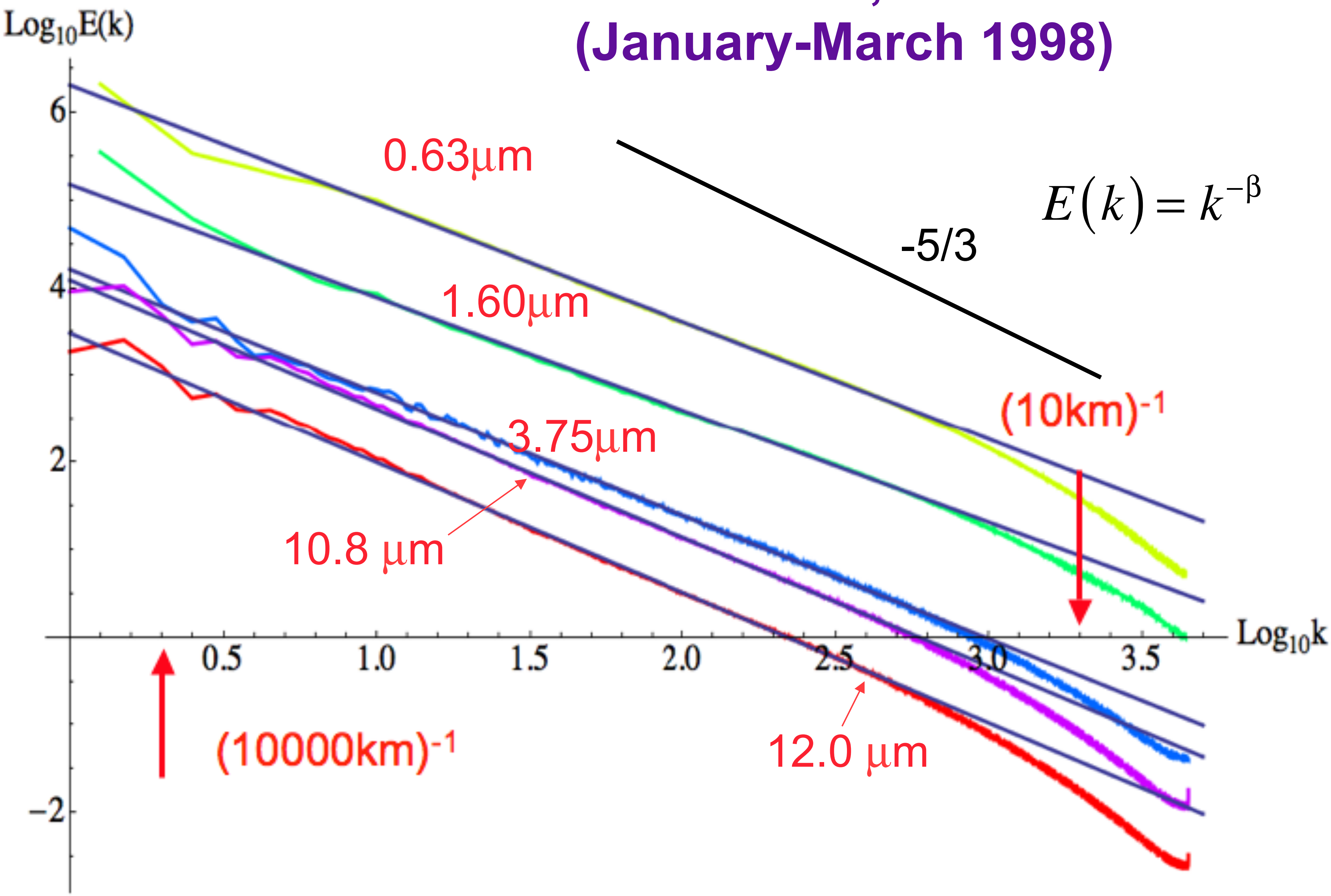



H = 0.4
H = -0.4
Scale increasing
Fluctuations increase with scale
Fluctuations decrease with scale
Weather and climate
Macroweather

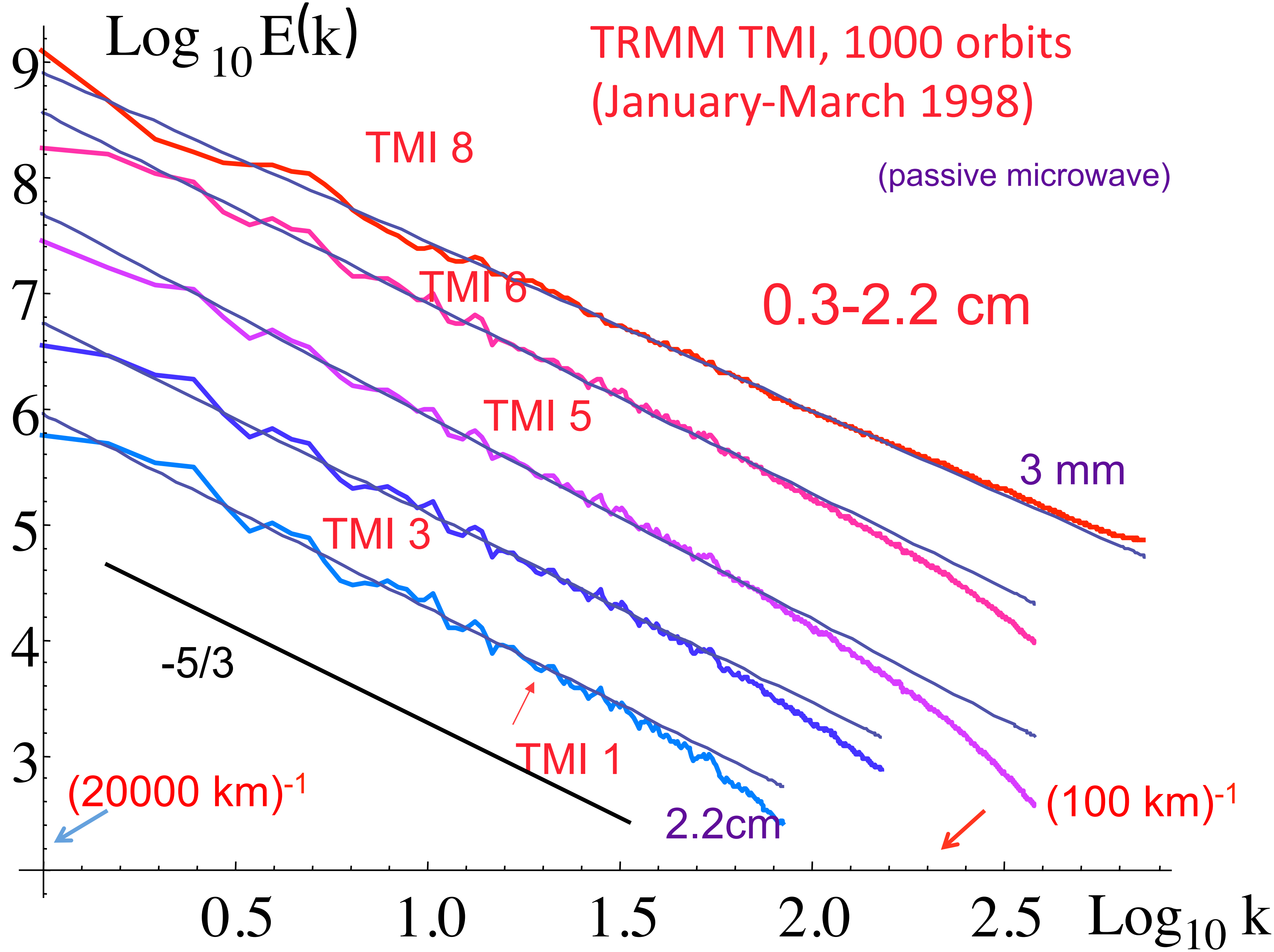
The weather regime:
The emergent laws hold up to planetary scales
(Horizontal scaling)

$$E(k) = k^{-\beta}$$

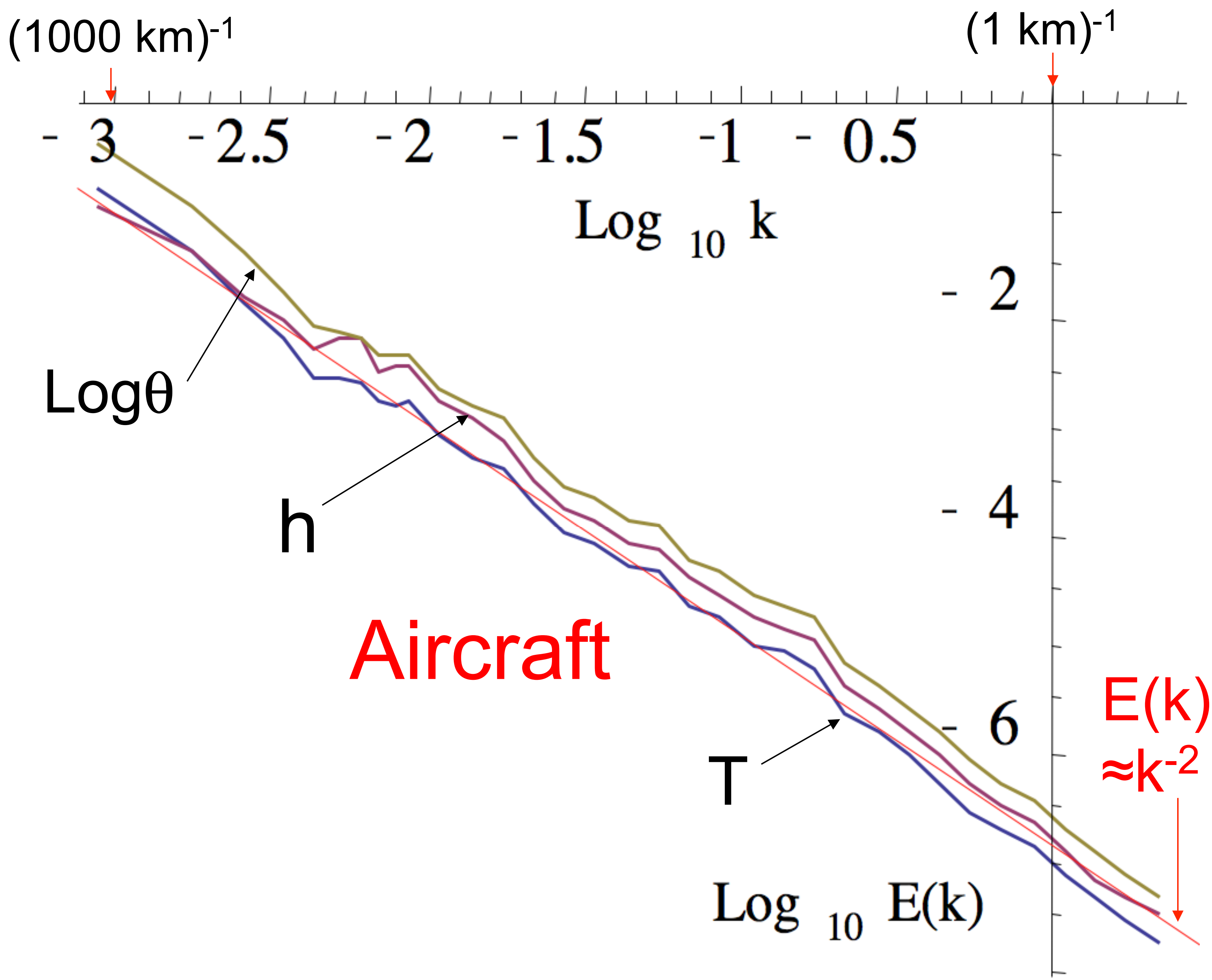
TRMM VIRS, 1000 orbits (January-March 1998)



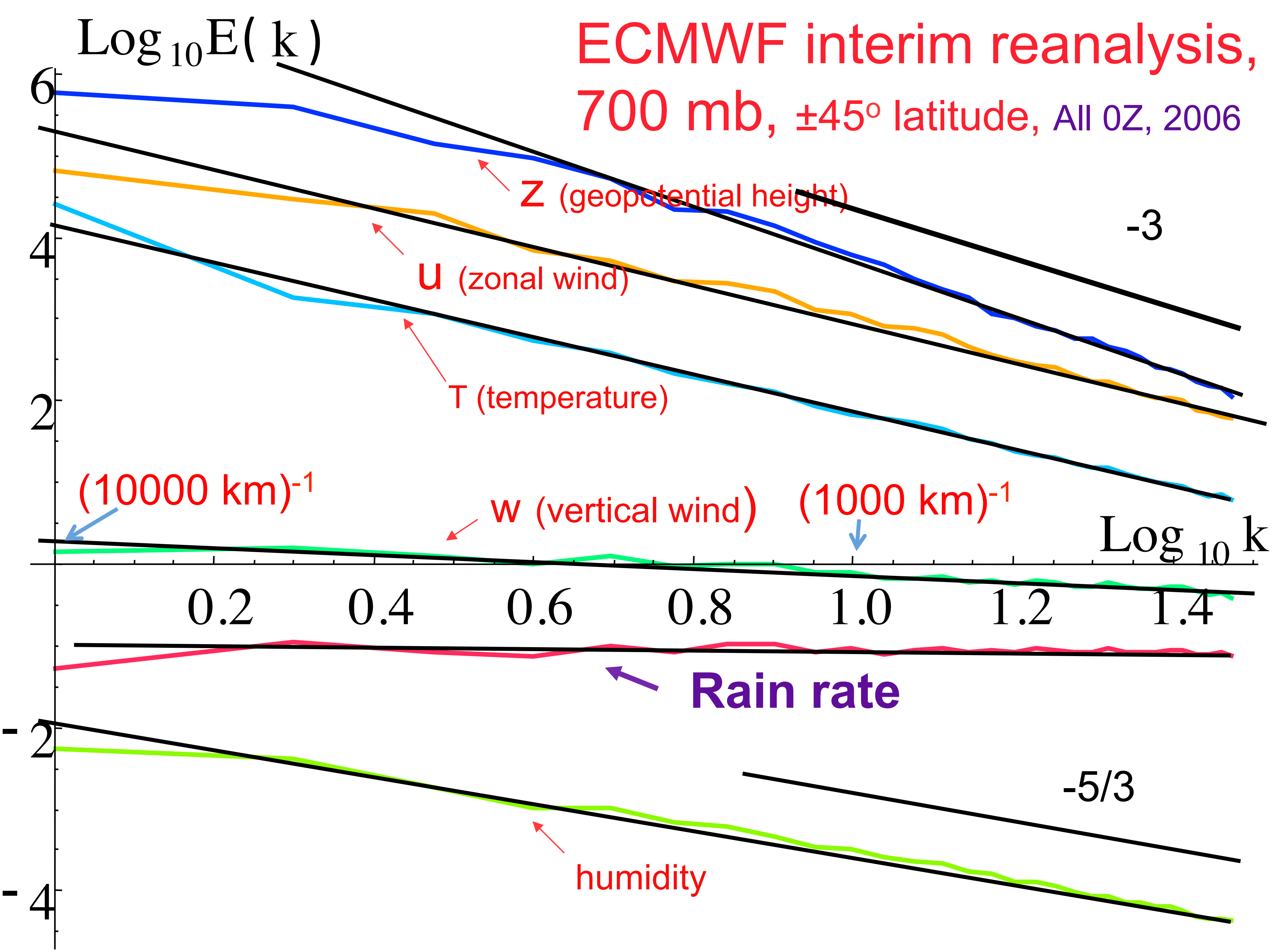
Visible, near infra red, thermal infra red



Log 10 E(k)
TRMM TMI, 1000 orbits
(January-March 1998)
(passive microwave)
TMI 8
TMI 6
TMI 5
TMI 3
TMI 1
0.3-2.2 cm
3 mm
2.2cm
-5/3
(20000 km)-1
(100 km)-1
9
8
7
6
5
4
3
0.5
1.0
1.5
2.0
2.5
Log10 k



(1000 km)-1
(1 km)-1
- 3
- 2.5
- 2
- 1.5
-1
- 0.5
Log 10 k
- 2
- 4
- 6
Logθ
h
Aircraft
T
E(k)
≈k-2
Log 10 E(k)

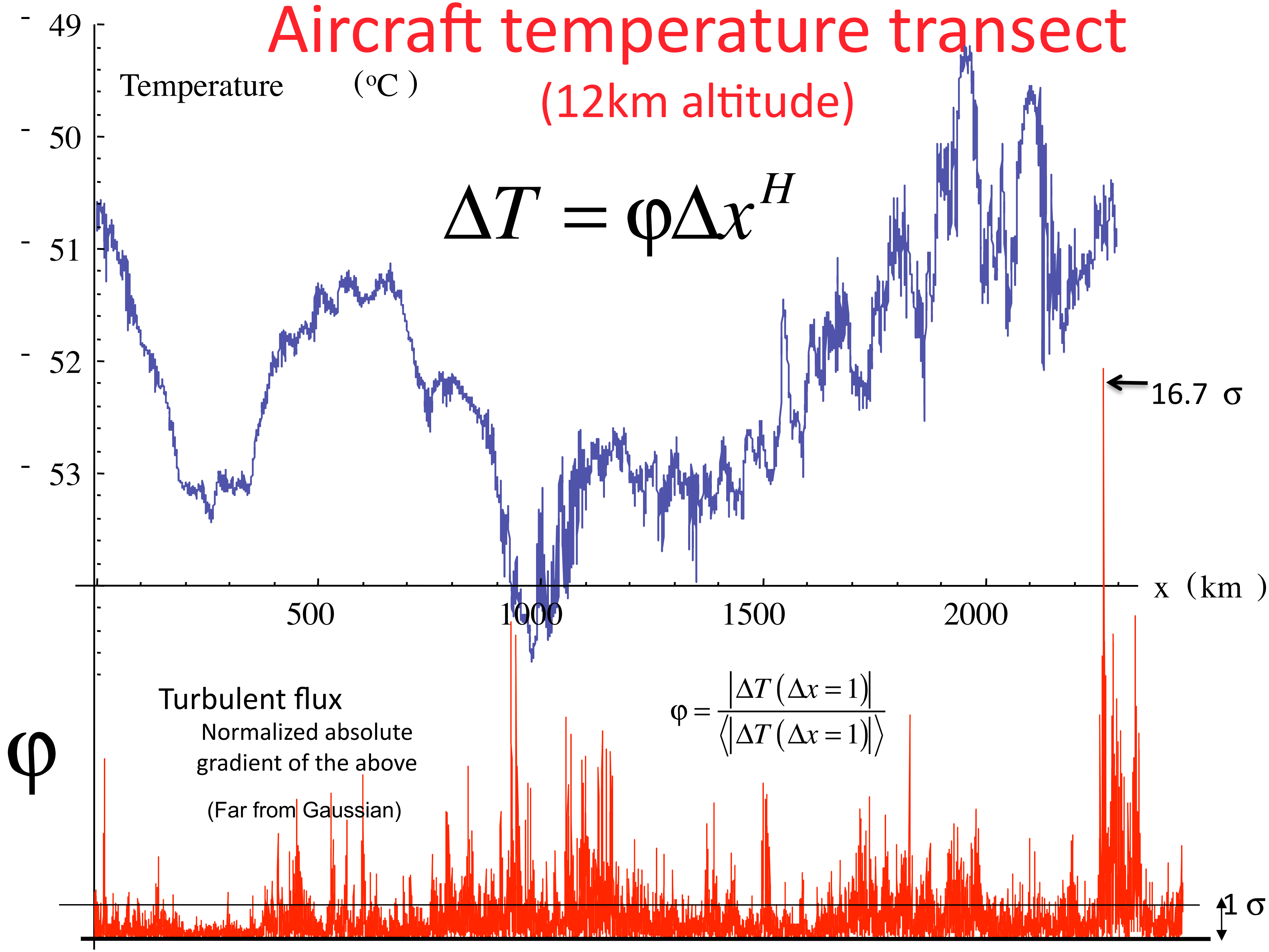


ECMWF interim reanalysis,
700 mb, ±45° latitude, All 0Z, 2006
Log 10 E(k)
6
4
2
-2
-4
Z (geopotential height)
u (zonal wind)
T (temperature)
w (vertical wind)
(10000 km)-1
(1000 km)-1
Log 10 k
0.2
0.4
0.6
0.8
1.0
1.2
1.4
-3
Rain rate
-5/3
humidity

Cascades and Multifractals

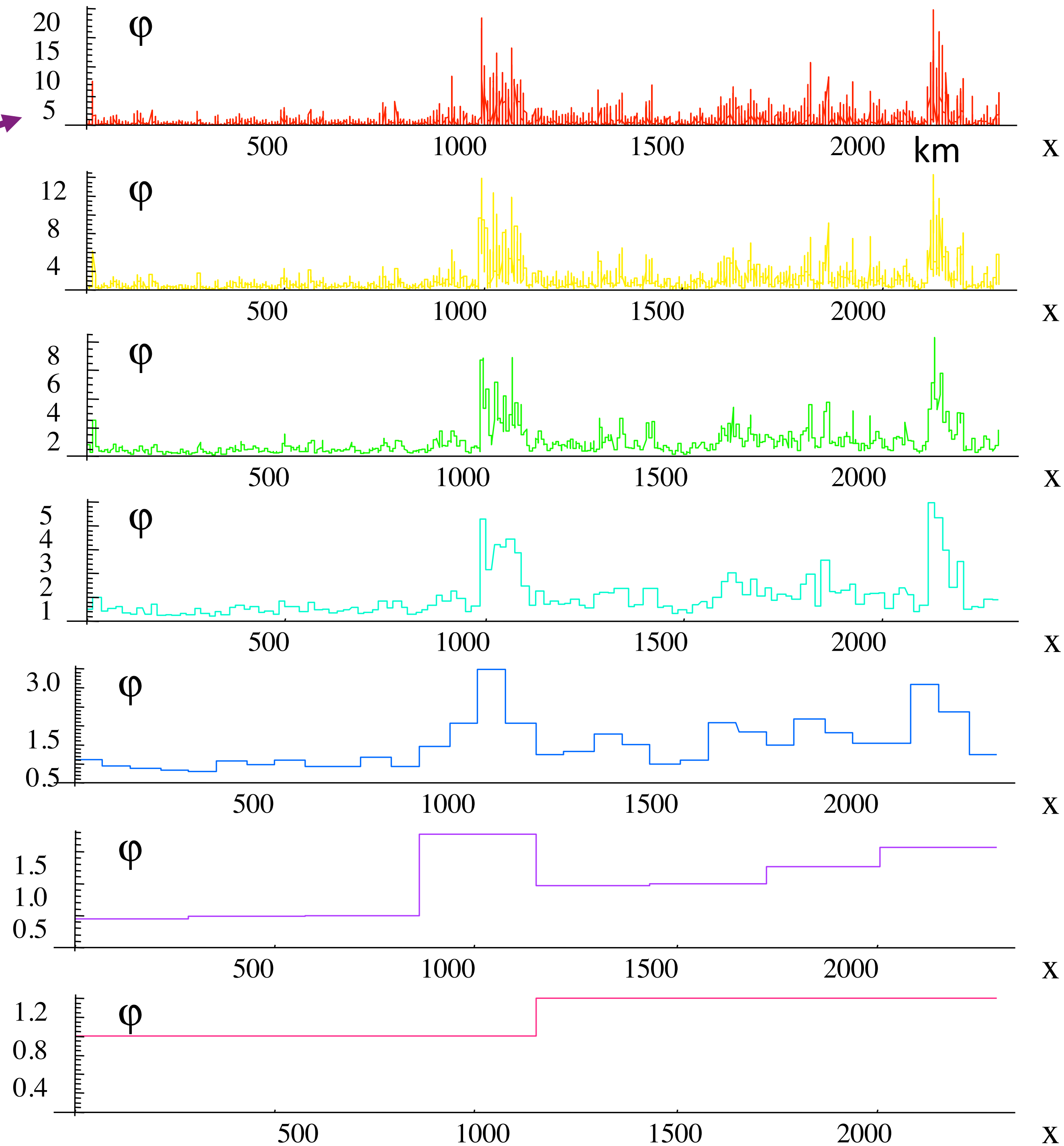
Aircraft temperature transect

(12km altitude)

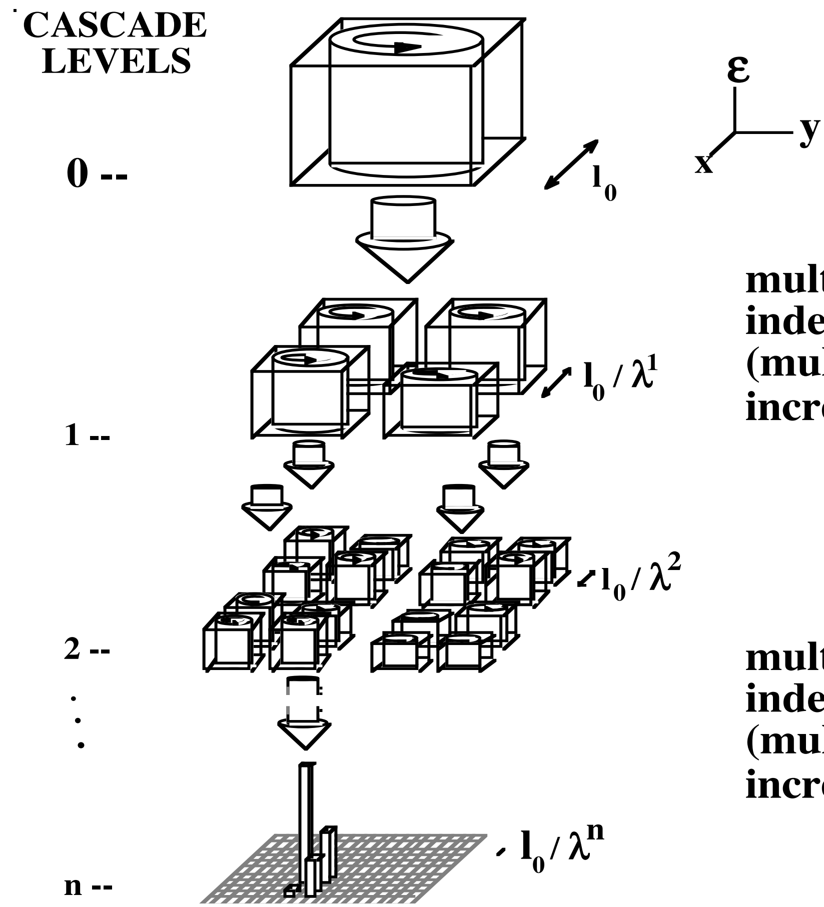


Temperature turbulent flux φ at 280m resolution

High to low Resolution: degrading by factors of 4



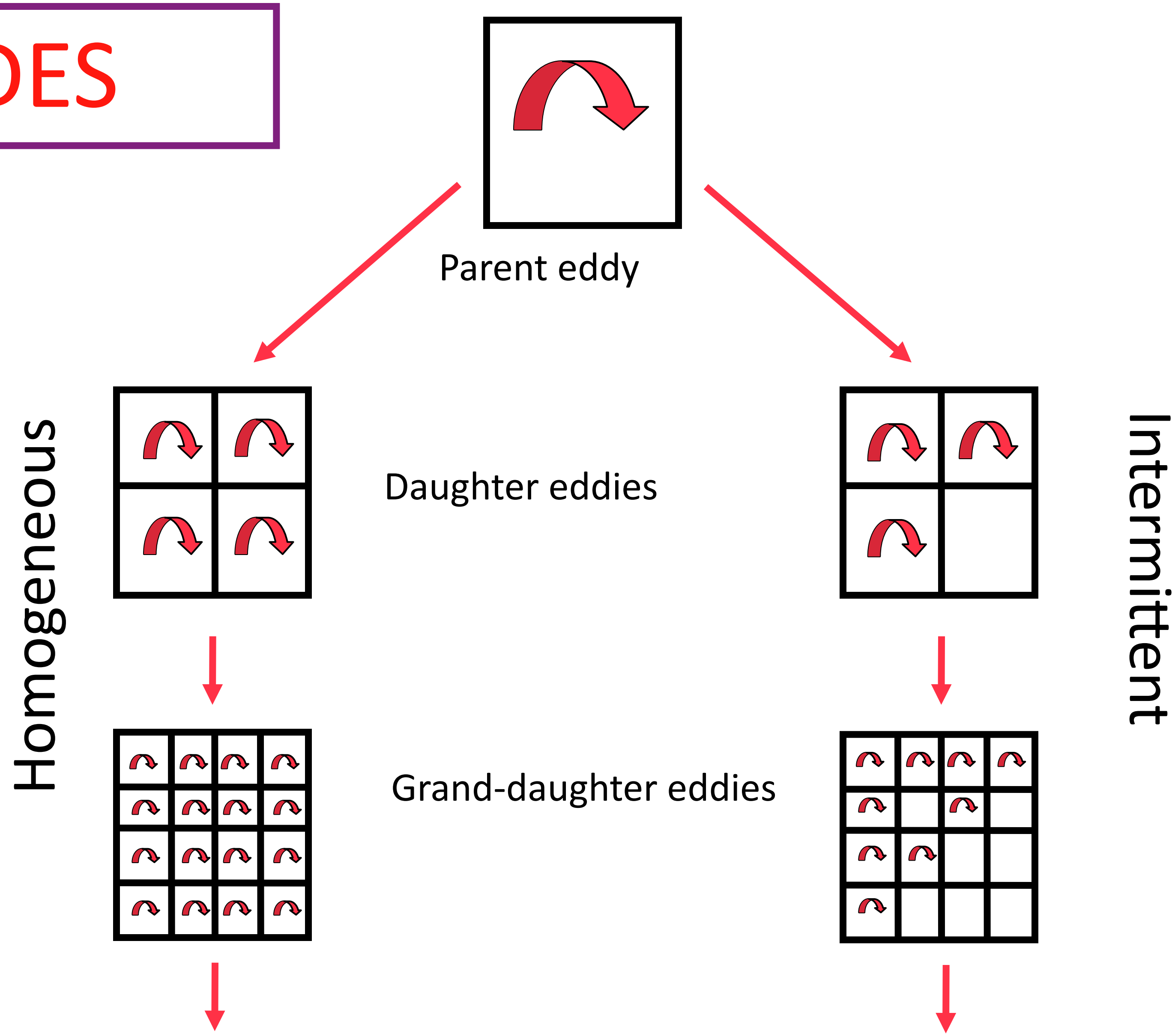
Scale by scale simplicity: cascades



multiplication by 4 independent random (multiplicative) increments

multiplication by 16 independent random (multiplicative) increments

CASCADES

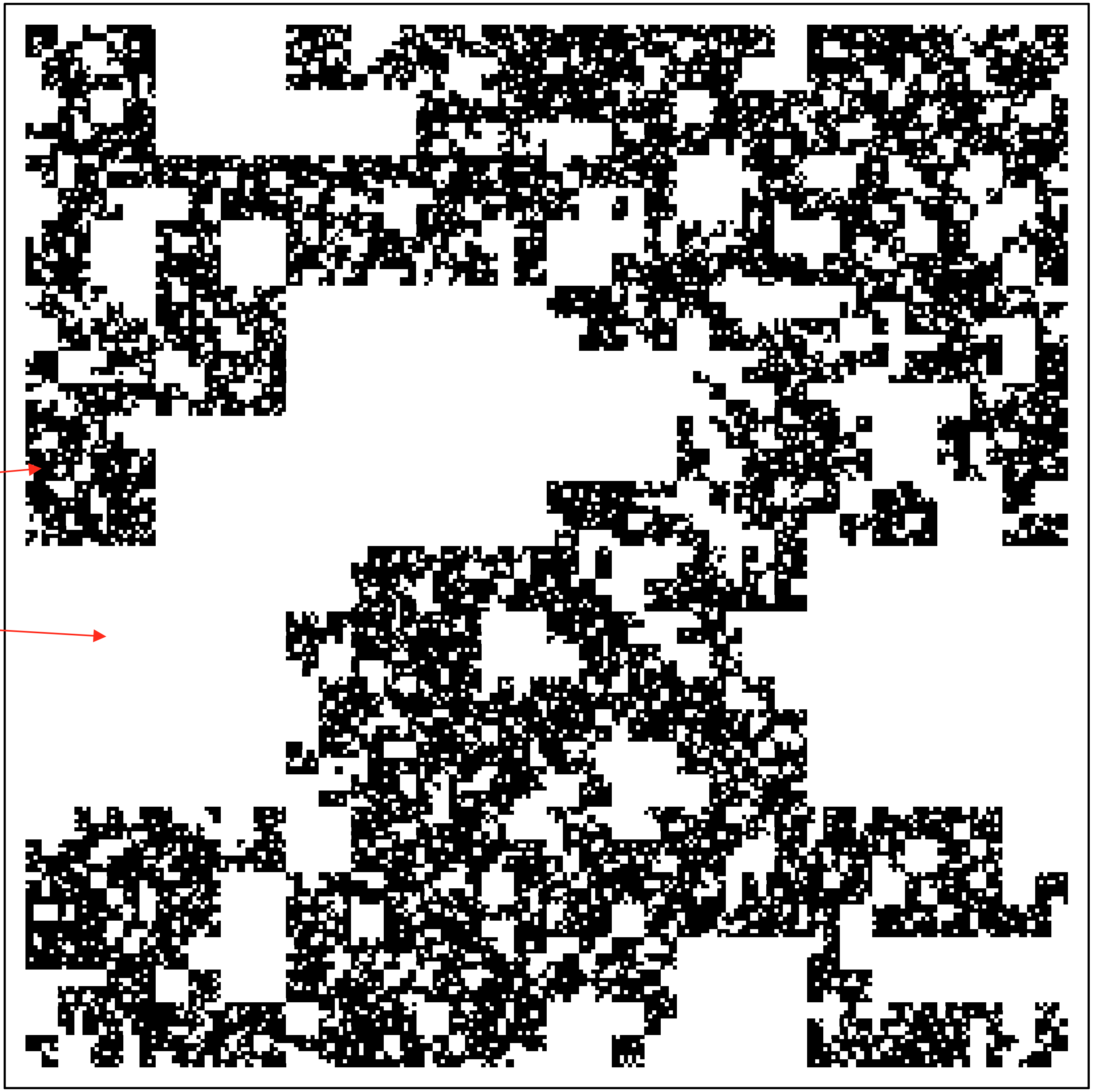

Parent eddy
Homogeneous
Daughter eddies
Intermittent
Grand-daughter eddies

β-model

Fractal set

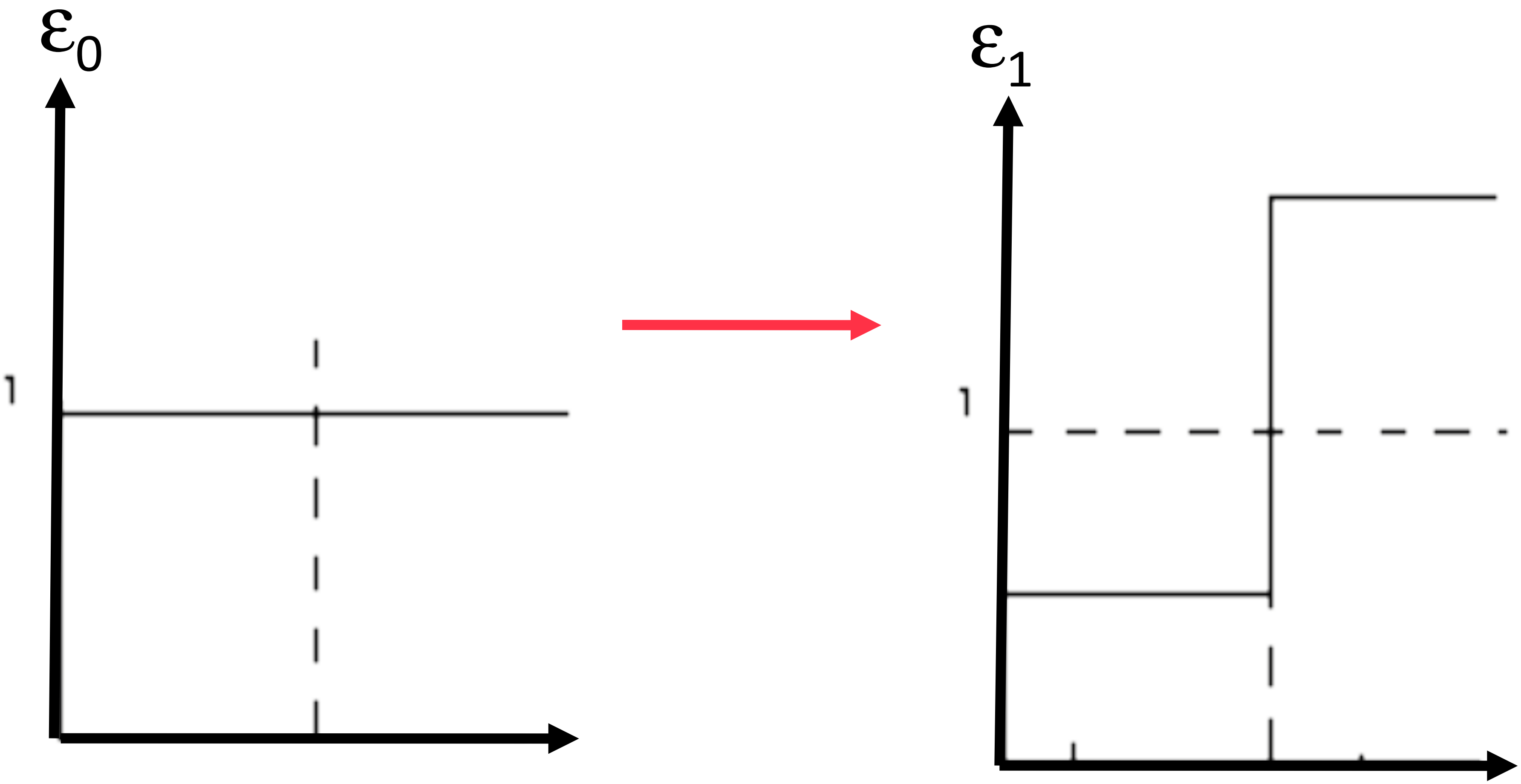
“active”

“calm”



Cascades and Multifractals

Simulations: adding small scale details
(low resolution to high)



("α model")

Multiplicative Cascades

Generic statistical behaviour:

scaling

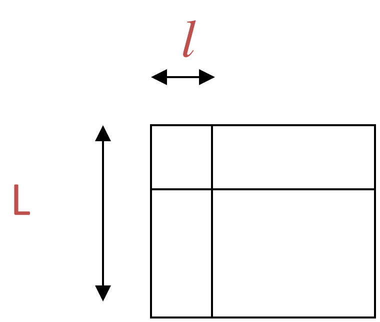
Turbulent flux

Scale invariant

$$\left\langle \varepsilon_{\lambda}^{q} \right\rangle \approx \lambda^{K(q)}$$

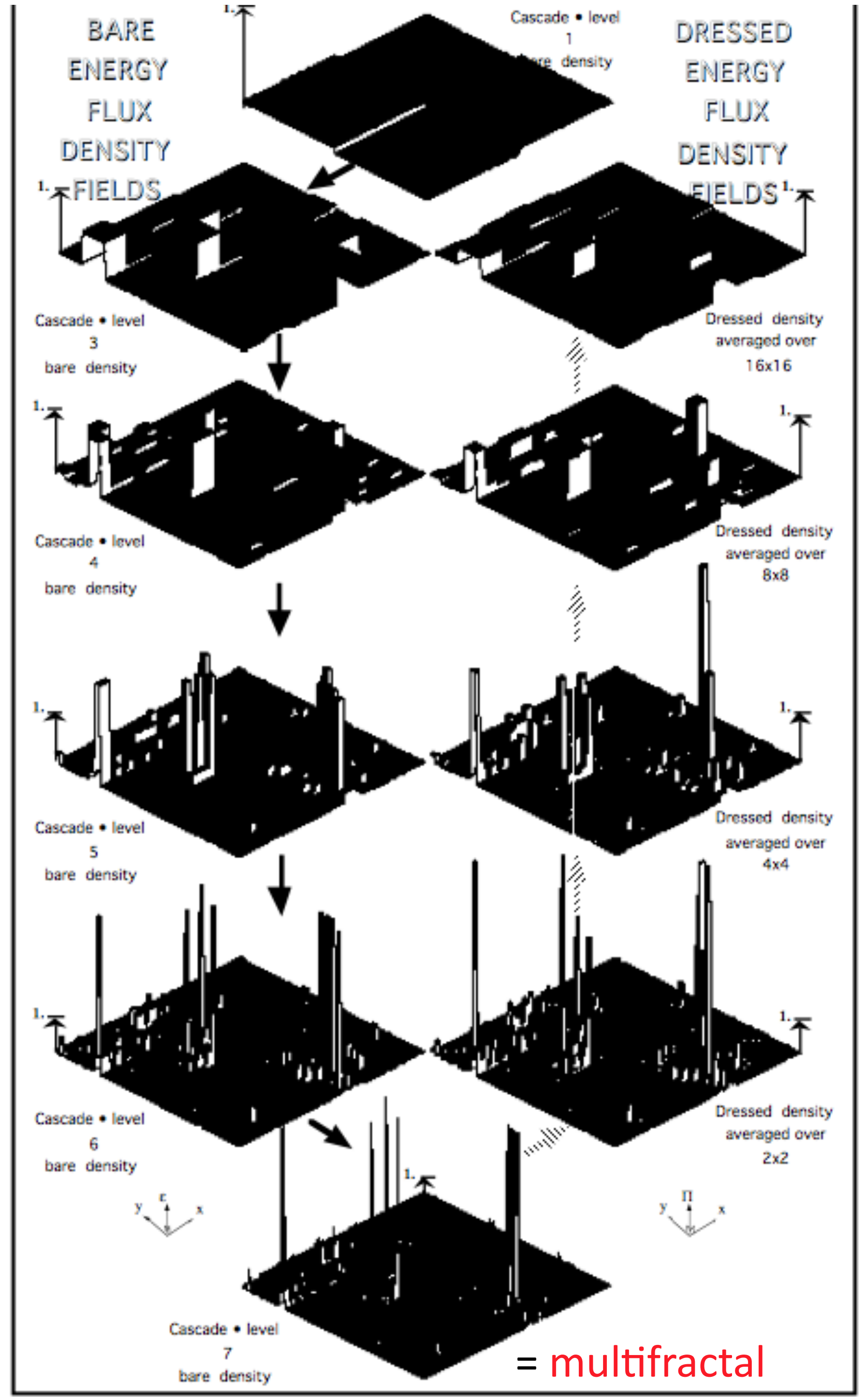
Statistical averaging

Resolution: ratio λ=L/l

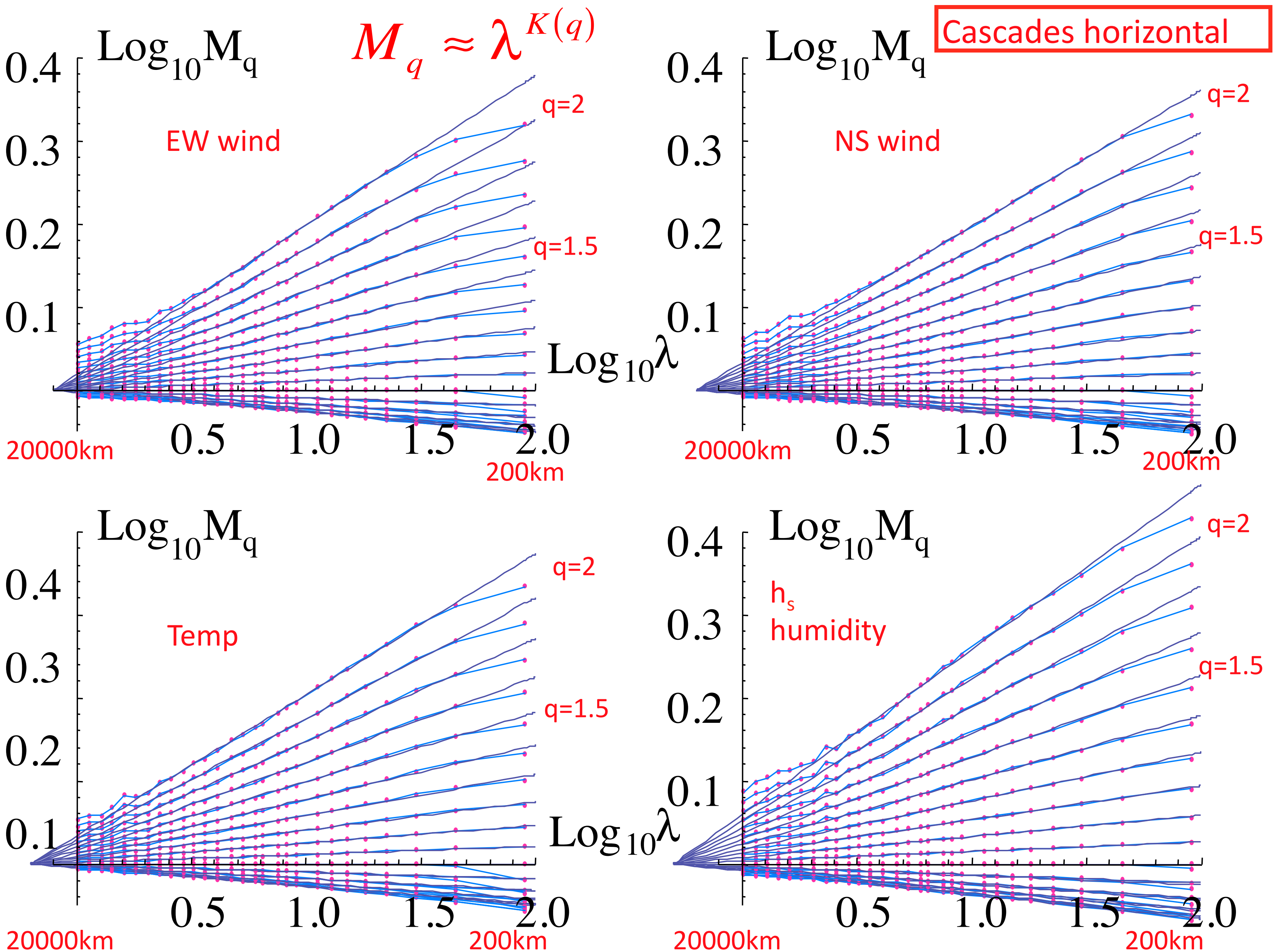


Probabilities:

$$\Pr\left(\varepsilon_{\lambda} > \lambda^{\gamma}\right) \approx \lambda^{-c(\lambda)}$$



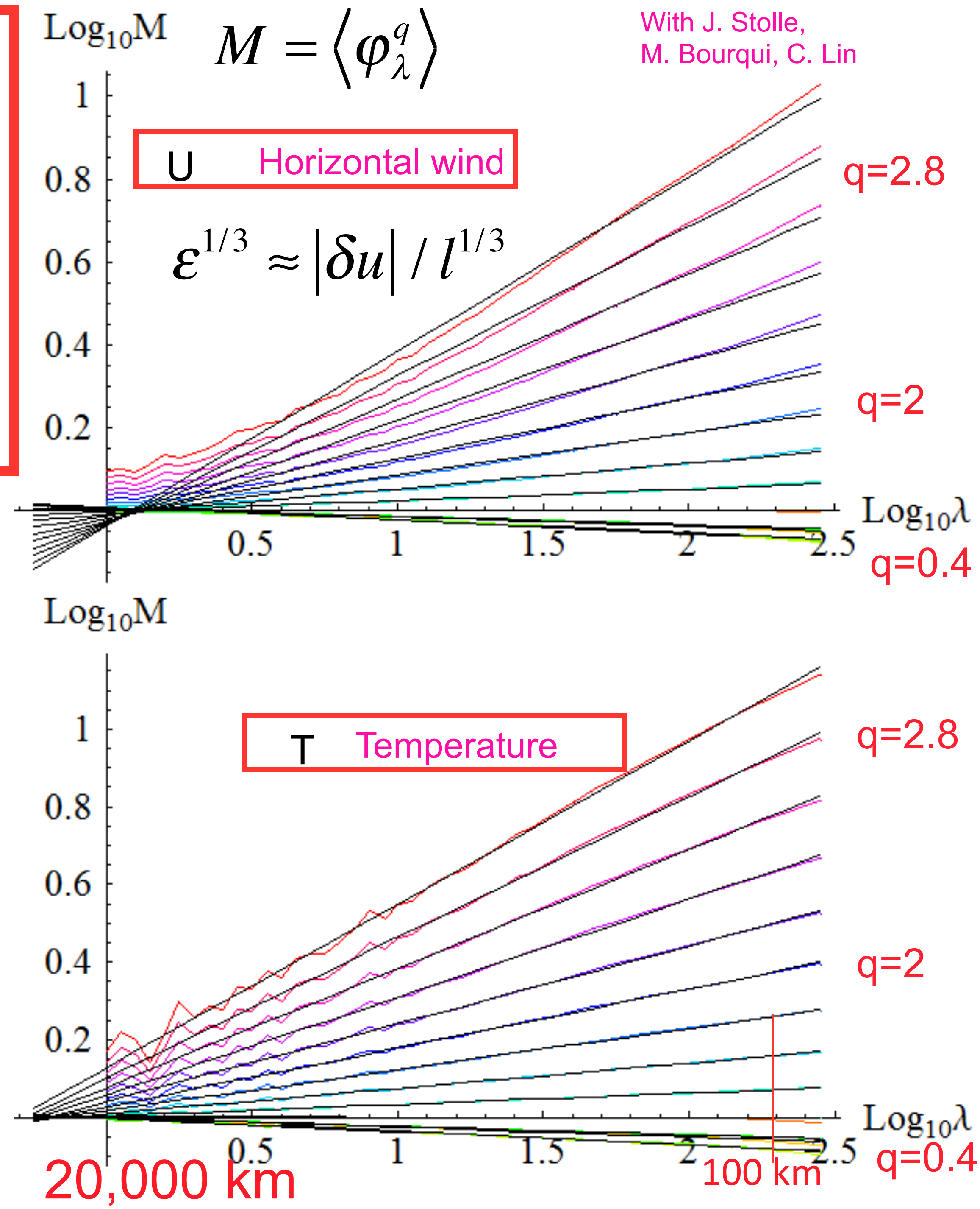
= multifractal


Cascades horizontal
$M_q \approx \lambda^{K(q)}$
Log10Mq
EW wind
NS wind
Temp
hs
humidity
q=2
q=1.5
Log10λ
0.4
0.3
0.2
0.1
0.5
1.0
1.5
2.0
20000km
200km

Global GEMS Model 00h

Analysis of four months U,T at 1000 mb

(48 h forecasts are almost the same)



Multiplicative processes

The process:

$$\varepsilon_\lambda = e^{\Gamma_\lambda}$$

Multifractal process over scale range λ= Multiplicative cascade

The generator= Additive process

Dimension of space

Auxiliary variable α'

$$\frac{1}{\alpha'}+\frac{1}{\alpha}=1$$

$$\Gamma(\underline{r}) \propto C_1^{1/\alpha}\gamma_\alpha(\underline{r}) * \left|\underline{r}\right|_{\left[1,\lambda^{-1}\right]}^{-D/\alpha'}$$

$$\tilde{\Gamma}(\underline{k}) = \tilde{\gamma}_\lambda(\underline{k})\left|\underline{k}\right|^{-D/\alpha}$$

The codimension of the mean= Amplitude of the generator, sparseness of ε

i.i.d. Levy noise index α= subgenerator

Convolution with singularity over range 1 to λ^{-1}

Fourier space= power law filter

The statistics:

$$\left\langle \varepsilon_\lambda^q \right\rangle = \lambda^{K(q)}$$

General multifractal statistics, convex K(q)

$$K(q) = \frac{C_1}{\alpha-1}\left(q^\alpha - q\right)$$

Universal multifractals

Fractionally Integrated Flux (FIF) model (both additive and multiplicative)

The process

$$I(\underline{r}) = \varepsilon_\lambda(\underline{r}) * |\underline{r}|^{-(D-H)} \longleftrightarrow \tilde{I}(\underline{k}) = \tilde{\varepsilon}_\lambda(\underline{k})|\underline{k}|^{-H}$$

Convolution= fractional integration order H

Fourier space= power law filter

The statistics

$$S_q(\underline{\Delta r}) = \left\langle \Delta I(\underline{\Delta r})^q \right\rangle = \left\langle \varepsilon_\lambda^q \right\rangle |\underline{\Delta r}|^{qH} = |\underline{\Delta r}|^{\xi(q)}$$

$$\xi(q) = qH - K(q)$$

q^{th} order structure function

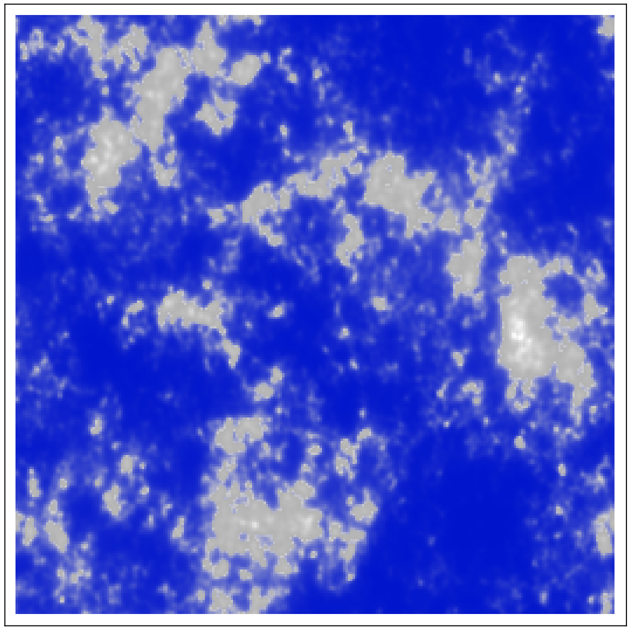
fluctuation

Note:
$\lambda = L/|\underline{\Delta r}|$
$\left\langle \varepsilon_\lambda^q \right\rangle = \lambda^{K(q)}$

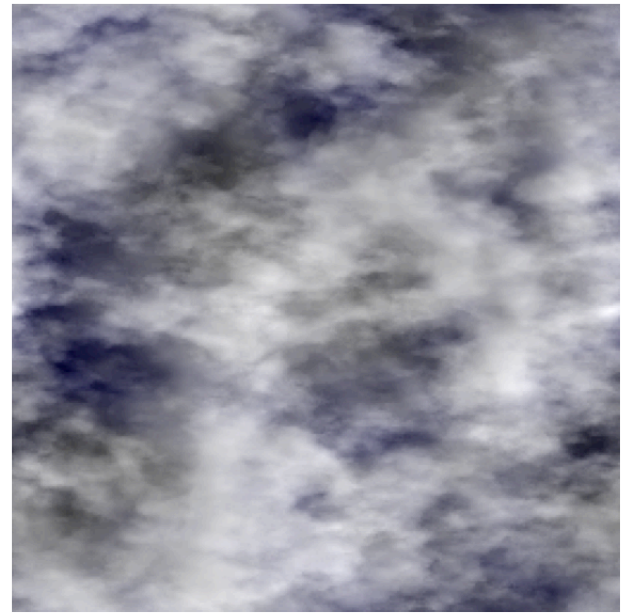
structure function exponent

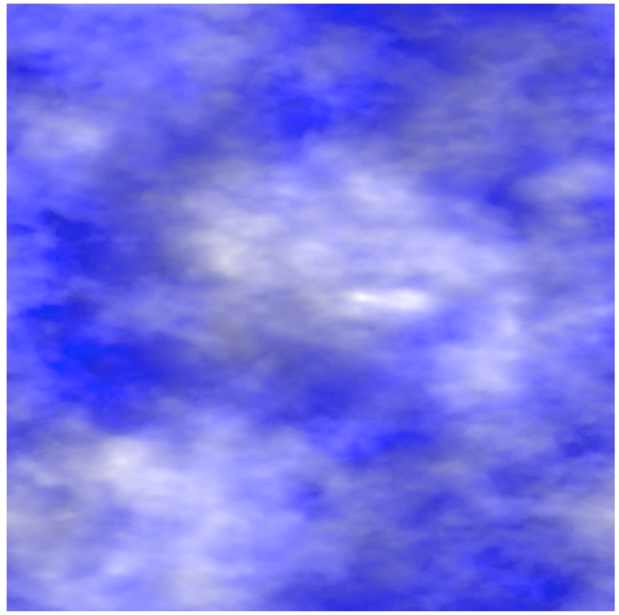
FIF modeling: clouds and radiative transfer

Cloud liquid water (top)

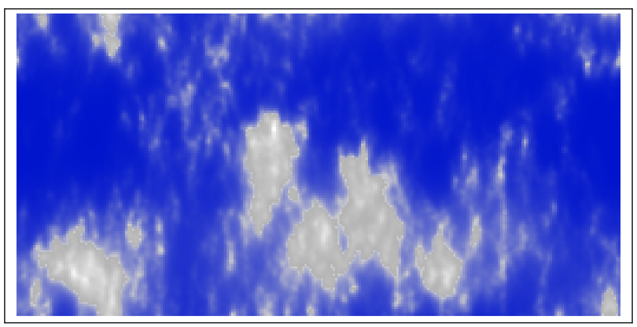


Cloud top visible

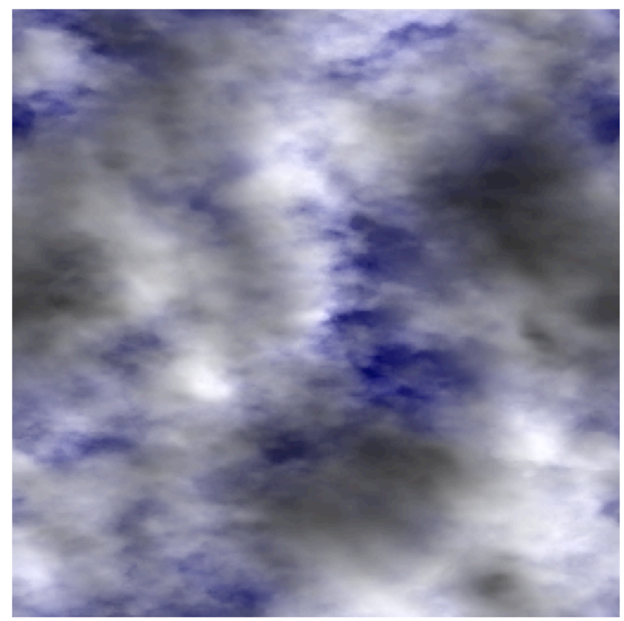




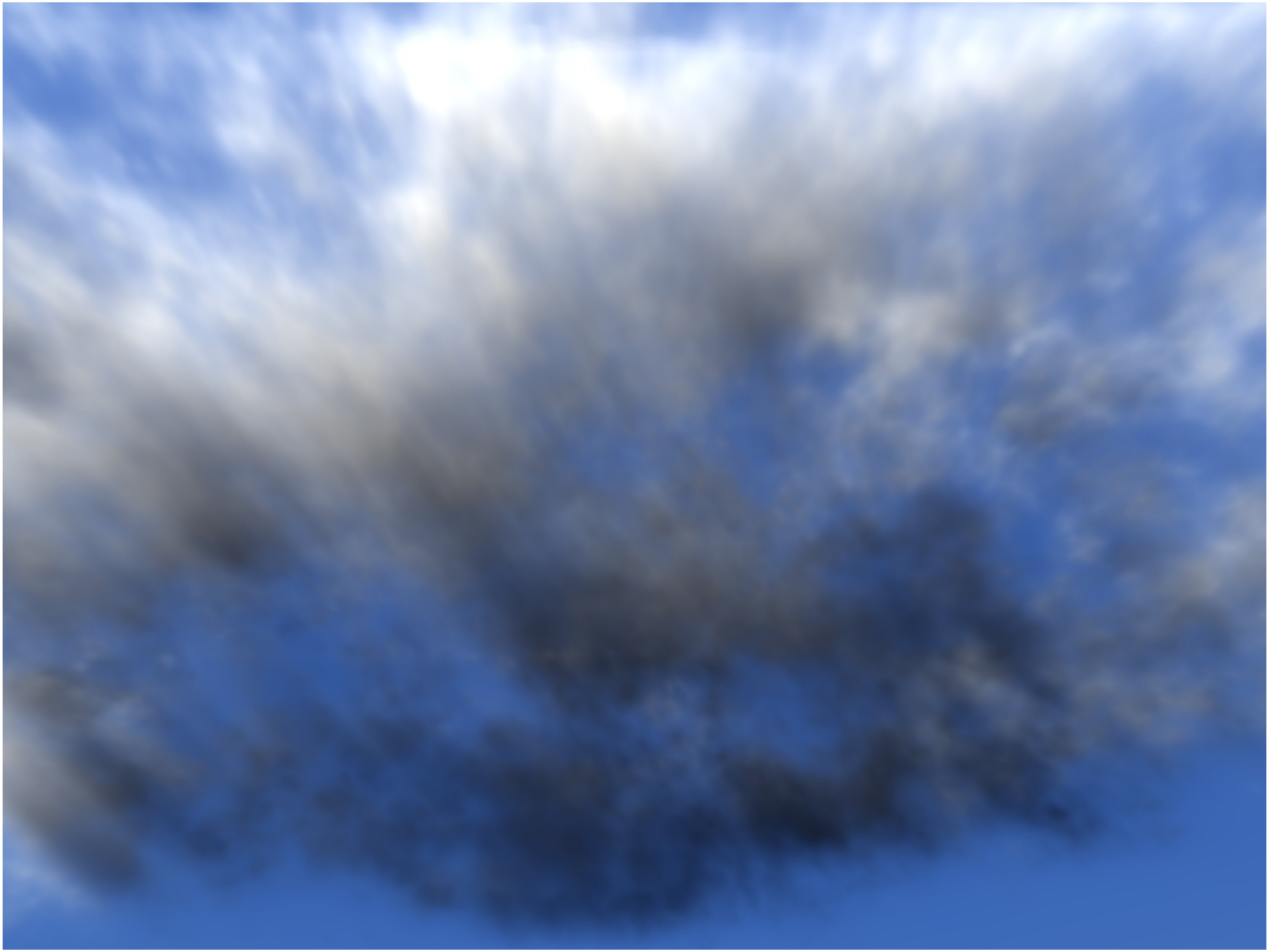
Cloud top, infra red



Cloud liquid water (side)



Cloud bottom visible



Extensions to the vertical (scaling stratification)

The physical scale function and differential scaling

$$\left|\underline{\Delta r}\right| \rightarrow \left\|\underline{\Delta r}\right\|$$

Usual distance (=vector norm)

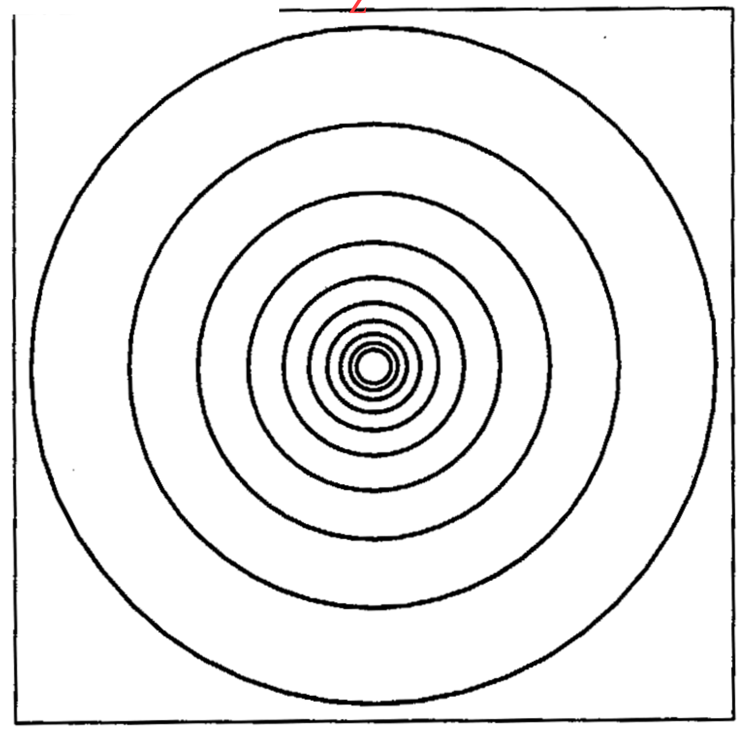
Scale function (scale notion)

"canonical" scale function:

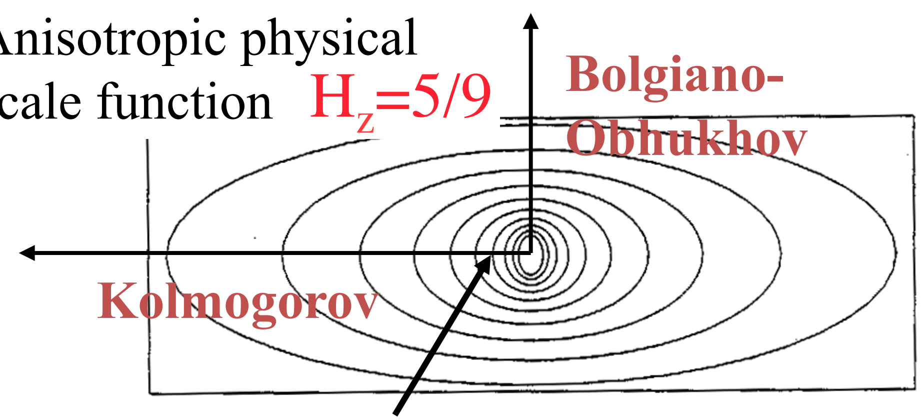
$$\left\|(\Delta x, \Delta z)\right\| = l_s \left(\left(\frac{\Delta x}{l_s} \right)^2 + \left(\frac{\Delta z}{l_s} \right)^{2/H_z} \right)^{1/2}$$

Vertical sections

Isotropic function $H_z=1$



Anisotropic physical scale function $H_z=5/9$

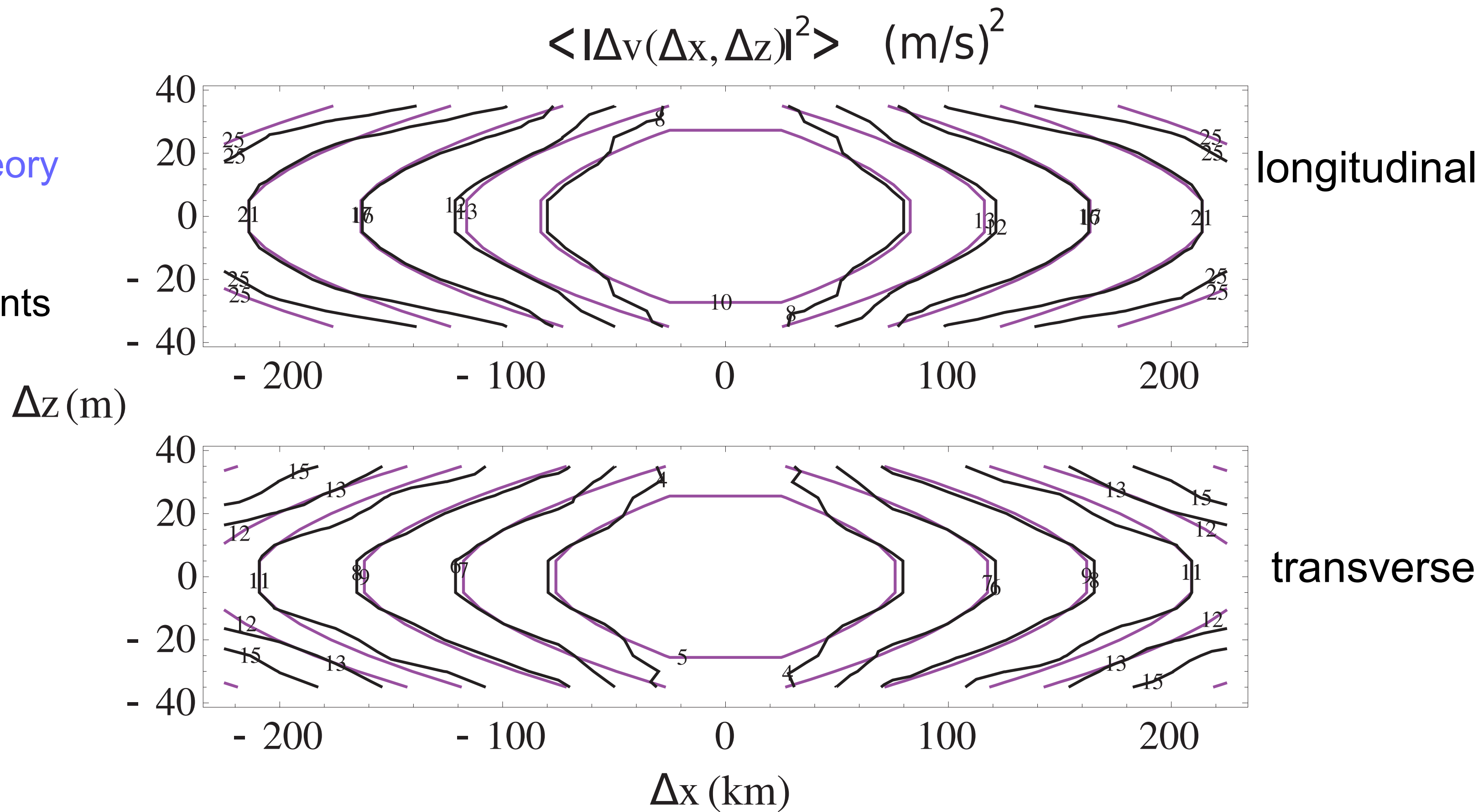


Pinel et al 2012

14500 aircraft flights: 5-5.5km altitude, 2009, US (TAMDAR data)

Purple = theory

Black= measurements



Velocity structure function

$$\left\langle \Delta v^2 (\Delta x, \Delta z) \right\rangle = C \left\| (\Delta x, \Delta z) \right\|^{\xi(2)}$$

$$\xi(2) \approx 0.80$$

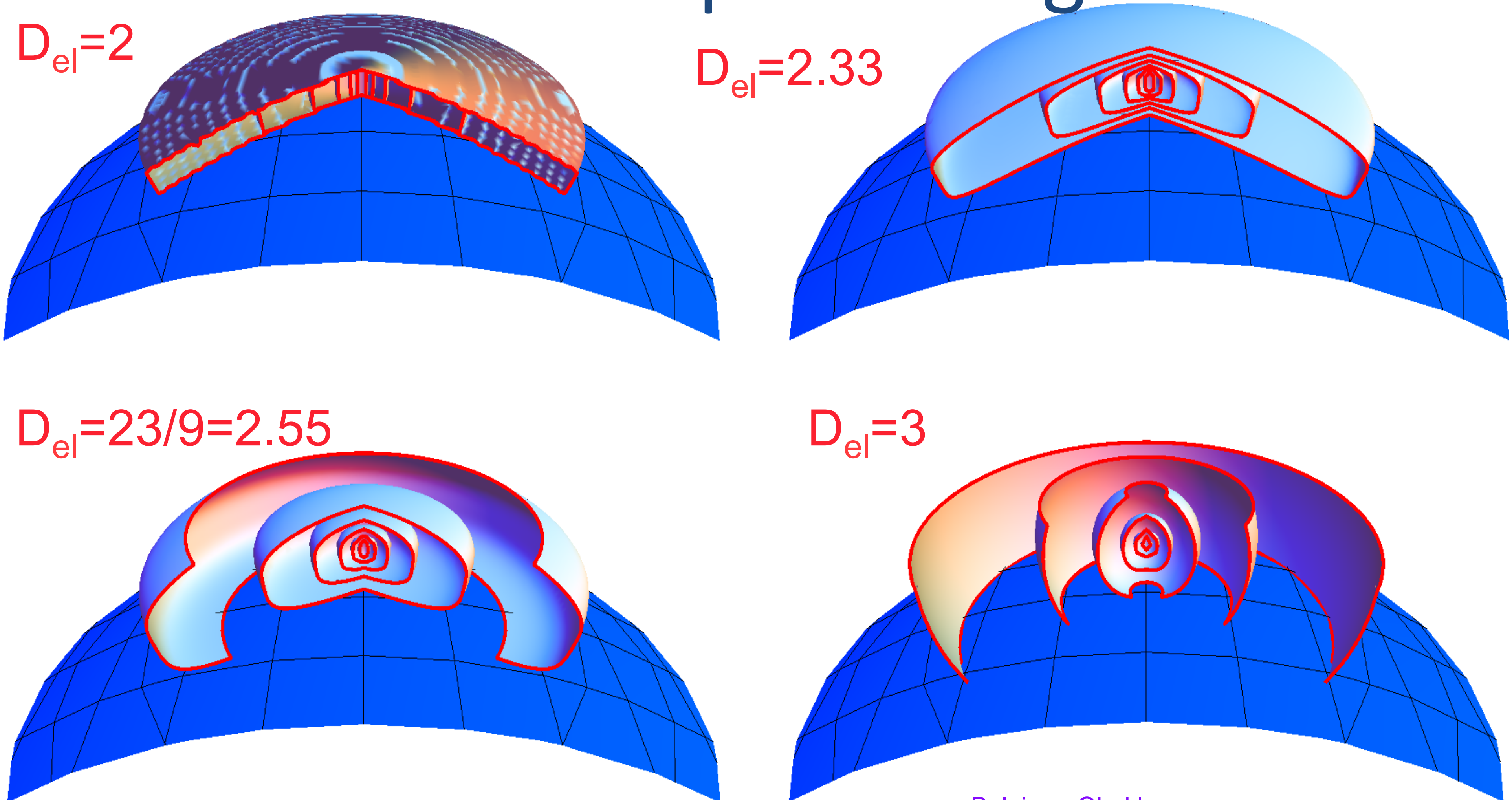
Canonical scale function

$$\left\| (\Delta x, \Delta z) \right\| = \left(\left(\frac{\Delta x}{l_s} \right)^2 + \left(\frac{\Delta z}{l_s} \right)^{2/H_z} \right)^{1/2}$$

$$H_z \approx 0.57 \pm 0.01$$

(Theory: 5/9=0.555…)

Anisotropic Scaling



The 23/9D model:

Bolgiano-Obukhov

$$\underbrace{\Delta v(\Delta x) = \varepsilon^{1/3}\Delta x^{1/3}}_{\text{Kolmogorov}}; \quad \overbrace{\Delta v(\Delta z) = \phi^{1/5}\Delta z^{3/5}}^{\text{Bolgiano-Obukhov}}$$

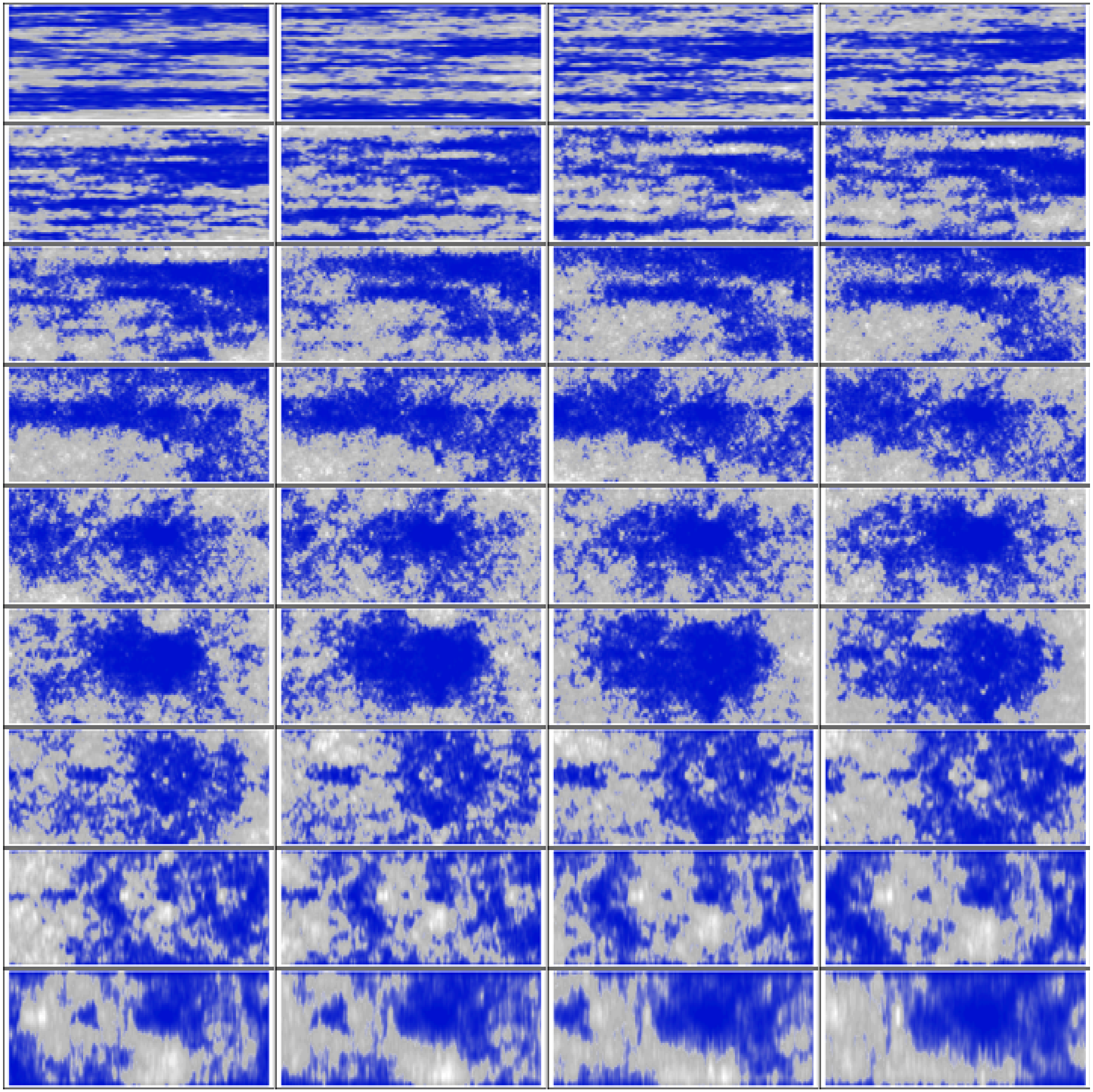
$H_z=(1/3)/(3/5)=5/9$

Kolmogorov

Volume≈LxLxL^{Hz}≈L^{Del}

$D_{el}=2+H_z=23/9$

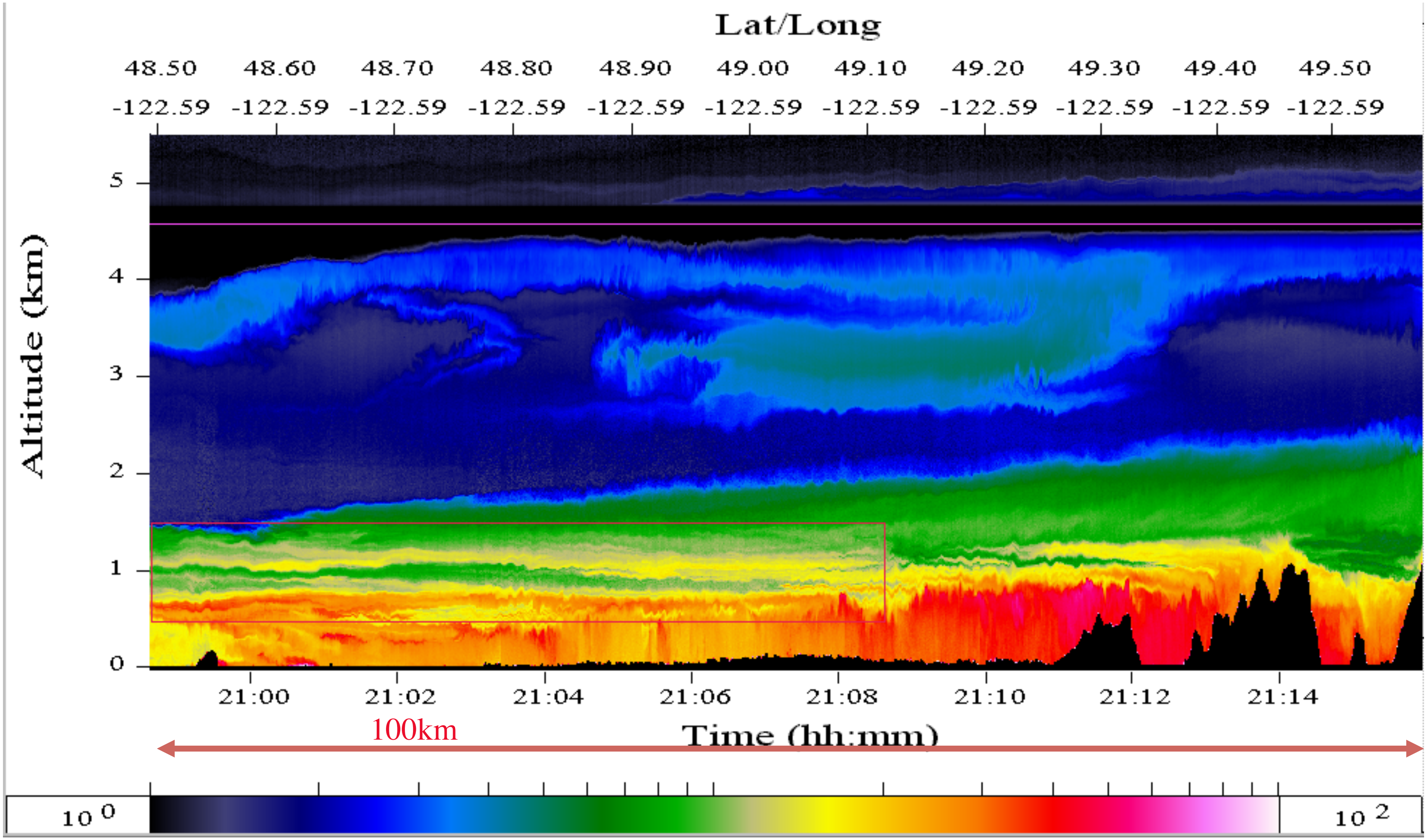
Zoom factor 1000



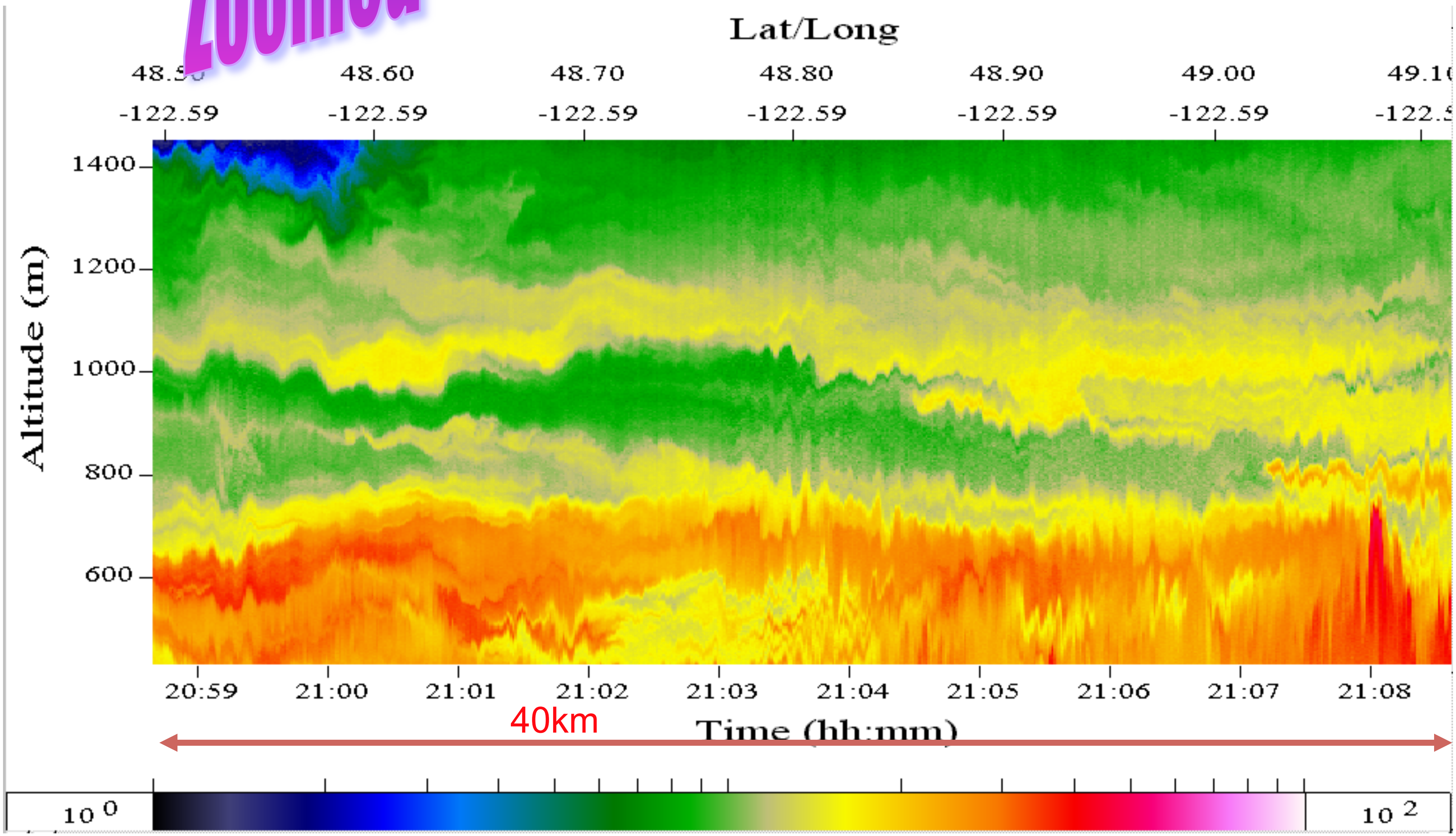
Vertical cross-section

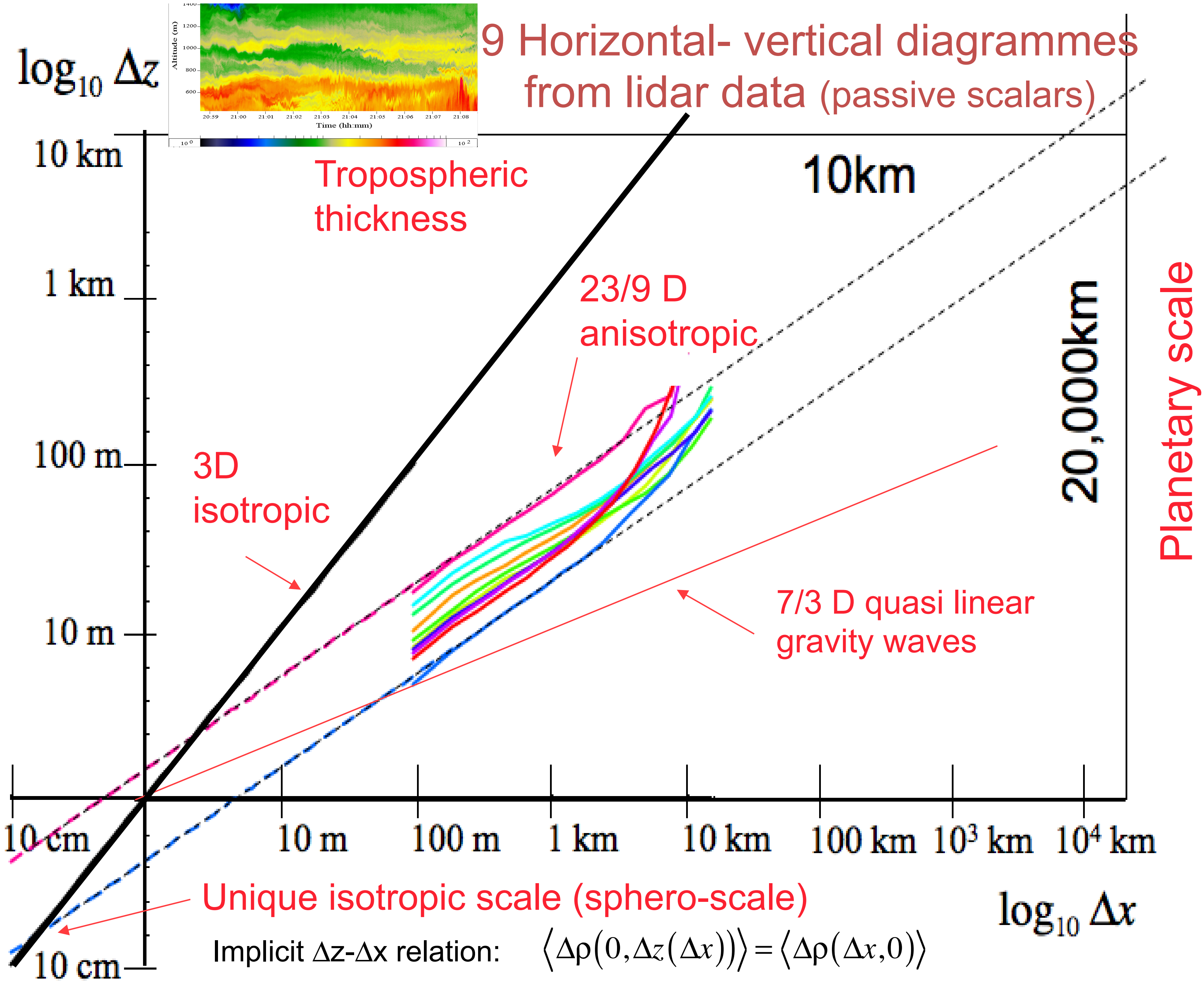
AERIAL Lidar Data

(courtesy of K. Strawbridge)



Zoomed

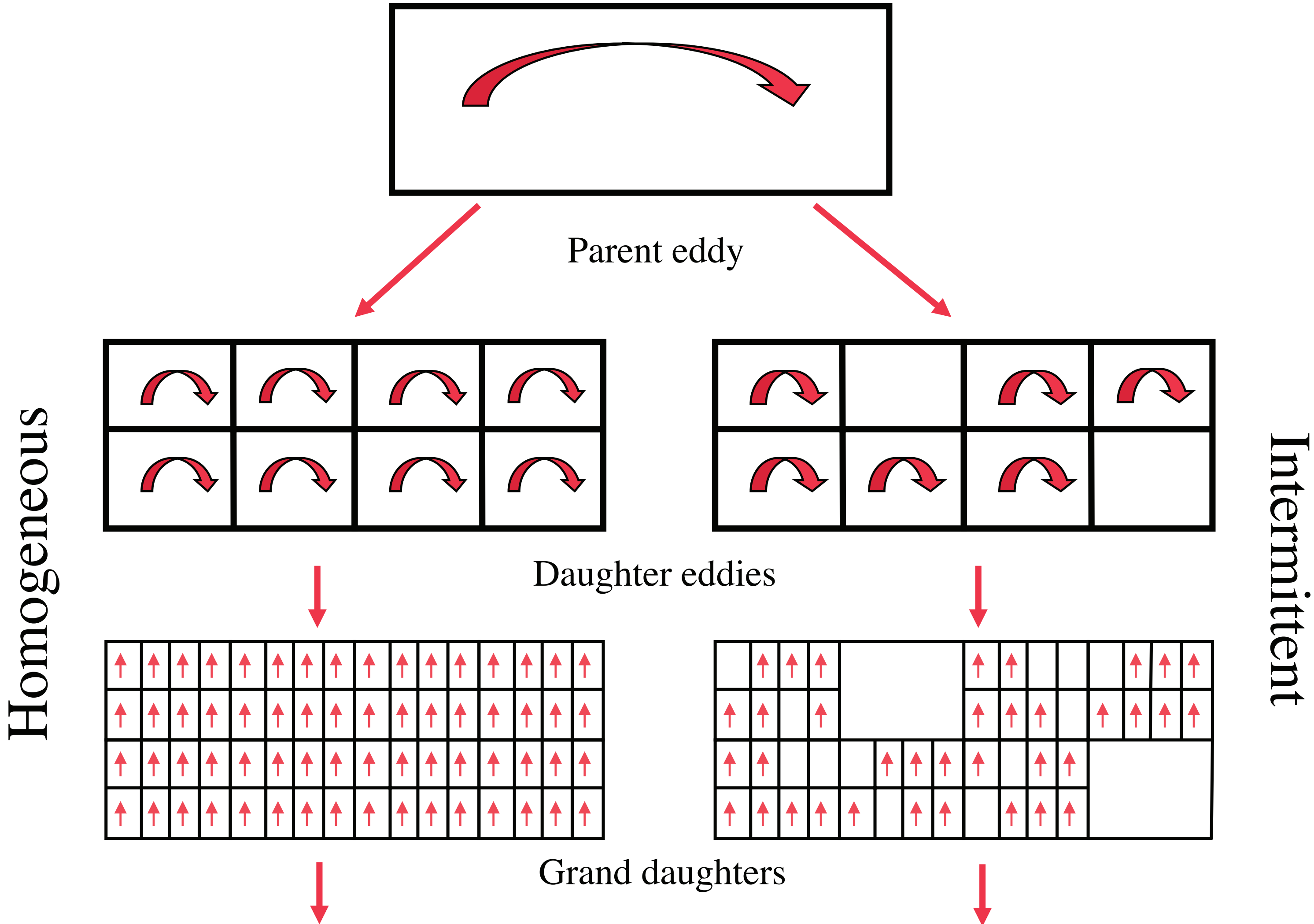

Lat/Long
48.60
48.70
48.80
48.90
49.00
-122.59
-122.59
-122.59
-122.59
-122.59
-122.59
1400
1200
1000
800
600
Altitude (m)
20:59
21:00
21:01
21:02
21:03
21:04
21:05
21:06
21:07
21:08
Time (hh:mm)
40km
10^0
10^2



9 Horizontal- vertical diagrammes
from lidar data (passive scalars)
$\log_{10} \Delta z$
10 km
1 km
100 m
10 m
10 cm
Tropospheric
thickness
10km
20,000km
Planetary scale
23/9 D
anisotropic
3D
isotropic
7/3 D quasi linear
gravity waves
10 cm
10 m
100 m
1 km
10 km
100 km
10^3 km
10^4 km
$\log_{10} \Delta x$
Unique isotropic scale (sphero-scale)
Implicit Δz-Δx relation: $\langle \Delta\rho(0,\Delta z(\Delta x)) \rangle = \langle \Delta\rho(\Delta x,0) \rangle$

Fly by of anisotropic (multifractal, cascade) cloud

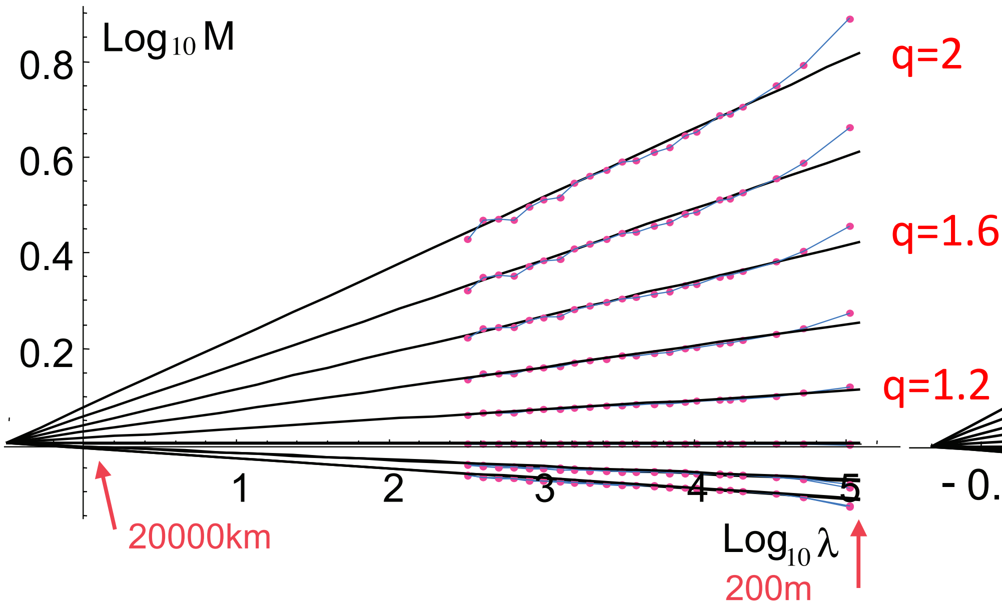
Stratified CASCADES



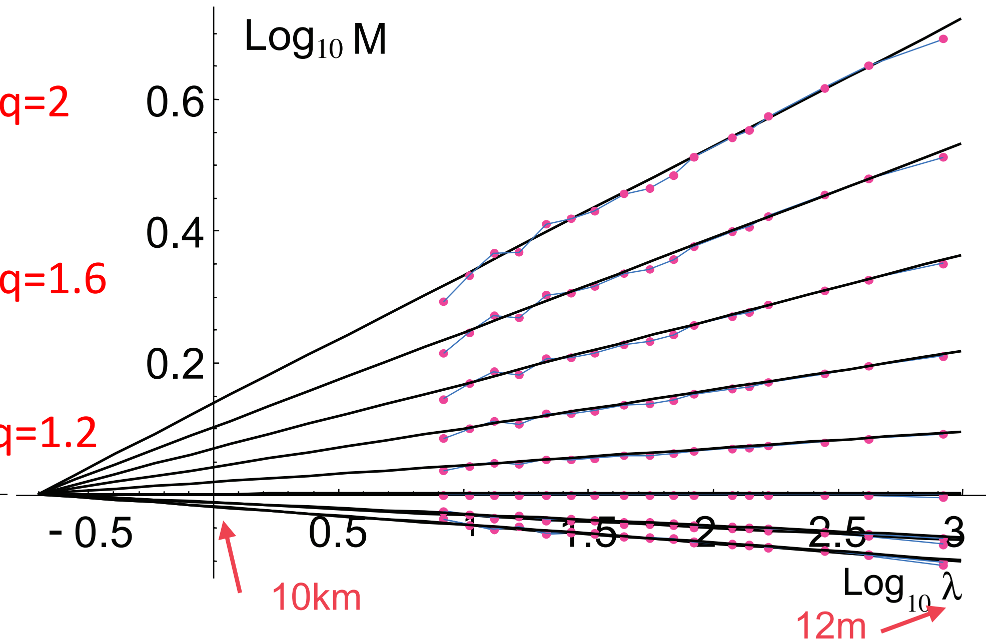
$$D = \frac{Log 8}{Log 4} = \frac{3}{2}$$

$$D = \frac{Log 6}{Log 4} = 1.29...$$

Lidar Backscatter



Horizontal

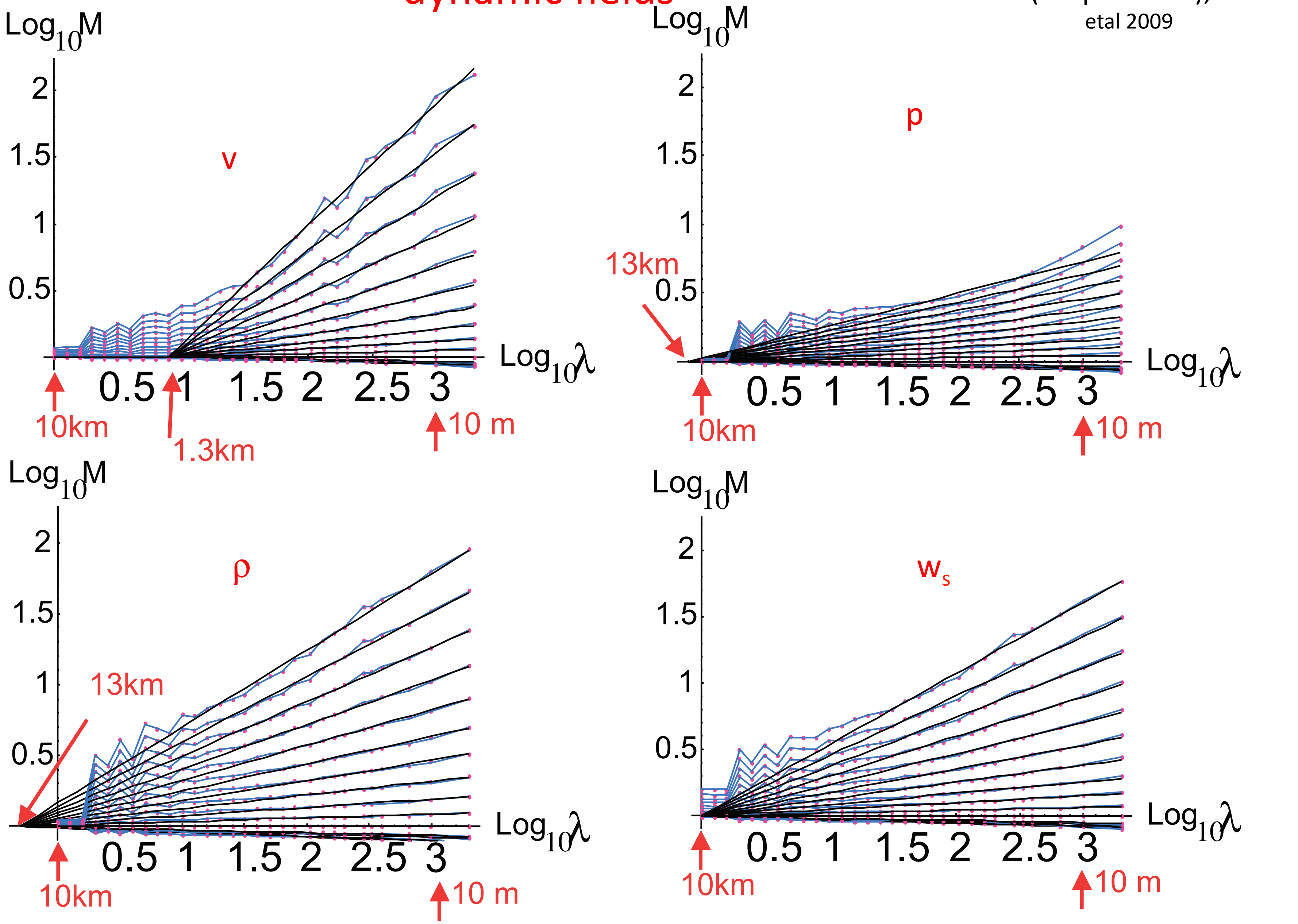


Vertical

Vertical cascades:

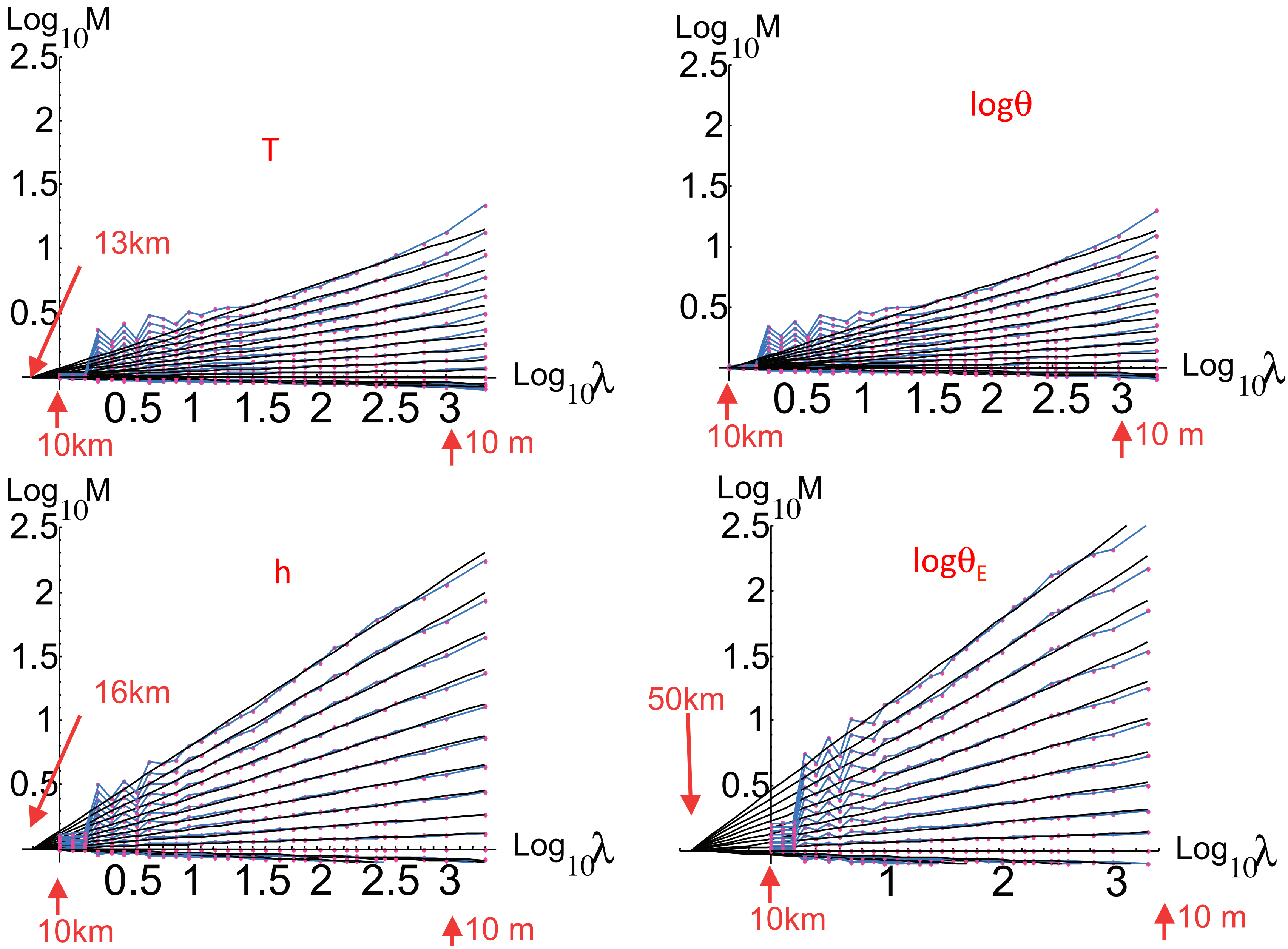
dynamic fields

(drop sondes), L etal 2009



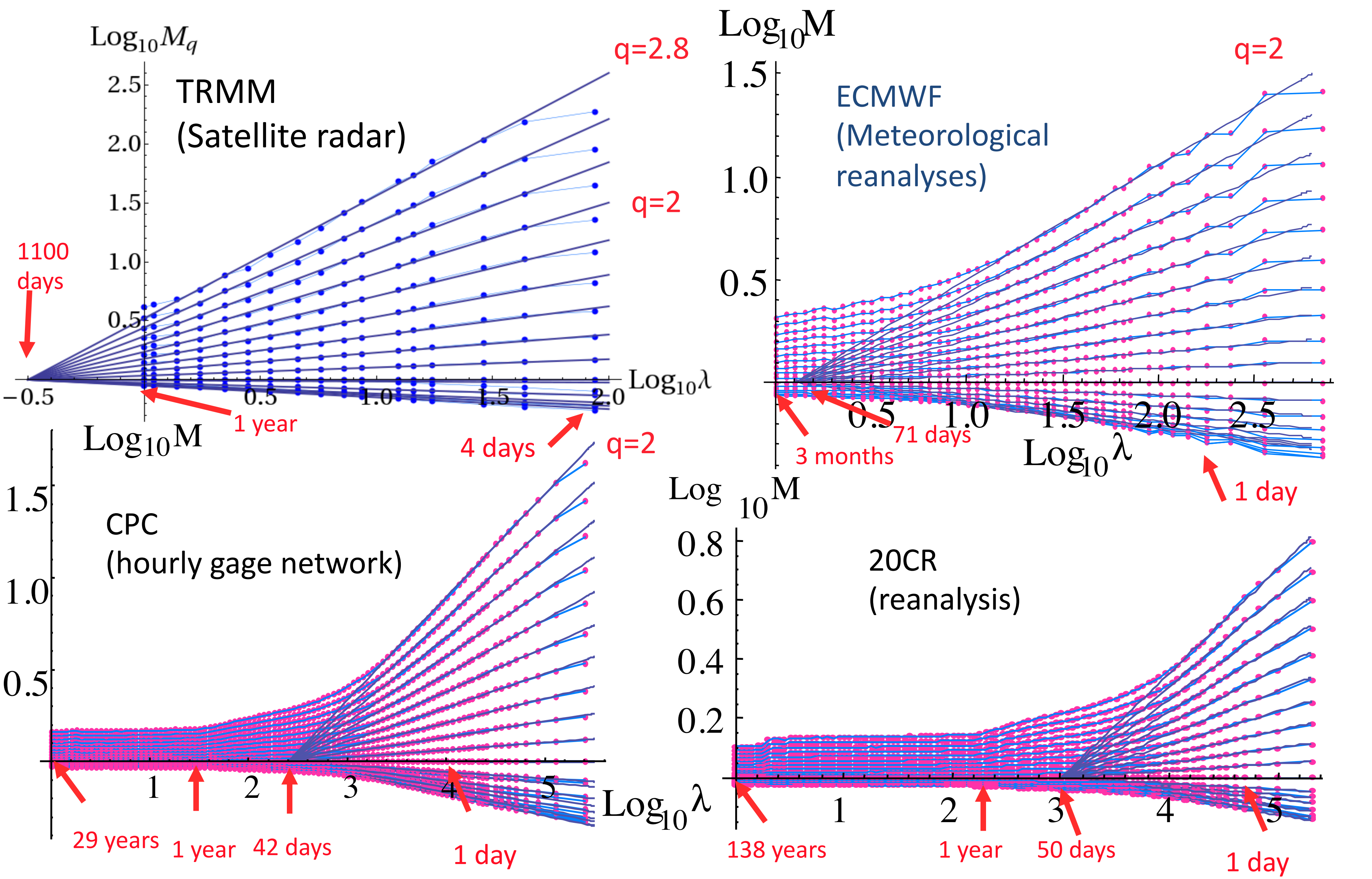
Vertical cascades:

Thermodynamic fields



$M = \langle \phi_\lambda^q \rangle / \langle \phi \rangle^q$

Rainrate Moments: (time)



Extension from space to space-time (including waves)

Turbulence in Space-time (horizontal)

Theory (assuming largest eddies "sweep" smaller ones)

Observable → I; Turbulent flux forcing → φ

$$g_I^{-1}(\underline{r},t) * I(\underline{r},t) = \varphi(\underline{r},t)$$

$$g_I(\underline{r},t) \overset{F.T.}{\leftrightarrow} \tilde{g}_I(\underline{k},\omega)$$

propagator → g_I

$$\omega' = (\omega + \underline{k}\cdot\underline{\mu})\sigma^{-1} \qquad \|\underline{k}\| = \left(k_x^2 + k_y^2 / a^2\right)^{1/2}$$

$$\sigma = \left(1 - \left(\mu_x^2 + a^2\mu_y^2\right)\right)^{1/2}$$

$$\underline{\mu} = \left(\overline{v_x}, \overline{v_y}\right) / V_w \qquad V_w = \varepsilon_{L_e} L_e^{1/3}$$

EW/NS aspect ratio = a
mean horizontal wind= $\left(\overline{v_x}, \overline{v_y}\right)$
Mean planetary scale energy flux ε_{L_e}
Planet size: L_e = 20000 km

$$\tilde{I}(\underline{k},\omega) = \tilde{g}_I(\underline{k},\omega)\tilde{\varphi}(\underline{k},\omega)$$

$$\tilde{g}_{tur}(\underline{k},\omega) = \left(i\omega' + \|\underline{k}\|\right)^{-H_{tur}}$$

← Pure (localized) turbulence propagator

Turbulence and waves

Turbulence forcing

Turbulent flux

$$\tilde{I}(\underline{k},\omega) = \tilde{g}_I(\underline{k},\omega)\tilde{\varphi}(\underline{k},\omega)$$

Turbulence-waves

$$\tilde{g}_I(\underline{k},\omega) = \tilde{g}_{wav}(\underline{k},\omega)\tilde{g}_{tur}(\underline{k},\omega)$$

Wheeler Kiladis 1999
factorization

Propagator symmetries, constraints

Reality

$$\tilde{g}(\underline{k},\omega) = \tilde{g}^*(-\underline{k},-\omega)$$

Space-time scaling

$$\tilde{g}\left(\lambda^{-1}(\underline{k},\omega)\right) = \lambda^{-H}\tilde{g}(\underline{k},\omega)$$

Causality

Poles of g in ω plane are below real axis:

$\omega' = -i\|\underline{k}\|$ OK since $\|\underline{k}\| \geq 0$

Simple wave ansatz

Simple scaling wave propagator

Fractional (and anisotropic) wave equation propagator

$$\tilde{g}_{wav}(\underline{k},\omega)=\left(\omega'^2/v_{wav}^2-\left\|\underline{k}\right\|^2\right)^{-H_{wav}/2}$$

$$H=H_{tur}+H_{wav}$$

Dispersion relation:

$$\omega=-\underline{k}\cdot\underline{\mu}\pm\sigma v_{wav}\left\|\underline{k}\right\|$$

$$\omega'=\pm v_{wav}\left\|\underline{k}\right\|$$

Spectral density

Turbulent part

Wave part

$$P_I(\underline{k},\omega)=P_\varphi(\underline{k},\omega)\left|\widetilde{g_I}\right|^2$$

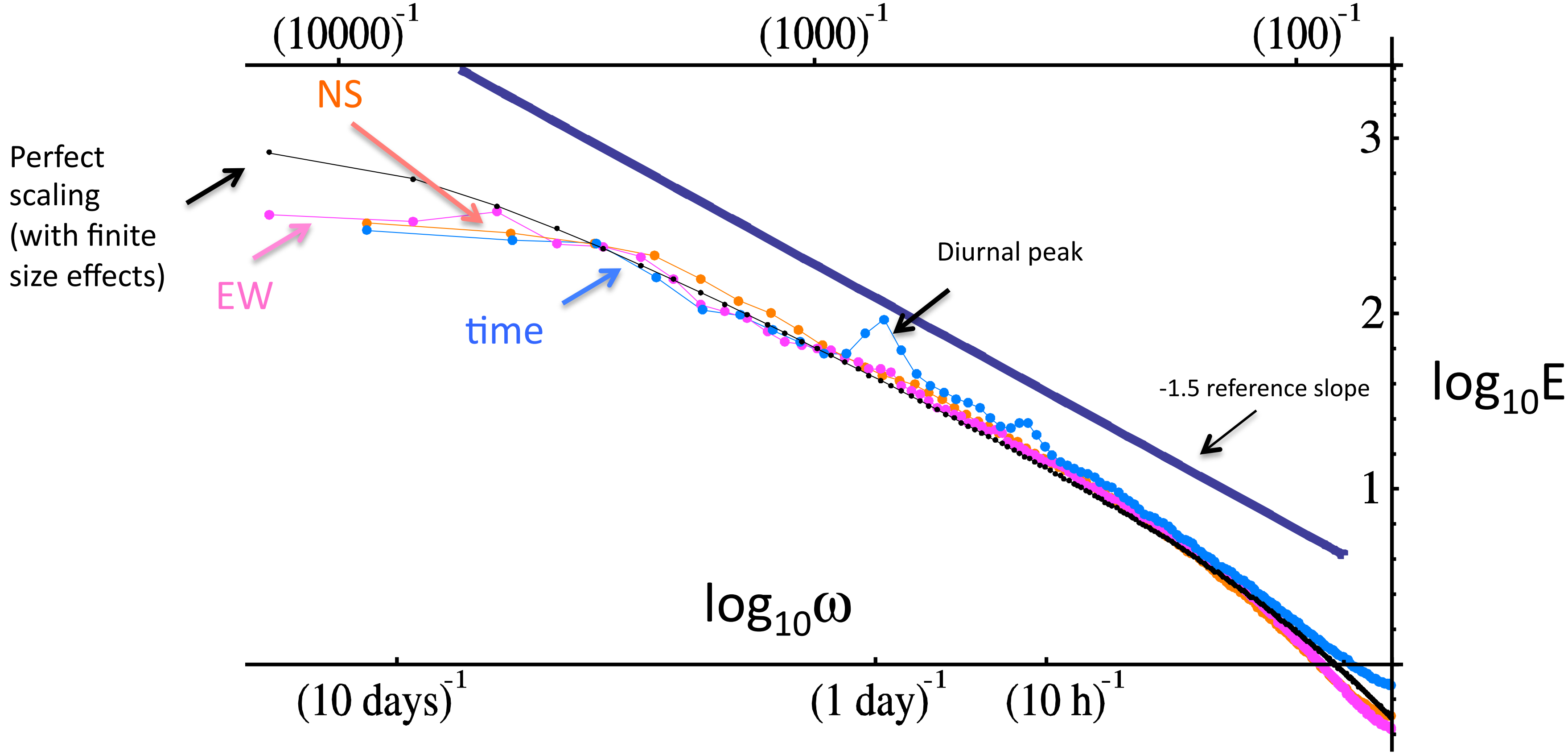
$$\left|\widetilde{g_I}\right|^2=\left|\widetilde{g_{tub}}\right|^2\left|\widetilde{g_{wav}}\right|^2=\left(\omega'^2+\left\|\underline{k}\right\|^2\right)^{-H_{tur}}\left(\omega'^2/v_{wav}^2-\left\|\underline{k}\right\|^2\right)^{-H_{wav}/2}$$

$$P_\varphi(\underline{k},\omega)=P_0\left(\omega'^2+\left\|\underline{k}\right\|^2\right)^{-s_\varphi/2}$$

Spectrum of turbulence forcing

1400 MTSAT IR images -30° to +40° latitude, Pacific (Spectrum, 1-D subspaces)

1 hour, 10 km resolution



Space-time scaling is accurately respected:

$\tilde{g}\left(\lambda^{-1}\left(\underline{k},\omega\right)\right)=\lambda^{-H}\tilde{g}\left(\underline{k},\omega\right)$ implies $E\left(k_x\right)\approx k_x^{-\beta};\quad E\left(k_y\right)\approx k_y^{-\beta};\quad E\left(\omega\right)\approx \omega^{-\beta}$

Spectrum, 2-D subspaces

(no waves)

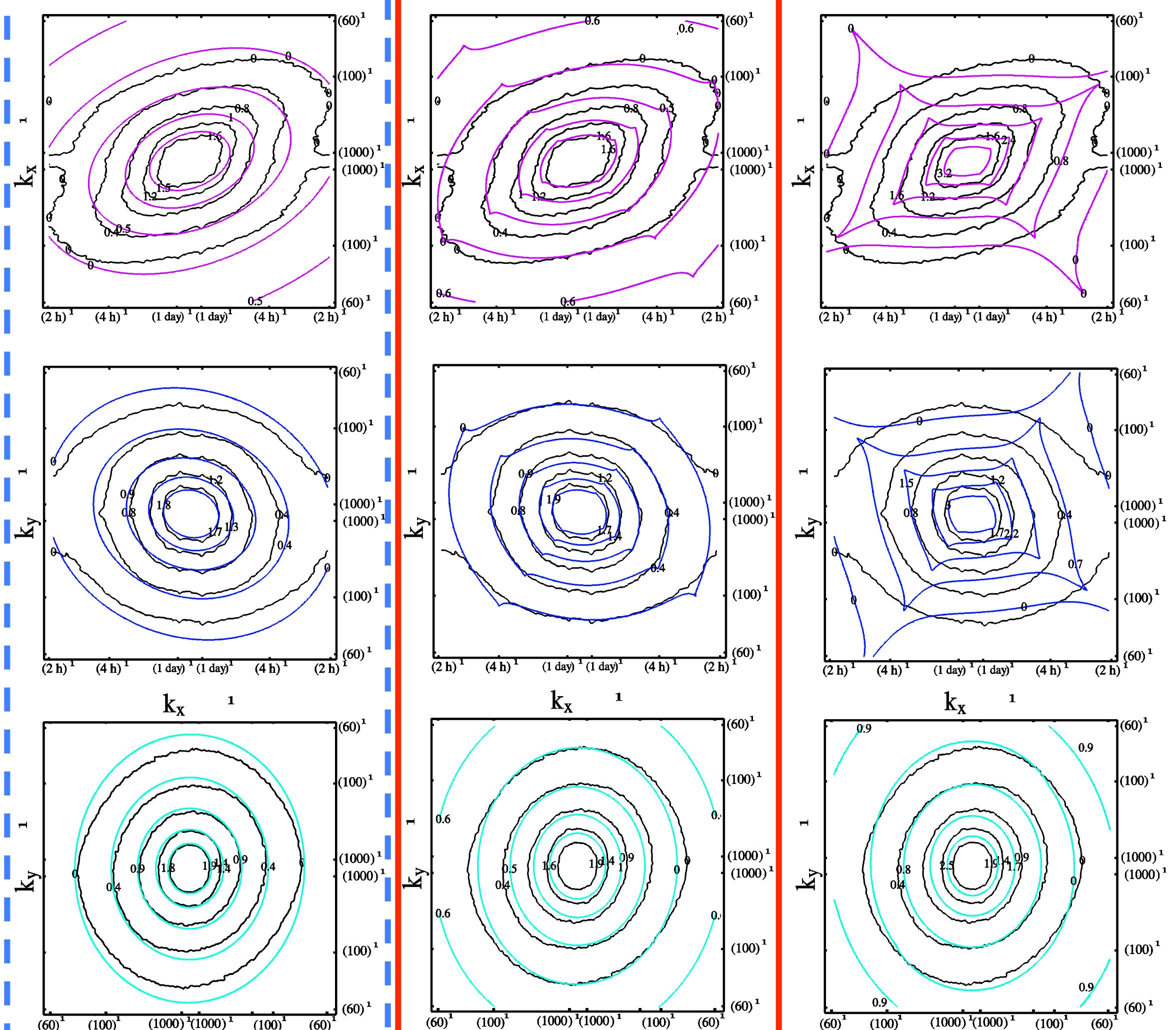
$H_{wav} = 0$ | $H_{wav} = 0.17$ | $H_{wav} = 1$

(Classical wave equation: H_{wav}=2)

(ω,k_x)

(ω,k_y)

(k_x,k_y)



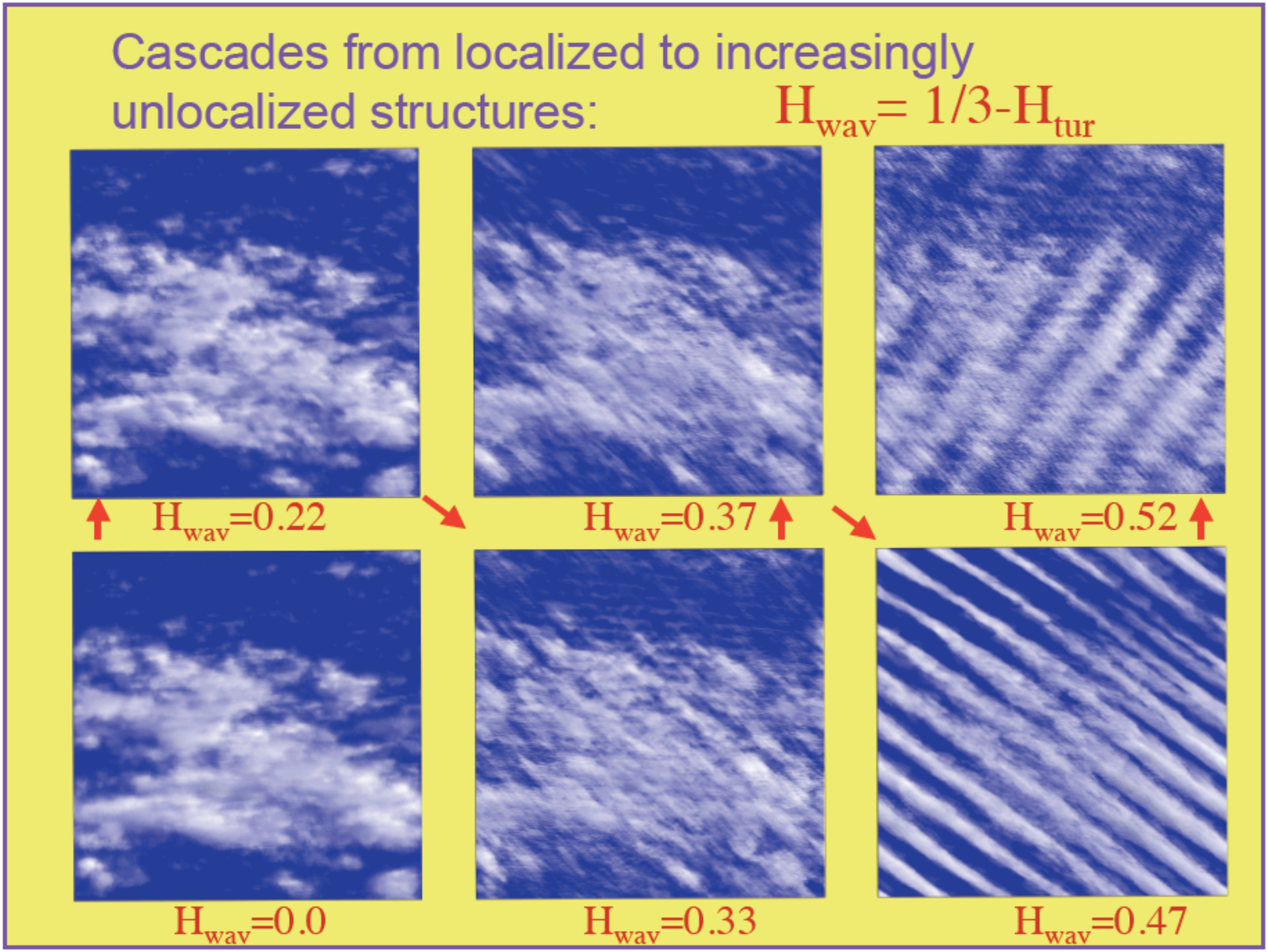
Parameters

$H_{wav} = 0.17 \pm 0.04$
$H_{tur} = 0.09 \pm 0.05$
$s_\varphi = 2.88 \pm 0.01$

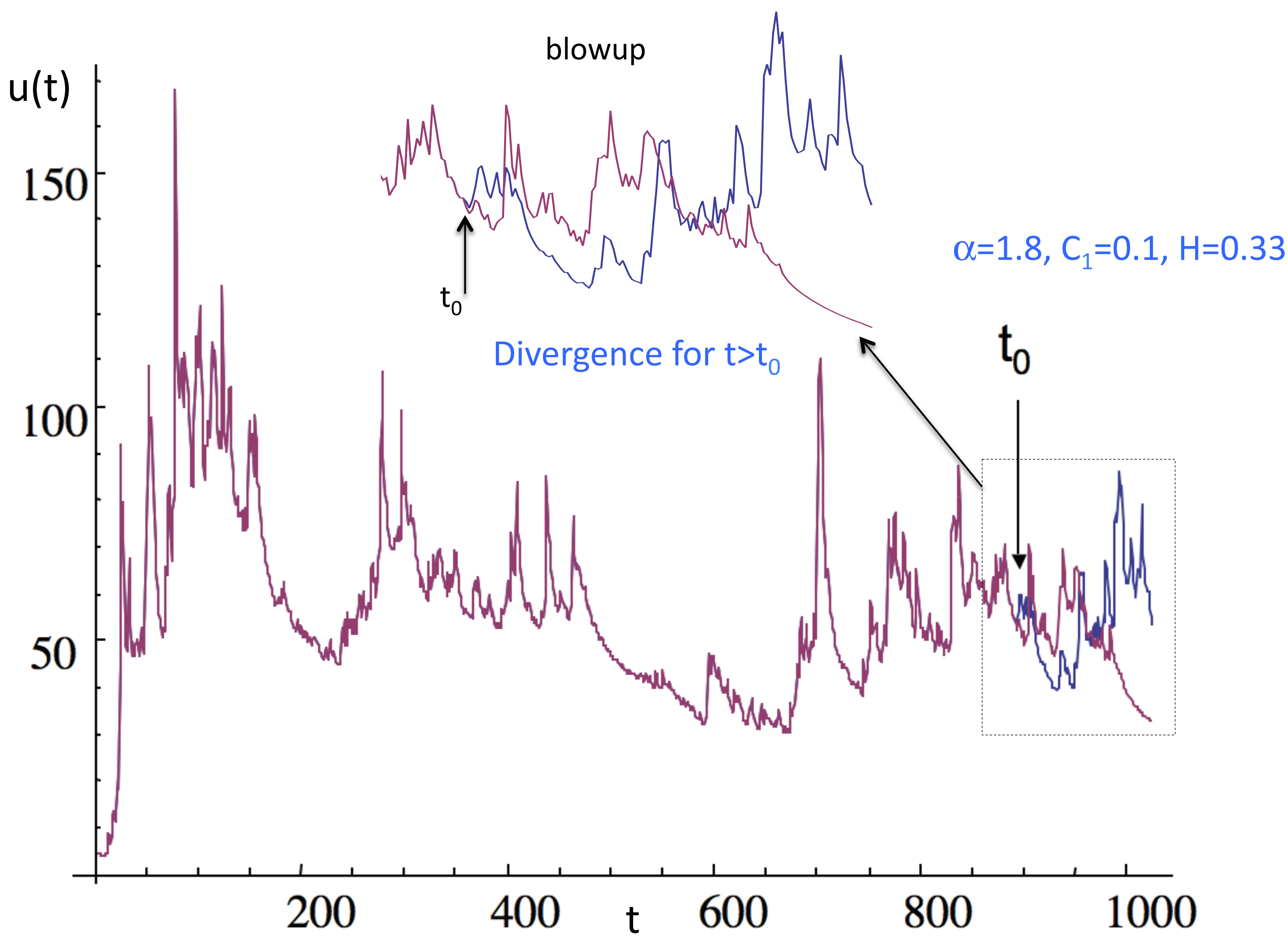
V_w = 41±3 km/h
$\tau_w = L_e/V_w \approx$ 20±1 days
$a \approx$1.2±0.1
$\mu_x \approx$ -0.3±0.1; (v_x -12±4 km/h)
$\mu_y \approx$ 0.10±0.08; $v_y \approx$ 4±3 km/h

Spectral density: $P_I(\underline{k},\omega) \propto \left(\omega'^2 + \|\underline{k}\|^2\right)^{-H_{tur}-s_\varphi} \left(\omega'^2 / v_{wav}^2 - \|\underline{k}\|^2\right)^{-H_{wav}/2}$;

$$H = H_{tur} + H_{wav}$$


Cascades from localized to increasingly unlocalized structures:
$H_{wav} = 1/3 - H_{tur}$
$H_{wav}=0.22$
$H_{wav}=0.37$
$H_{wav}=0.52$
$H_{wav}=0.0$
$H_{wav}=0.33$
$H_{wav}=0.47$

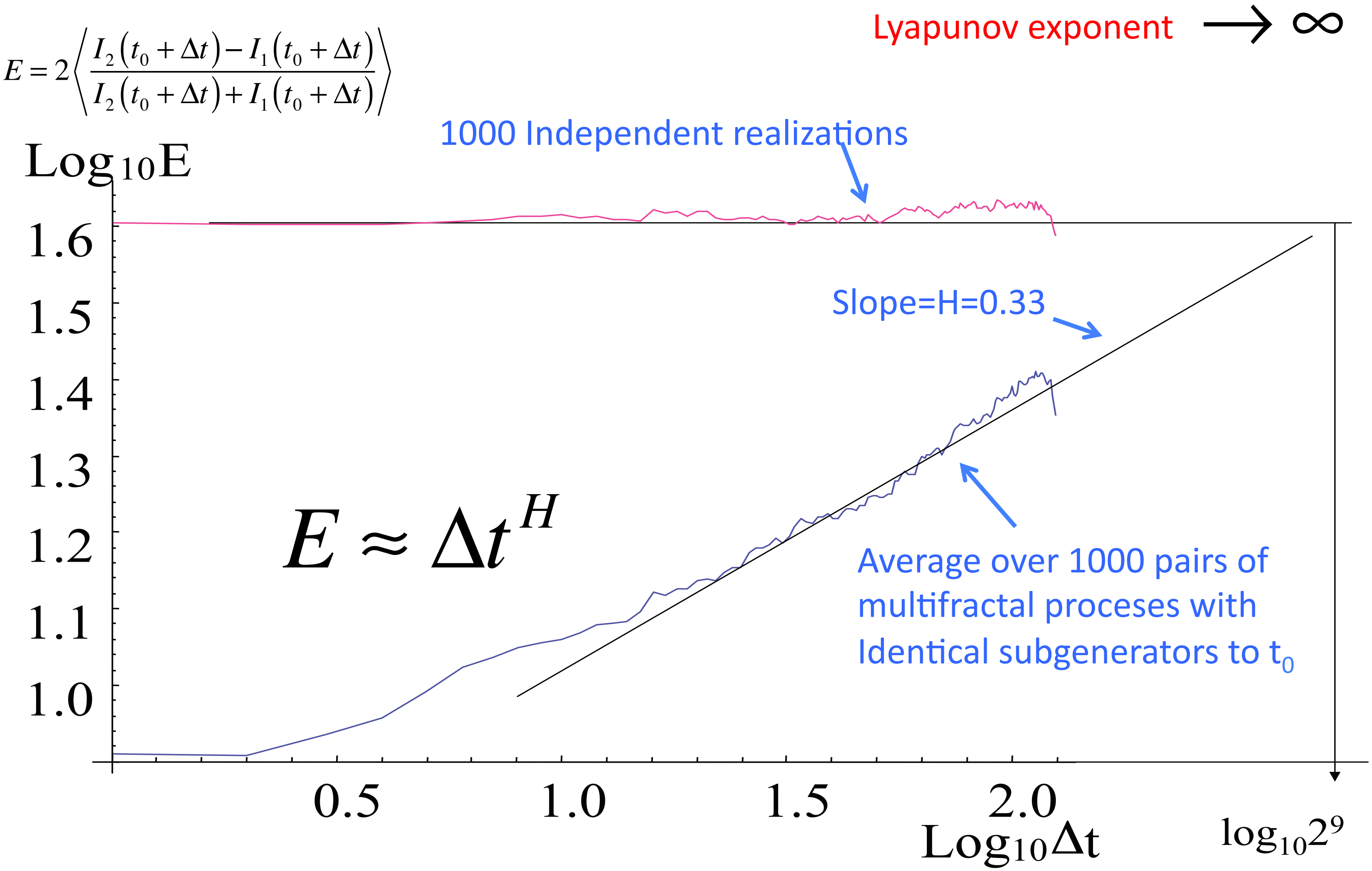
Predictability and stochastic forecasting



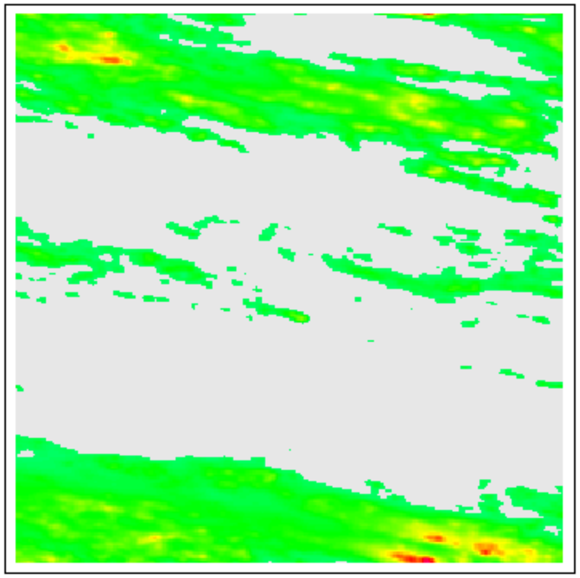
Two multifractal proceses with Identical subgenerators to t_0

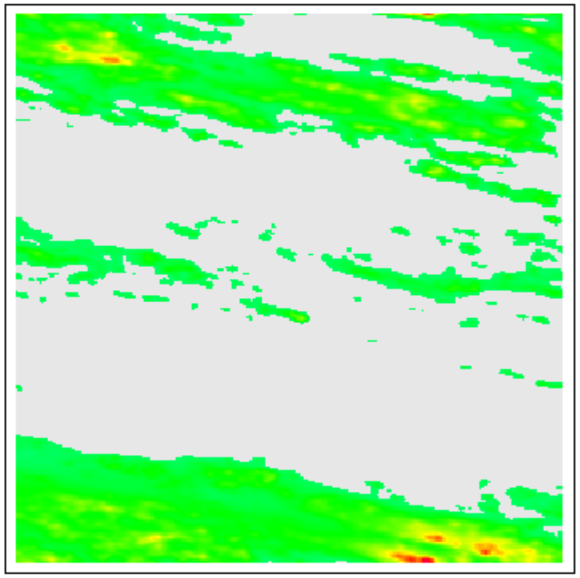
Algebraic divergence of realizations

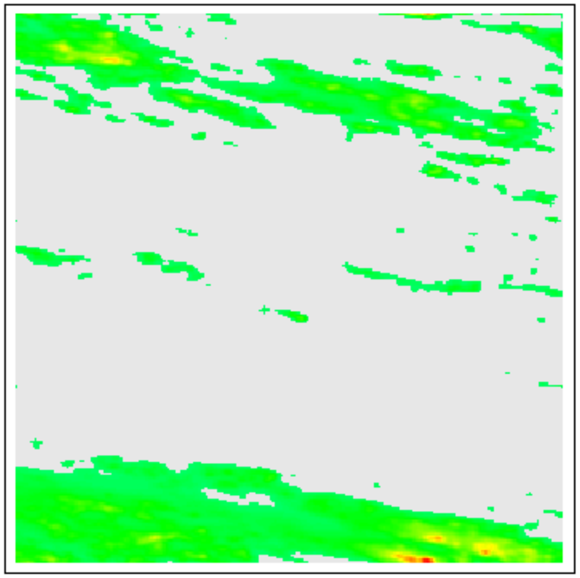
$$E = 2\left\langle \frac{I_2(t_0+\Delta t)-I_1(t_0+\Delta t)}{I_2(t_0+\Delta t)+I_1(t_0+\Delta t)} \right\rangle$$



Space-time Cascades, stochastic nowcasting (rain)







Realization A

Realization B

Forecast based on first 16 time steps

(all same initially)

The macroweather regime

Low frequency cascades

Time scales ≈>10 days ($\tau > \tau_w$)

…Predicting the spectral plateau / macroweather regime

"First principles" predictions Atmosphere:

The large scale winds and weather-climate transition scale

Power:

Solar heating, top of the atmosphere: $\approx 10^3$ W/m^2

Absorbed $\approx 2X10^2$ W/m^2

$\approx$2% Converted to K.E.$\approx$ 4 W/m^2

Energy flux:

If power is distributed over the troposphere, 10^4m thick, density, 0.75 Kg/m^3

$\varepsilon \approx 5x10^{-4}$ m^{2}s^{-3}

c.f. modern value 10^{-3} m^{2}s^{-3}

Prediction using horizontal relation:

$$\Delta v = \varepsilon^{1/3}\Delta x^{1/3}$$

Scales:

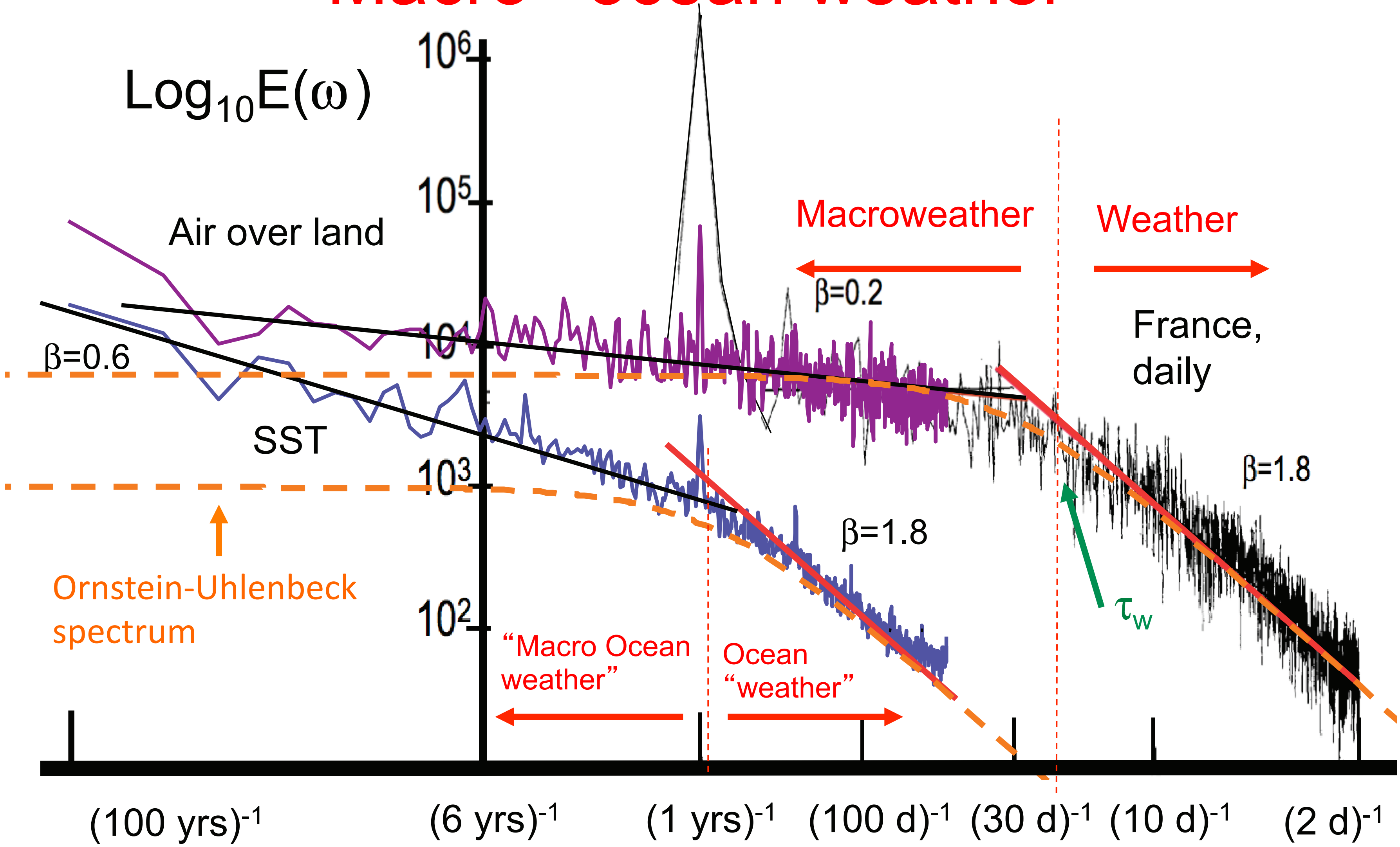
Length: $L \approx 2x10^7$ m

Velocity: $V \approx \varepsilon^{1/3} L^{1/3} \approx 21$ m/s

Time: $T = L/V \approx 10^6$ s = 11 days

c.f. empirical antipodes velocity difference 17.4±5.7 m/s

Macroweather,
Macro "ocean weather"



Ocean Drifter data: $\varepsilon_o \approx 10^{-8}$ m^2/s^3
$\tau_o \approx \varepsilon_o^{1/3} L^{-2/3} \approx 1$ yr

Reanalyses:
$\varepsilon_w \approx 10^{-3}$ m^2/s^3
$\tau_w \approx \varepsilon_w^{1/3} L^{-2/3} \approx 10$ dys

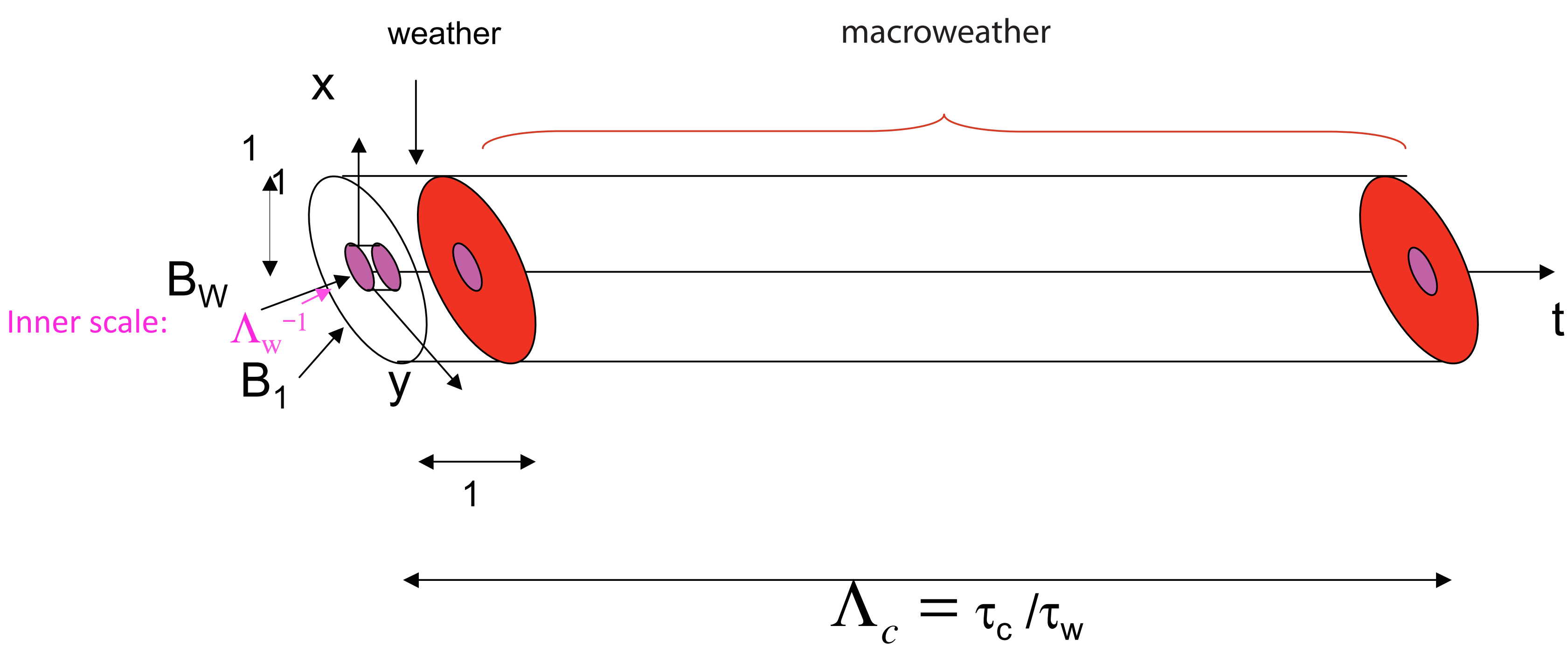
From the Weather to macroweather: a dimensional transition

$$\Gamma(\underline{r},t)=\underbrace{\int_{\Lambda_w^{-1}}^{1}\int_{B_w}^{B_1}\gamma(\underline{r}-\underline{r}',t-t')g(\underline{r}',t')d\underline{r}'dt'}_{\Gamma_w}+\underbrace{\int_{1}^{\Lambda_c}\int_{B_w}^{B_1}\gamma(\underline{r}-\underline{r}',t-t')g(\underline{r}',t')d\underline{r}'dt'}_{\Gamma_{mw}}$$

$$\varepsilon_\lambda = e^{\Gamma_\lambda}$$

The generator=
Additive process

Space-time region of generator interactions:



For $\tau_c \gg \tau_w$, space-time interaction domain becomes pencil-like (1-D), not 3-D:
Dimensional transition

Implications of the FIF model for $\tau > \tau_w$

(the macroweather regime)

$$\varepsilon_{\Lambda_w,\Lambda_c}(\underline{r},t) \approx e^{\Gamma_w(\underline{r},t)+\Gamma_{mw}(\underline{r},t)} = \varepsilon_{\Lambda_w}(\underline{r},t)\varepsilon_{\Lambda_c}(t)$$

Weather-macroweather factorization

Observable:

$$I = \varepsilon * [\![\Delta r, \Delta t]\!]^{-(D-H)}$$

Autocorrelations:

$$\left\langle \Delta I_w(\Delta t)^2 \right\rangle \propto \Delta t^{2H-K(2)}$$

Weather regime

$$\left\langle I_{mw}(t) I_{mw}(t-\Delta t) \right\rangle \propto \Delta t^{-1}$$

macroweather regime

Low frequency divergence

Spectra:

$$E(k) \approx k^{-\beta_w}; \qquad k > L_w^{-1}$$

$$E(\omega) \approx \omega^{-\beta_w}; \qquad \omega > \tau_w^{-1}$$

Weather regime

$$E(\omega) \approx \omega^{-\beta_{mw}}; \qquad \tau_c^{-1} < \omega < \tau_w^{-1}$$

macroweather regime

Exponents:

$$\beta_w = 1 + 2H - K(2)$$

$$0.2 < \beta_{mw} < 0.4$$

Independent of H,C_1, weak dependence on α, scale range Λ_c

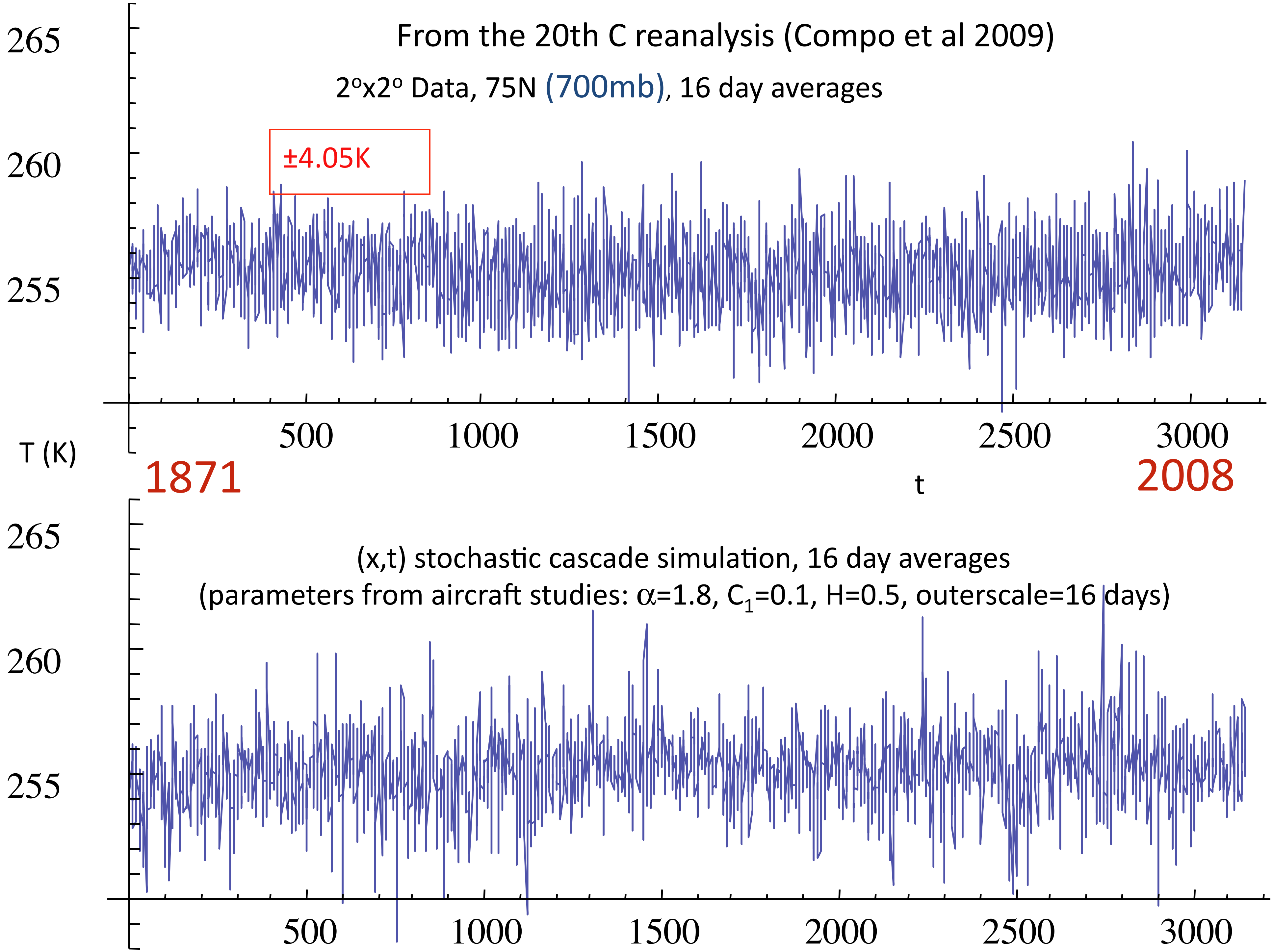
Comparing the data at 75°N (20thC reanalysis) and the cascade simulations

The cascade simulations depend on the following parameters for the temperatures:

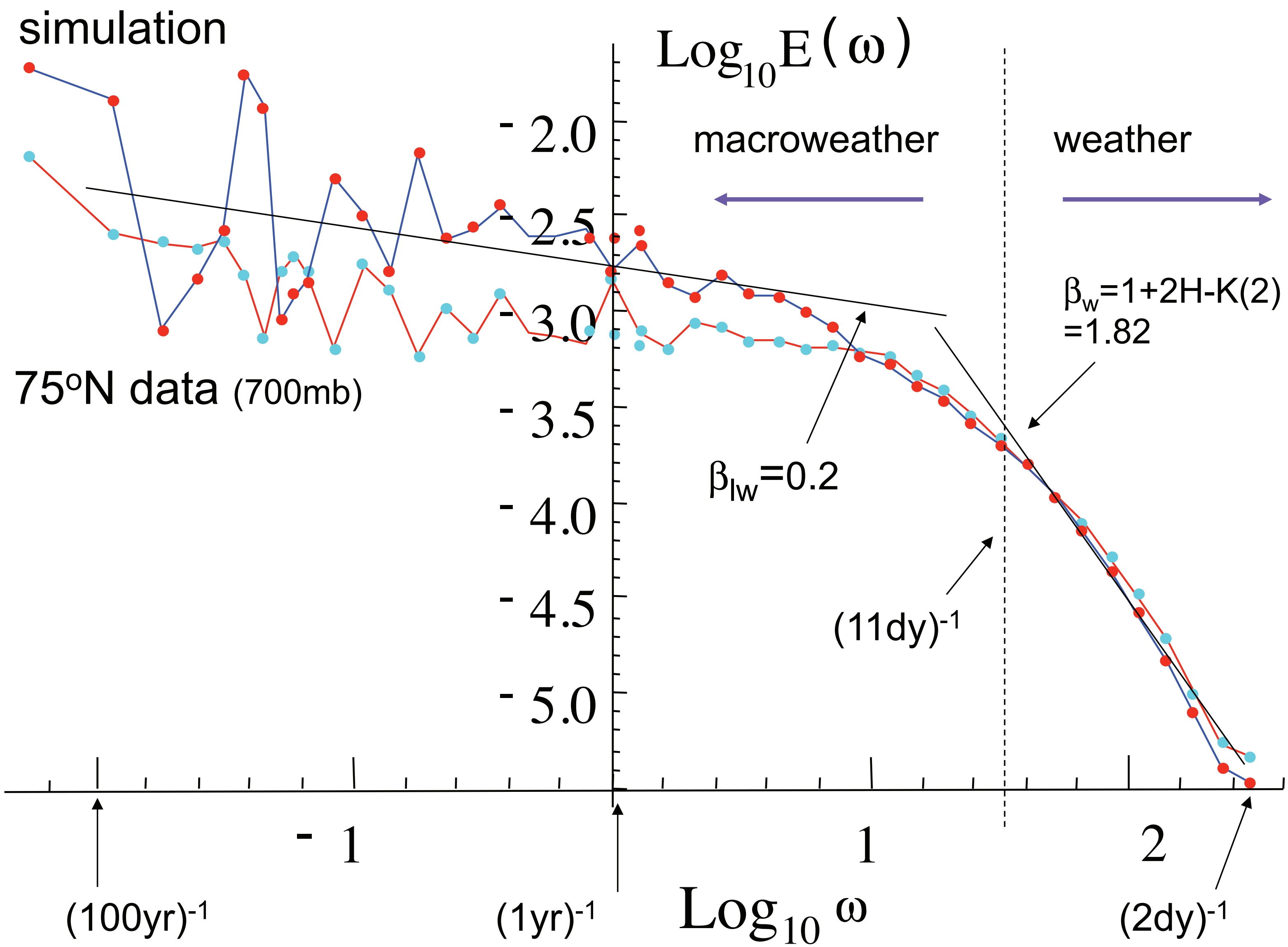
These following turbulent quantities were measured by the Pacific 2004 experiment using the Gulfstream 4 aircraft over the Northern Pacific ocean, at 200mb. The data were taken at 4Hz (≈280m) resolution:

Cascade exponents: α =1.8, C_1 = 0.1
Scale by scale nonconservation exponent: H = 0.5
Average energy flux: $5 \times 10^{-4} m^2/s^3$

The only parameter measured by the reanalysis was the standard deviation of the daily temperature at 75°N, 700 mb: ±4.05K


From the 20th C reanalysis (Compo et al 2009)
2°x2° Data, 75N (700mb), 16 day averages
±4.05K
265
260
255
500
1000
1500
2000
2500
3000
T (K)
1871
t
2008
(x,t) stochastic cascade simulation, 16 day averages
(parameters from aircraft studies: α=1.8, C1=0.1, H=0.5, outerscale=16 days)
265
260
255
500
1000
1500
2000
2500
3000

The weather cascade spectra calibrated with aircraft data



Summary of weather-macroweather

Classical laws

$$\Delta v(\underline{\Delta r}) = \varphi |\underline{\Delta r}|^H$$

e.g. Kolmogorov: $\varphi = \varepsilon^{1/3}$, $H=1/3$

$|\underline{\Delta r}| <\approx 100m$

isotropic

$\varphi \approx$ constant

Generalization 1)

Scaling Anisotropy:

$$|(\underline{\Delta r}, \Delta t)| \rightarrow [\![(\underline{\Delta r}, \Delta t)]\!]$$

Vector norm   Scale function

Generalization 2)

Intermittency: $\varepsilon \approx$ constant → $\langle \varepsilon_\lambda^q \rangle = \lambda^{K(q)}$

Cascade processes, multifractals

Weather regime:

Planetary scales, 10 days

Generalization 3)

Dimensional transition, low frequency weather regime

Spectra:

$E(k) \approx k^{-\beta_w}$; $k > L_w^{-1}$

$E(\omega) \approx \omega^{-\beta_w}$; $\omega > \tau_w^{-1}$

Weather regime

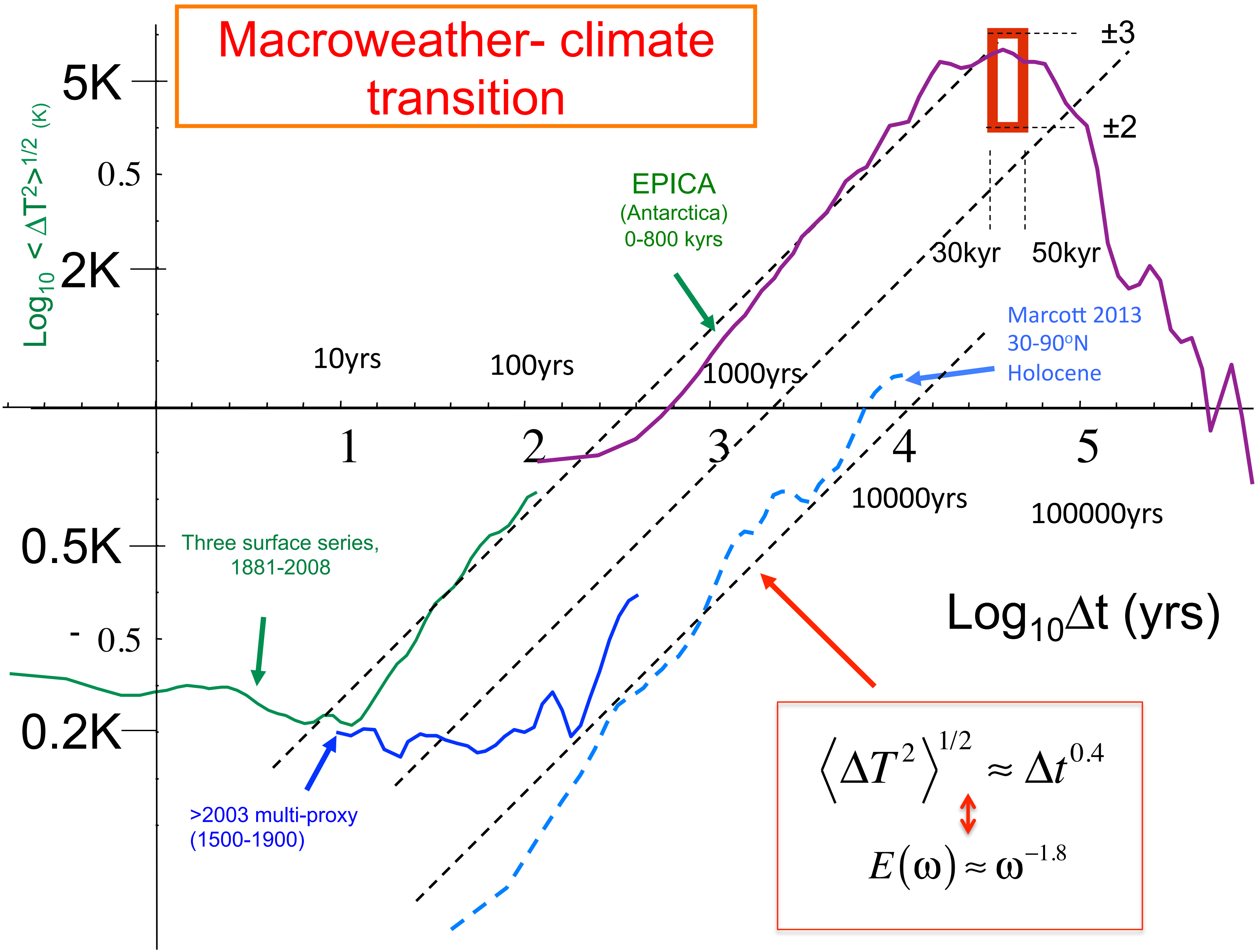
$E(\omega) \approx \omega^{-\beta_{wc}}$; $\tau_c^{-1} < \omega < \tau_w^{-1}$ macroweather regime

Exponents:

Weather regime → $\beta_w = 1 + 2H - K(2)$

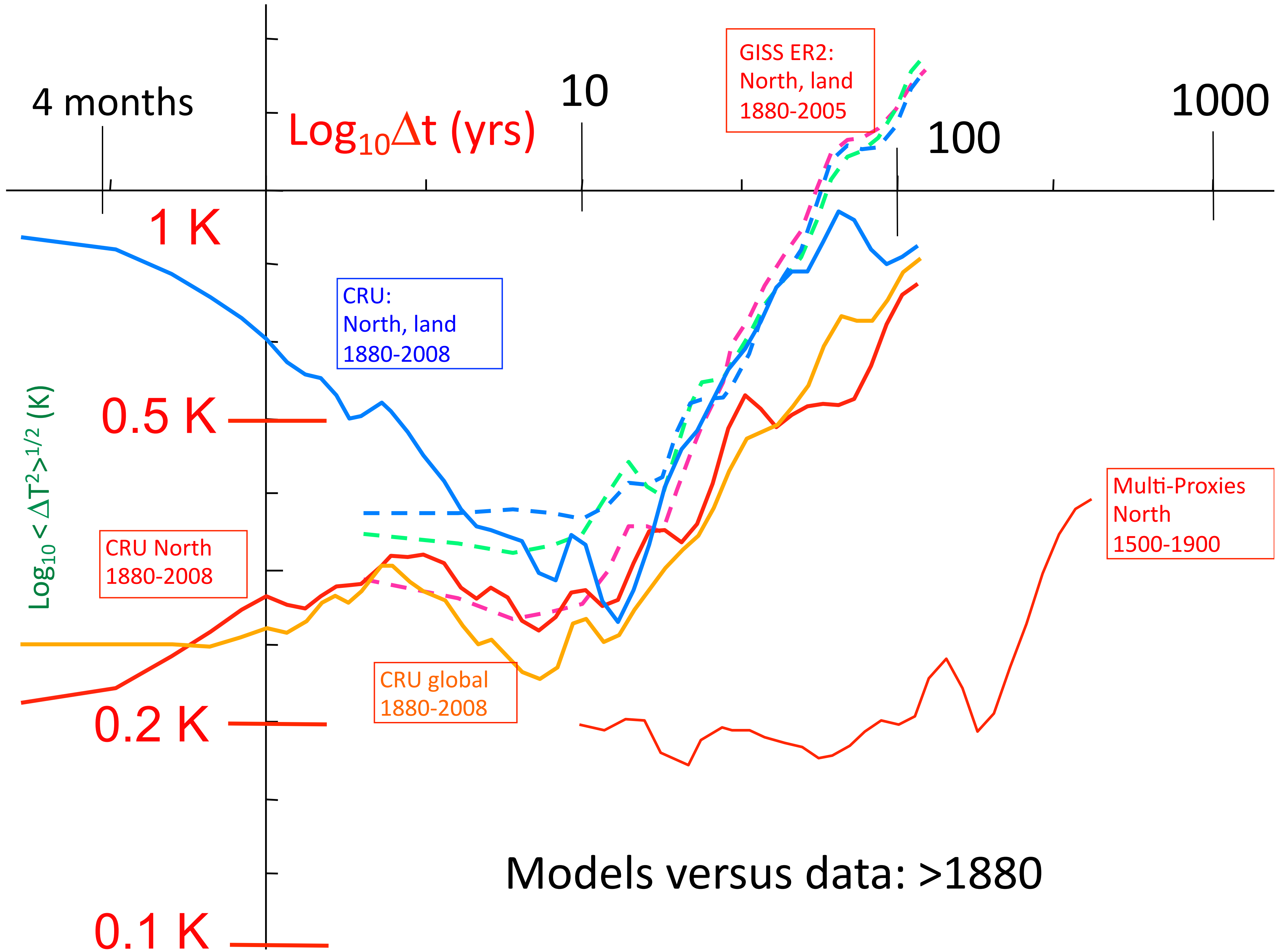
$0.2 < \beta_{wc} < 0.4$ ← Independent of H, C_1, weak dependence on α, scale range Λ_c

The climate

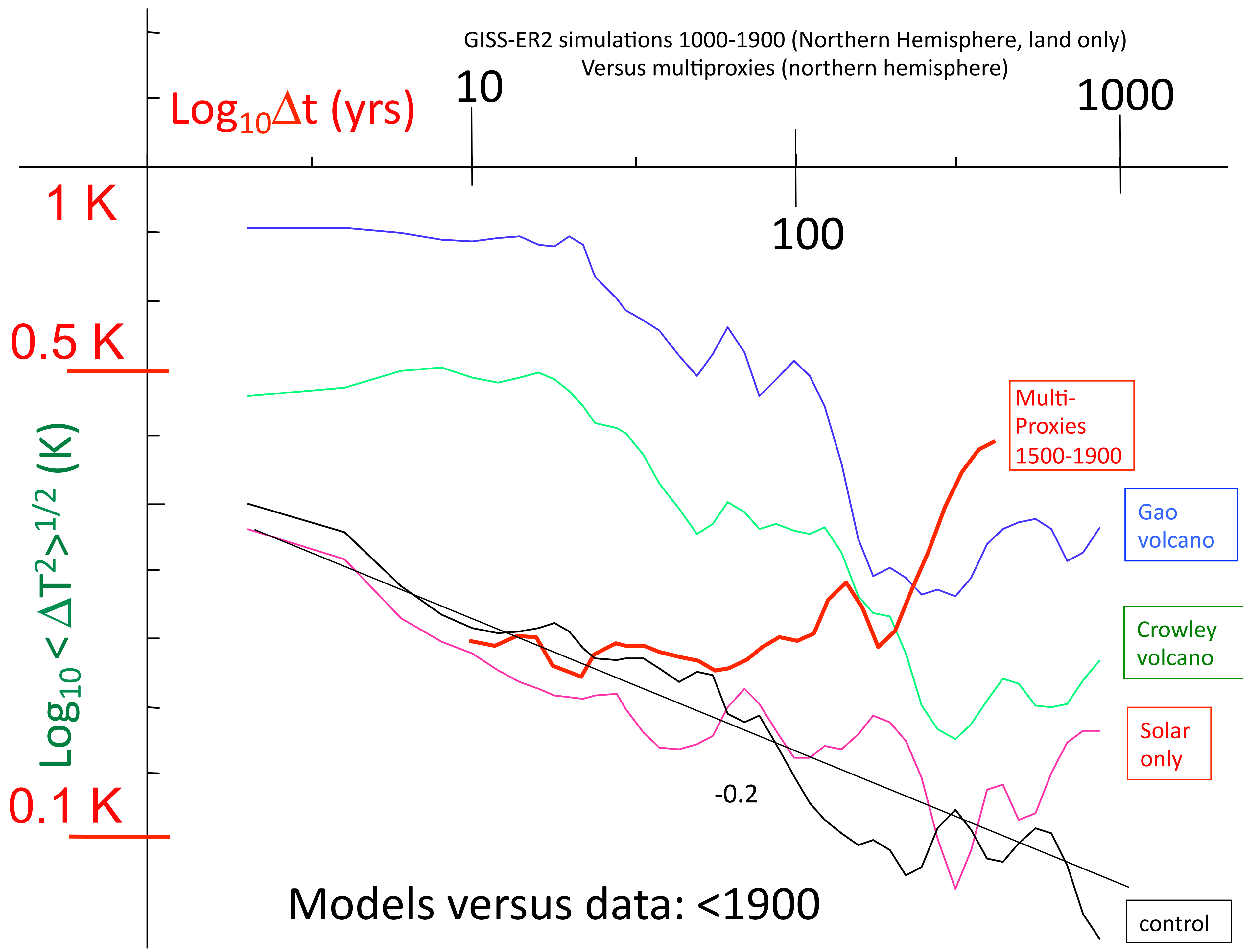


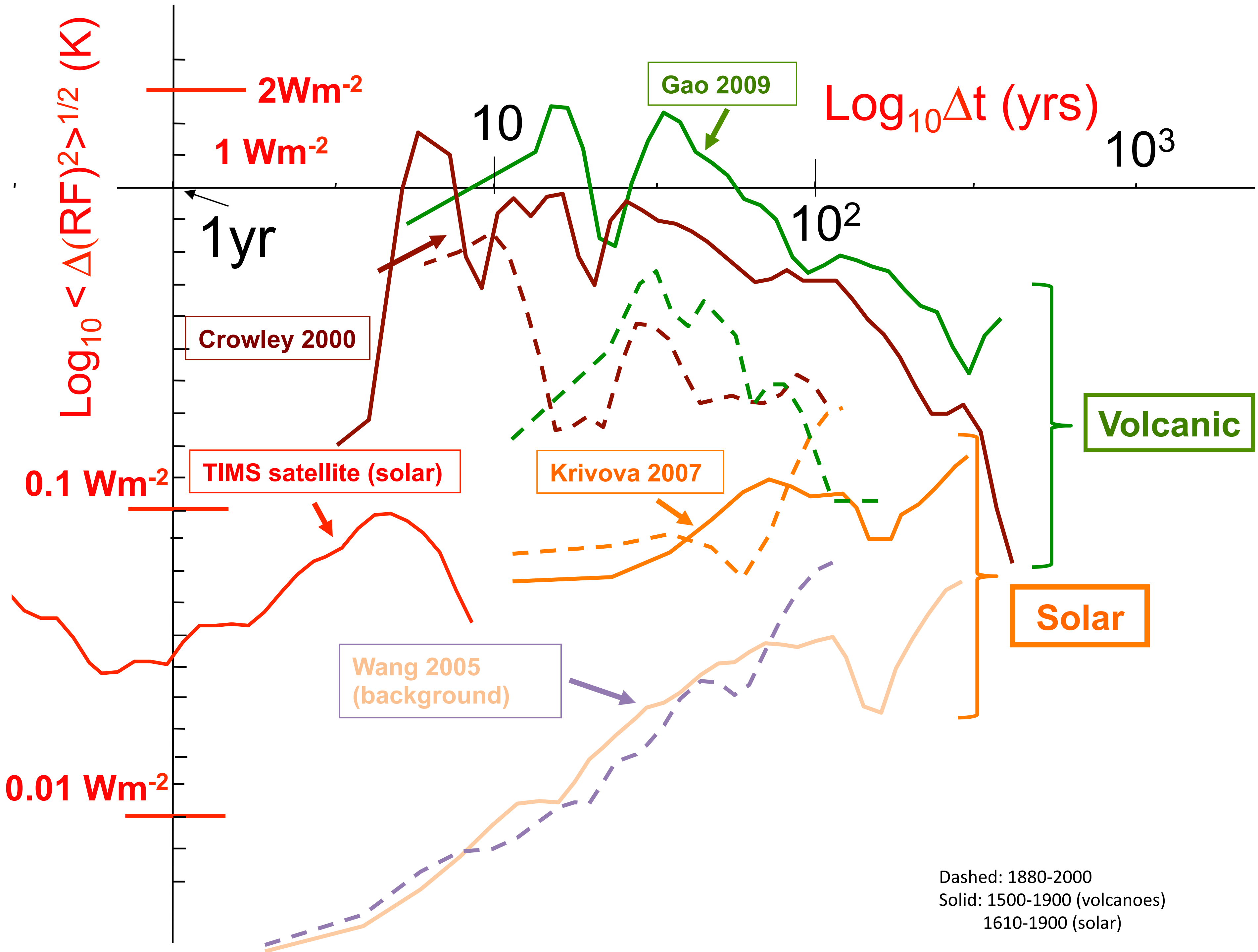
Macroweather- climate transition
$\mathrm{Log}_{10} < \Delta T^2 >^{1/2}$ (K)
5K
0.5
2K
0.5K
- 0.5
0.2K
10yrs
100yrs
1000yrs
10000yrs
100000yrs
1
2
3
4
5
$\mathrm{Log}_{10}\Delta t$ (yrs)
EPICA
(Antarctica)
0-800 kyrs
±3
±2
30kyr
50kyr
Marcott 2013
30-90°N
Holocene
Three surface series,
1881-2008
>2003 multi-proxy
(1500-1900)
$\left\langle \Delta T^2 \right\rangle^{1/2} \approx \Delta t^{0.4}$
$E(\omega) \approx \omega^{-1.8}$

Do GCM's predict the climate…. Or Macroweather?



4 months
10
100
1000
Log10Δt (yrs)
1 K
0.5 K
0.2 K
0.1 K
Log10 < ΔT2>1/2 (K)
GISS ER2:
North, land
1880-2005
CRU:
North, land
1880-2008
CRU North
1880-2008
CRU global
1880-2008
Multi-Proxies
North
1500-1900
Models versus data: >1880


GISS-ER2 simulations 1000-1900 (Northern Hemisphere, land only)
Versus multiproxies (northern hemisphere)
Log10Δt (yrs)
10
100
1000
1 K
0.5 K
0.1 K
Log10 <ΔT2>1/2 (K)
Multi-Proxies 1500-1900
Gao volcano
Crowley volcano
Solar only
control
-0.2
Models versus data: <1900



Log10 < Δ(RF)2>1/2 (K)
Log10Δt (yrs)
2Wm-2
1 Wm-2
0.1 Wm-2
0.01 Wm-2
1yr
10
10^2
10^3
Gao 2009
Crowley 2000
TIMS satellite (solar)
Krivova 2007
Wang 2005 (background)
Volcanic
Solar
Dashed: 1880-2000
Solid: 1500-1900 (volcanoes)
1610-1900 (solar)

Climate

Overall weather – climate process

$$\varepsilon_{w,c}\left(\underline{r},t\right)=\varepsilon_{w,mw}\left(\underline{r},t\right)\varepsilon_{c}\left(\underline{r},t\right)$$

Weather- macroweather cascade process (previous)

Low frequency space-time climate flux (new)

Weather/macroweather flux factorization:

$$\varepsilon_{w,mw}\left(\underline{r},t\right)\approx\varepsilon_{w}\left(\underline{r},t\right)\varepsilon_{mw}\left(t\right)$$

Macroweather: temporal variability only

Space-time Macroweather-climate statistical factorization

$$\varepsilon_{\tau}(\underline{r},t)=\frac{1}{\tau}\int_{t}^{t+\tau}\varepsilon(\underline{r},t')dt'$$

The flux at resolution τ

Weather-climate process at resolution τ

$$\varepsilon_{w,c,\tau}(\underline{r},t)=\underbrace{\varepsilon_{w,\tau}(\underline{r},t)\varepsilon_{mw,\tau}(t)}_{\varepsilon_{w,mw,\tau}(\underline{r},t)}\varepsilon_{c,\tau}(\underline{r},t)$$

Weather variability averaged out

$$\varepsilon_{w,\tau}(\underline{r},t)\approx 1;\quad \tau>\tau_w$$

climate too slow

$$\varepsilon_{c,\tau}(\underline{r},t)\approx\varepsilon_{c,\tau}(\underline{r});\quad \tau<\tau_c$$

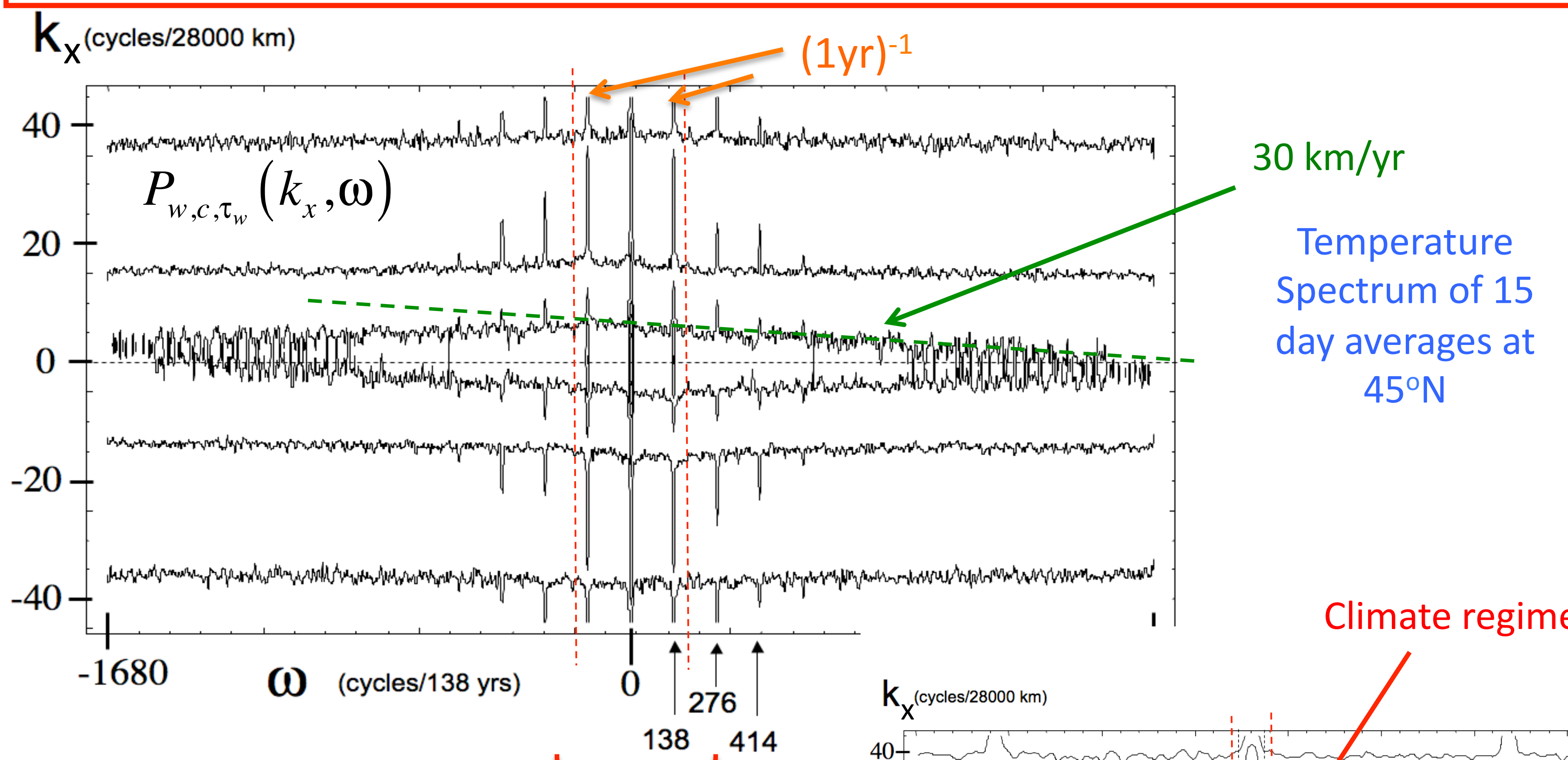
Spatial variability: climatic zones

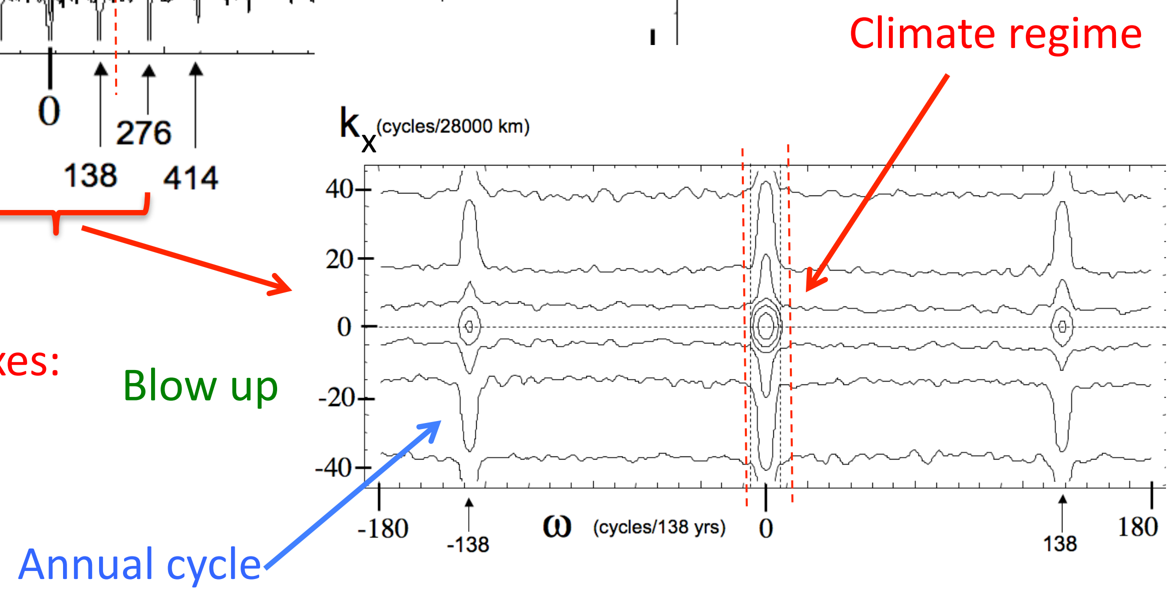
Prediction: Space-time Statistical factorization in the macroweather regime

$$\varepsilon_{w,c,\tau}(\underline{r},t)\approx\begin{cases}\varepsilon_{mw,\tau}(t)\varepsilon_{c,\tau_c}(\underline{r}); & \tau_w<\tau<\tau_c \quad \text{macroweather}\\ \varepsilon_{c,\tau}(\underline{r},t); & \tau>\tau_c \quad \text{climate}\end{cases}$$

Macroweather spectral Prediction: $P_{w,c,\tau_w}(k_x,\omega)\approx E_c(k_x)E_{mw}(\omega)\qquad \tau_w<\tau<\tau_c$

Empirical Macroweather space-time spectral factorization



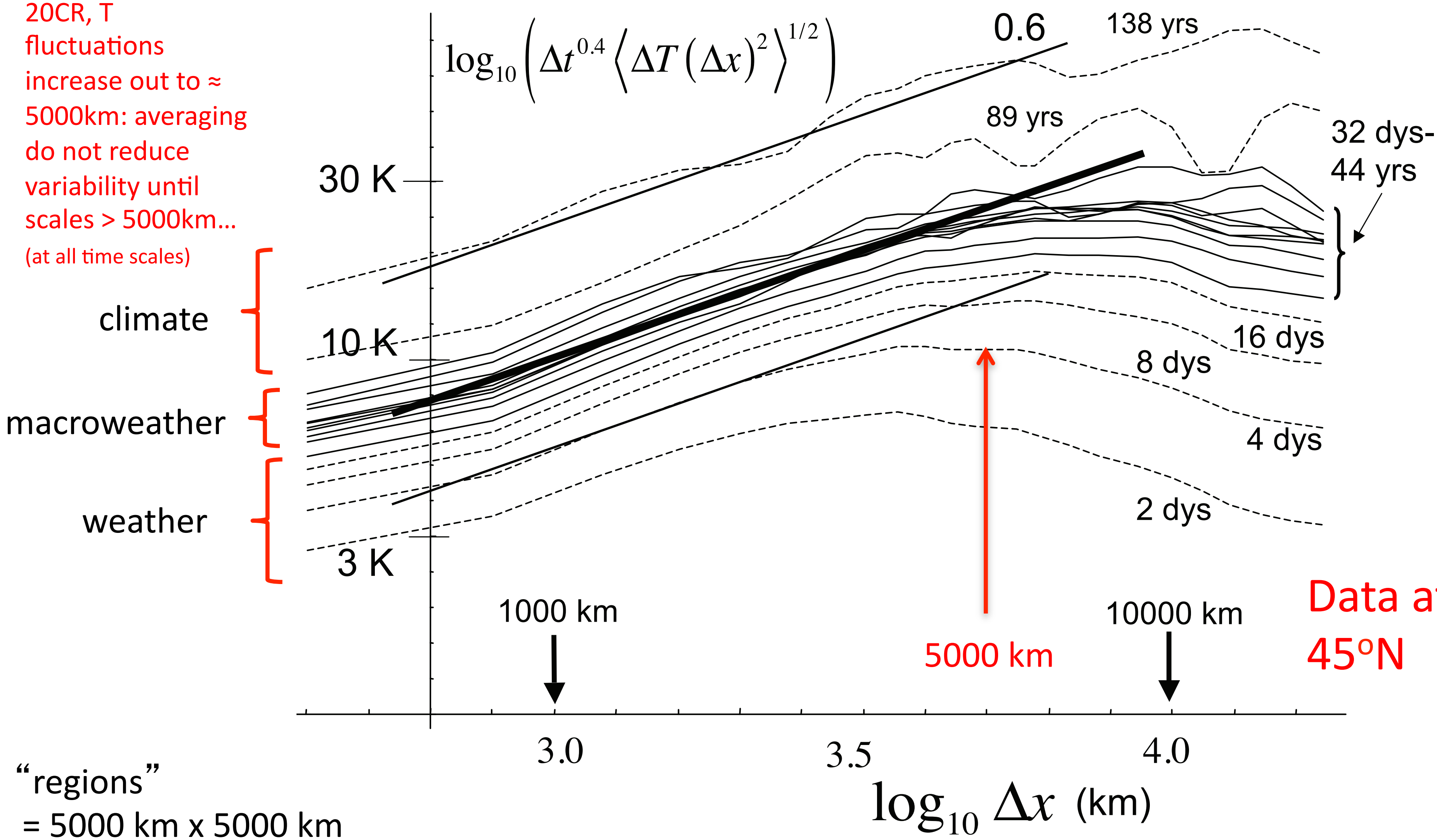


Isolines are nearly parallel to the axes:

$$P_{w,c,\tau_w}\left(k_x,\omega\right)\approx E_c\left(k_x\right)E_{mw}\left(\omega\right)$$

Space-time scaling of fluctuations

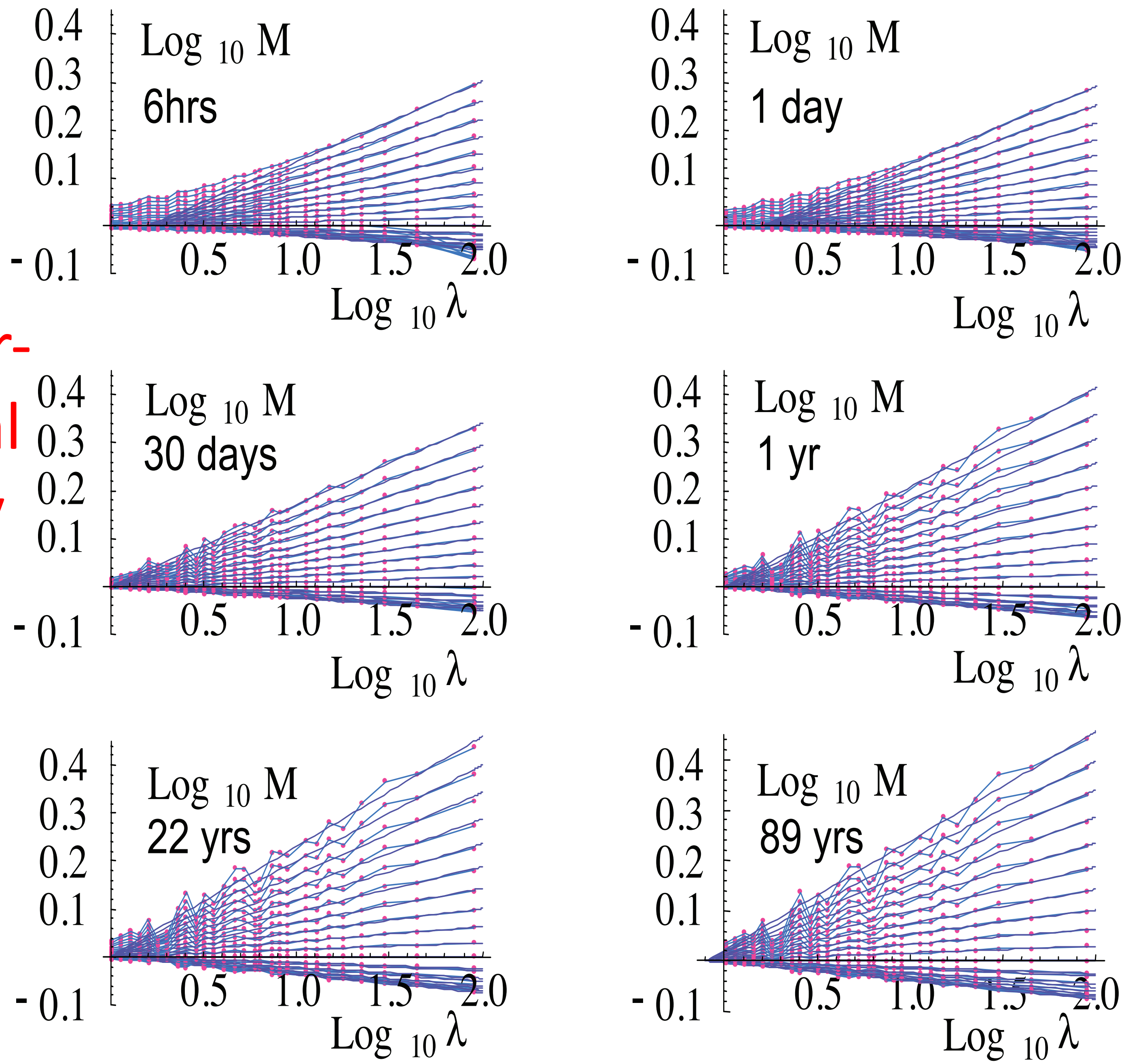
20CR, T fluctuations increase out to ≈ 5000km: averaging do not reduce variability until scales > 5000km…
(at all time scales)



Data at 45°N

"regions"
= 5000 km x 5000 km
=North America

Macroweather-Climate spatial Intermittency



	C_1	α	L_{eff}
6 *hrs*	0.095	1.65	6400 *km*
1 *day*	0.091	1.71	6400 *km*
30 *dys*	0.093	1.81	10000 *km*
1 *yrs*	0.118	1.75	10000 *km*
27 *yrs*	0.136	1.67	10000 *km*
89 *yrs*	0.130	1.63	13000 *km*

Atmospheric Statistics in a nutshell

Weather

$$S_{q,w}(\underline{\Delta r},\Delta t)=\left(s_{q,w}\right)^q [\![(\underline{\Delta r},\Delta t)]\!]_w^{qH_w-K_w(q)} \quad ; \quad \tau_i<\Delta t<\tau_w$$

$$[\![(\Delta x,0,0,\Delta t)]\!]_{w,can}=\left(\left(\frac{\Delta x}{L_e^*}\right)^2+\left(\frac{\Delta t}{\tau_w}\right)^2\right)^{1/2}$$

Macroweather

$$S_{q,mw}(\underline{\Delta r},\Delta t)=\left(s_{q,mw}\right)^q \|\underline{\Delta r}\|_c^{qH_c-K_c(q)}\left(\frac{\Delta t}{\tau_w}\right)^{qH_{mw}} \quad ; \quad \tau_w<\Delta t<\tau_c$$

$$\|(\Delta x,0,0)\|_{c,can}=\left|\frac{\Delta x}{L_e^*}\right|$$

Climate

$$S_{q,c}(\underline{\Delta r},\Delta t)=\left\langle \Delta T(\Delta x,\Delta t)^q\right\rangle_c=\left(s_{q,c}\right)^q [\![(\underline{\Delta r},\Delta t)]\!]_c^{qH_c-K_c(q)}$$

$$[\![(\Delta x,0,0,\Delta t)]\!]_{c,can}=\left(\left(\frac{\Delta x}{L_e^*}\right)^2+\left(\frac{\Delta t}{\tau_{lc}}\right)^{2/H_{c,t}}\right)^{1/2} \quad ; \quad H_{c,t}=H_c/H_{c,\tau}; \quad \tau_{lc}>\Delta t>\tau_c$$

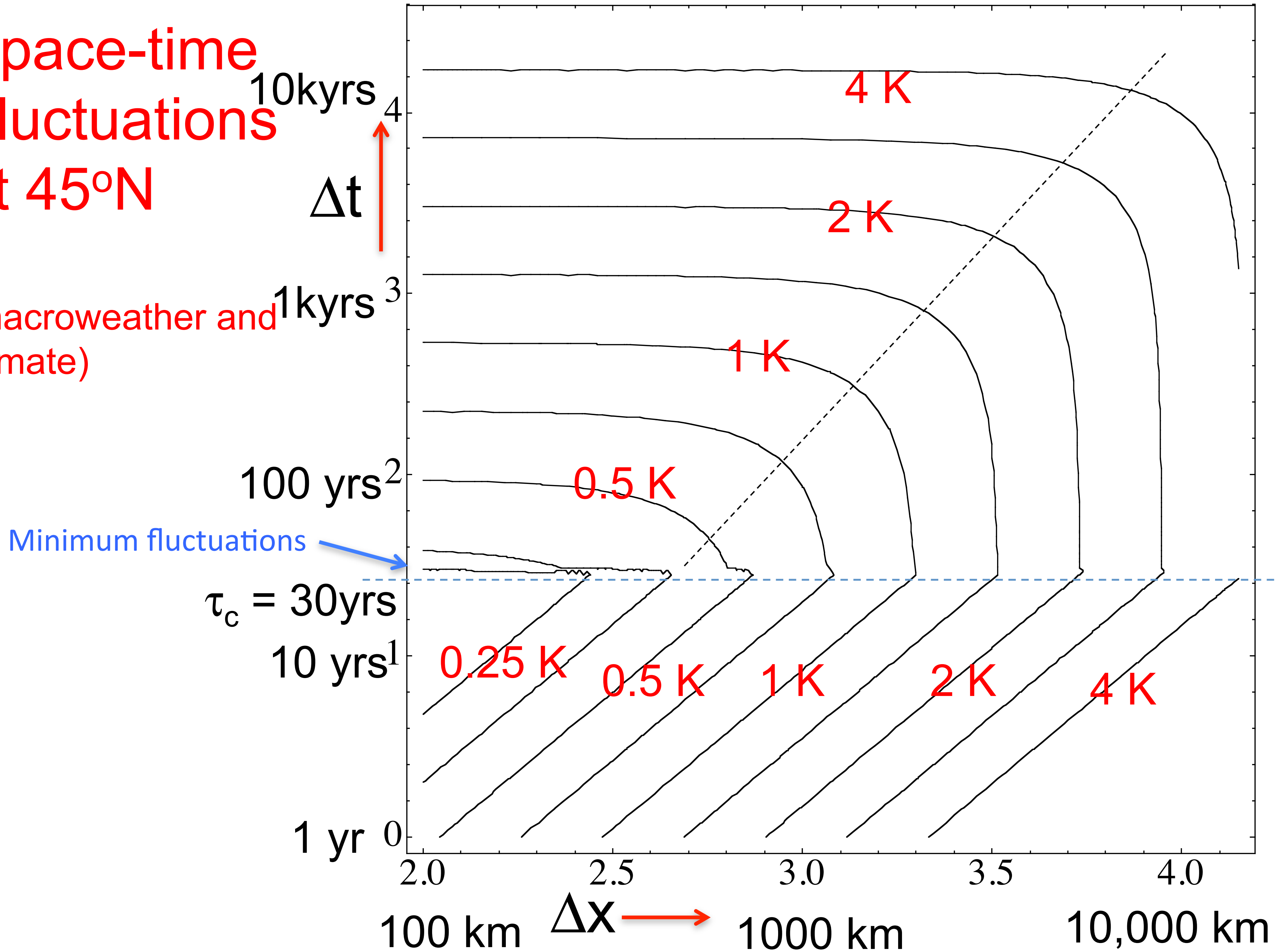
Parameters

$$K(q)=\frac{C_1}{\alpha-1}\left(q^\alpha-q\right)$$

Regime	H		C_1		α		Outer time scale
	Space	Time	Space	Time	Space	Time	
Weather	0.51		0.087		1.61		5-20 *dys*
Macro weather		-0.4		0		–	20-40 *yrs*
Climate	0.7	0.4	0.11	0.065	1.4	1.5	30-50 *kyrs*

Space-time Fluctuations at 45°N

(macroweather and climate)



Conclusions

1. High level stochastic turbulence laws emerge from (deterministic) continuum mechanics at strong nonlinearity

2. Regimes: Weather, macroweather, climate

3. Generalize classical laws: Intermittency using cascades

4. Generalize classical laws: wide range of scales using anisotropic scaling, stratification

5. Modelling with Fractionally Integrated Flux model

6. Consequences for space-time scaling: turbulent propagators and turbulence driven waves

7. Predictability and stochastic forecasting

8. At scales >≈30 years new scaling processes (anthropogenic at ≈ 10 yrs, natural at ≈ 100 yrs) with H>0 dominate up to ≈ 100 kyrs. Can be modelled in FIF framework.